



Mathematical Models of Multi-criteria Optimization of Subsystems of Higher Educational Institutions

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ABSTRACT

In new conditions an institution for successful work needs optimization of control, adaption of existing mathematical tools and development of new aspects. In the article it is suggested viewing the activities of higher educational institutions as production activities and higher educational institutions as complex organizational and economic systems consisting of a large number of subsystems. The relevance of the research is stipulated on the one hand by insufficient level of implementation of mathematical tools in the management process of both separate elements in the structure of sub-systems and sub-systems themselves, as well as institutions of higher education in general. On the other hand it is stipulated by the necessity to search for new forms and methods of organization and management of the educational process in the context of new educational standards. The conclusion about the interconnection and interdependence of the selected subsystems is made. The directions of further development within the frame of the designated problem are given.

Keywords: Decision Making, Higher Educational Institution, Model, Multi-criteria, Optimization

JEL Classifications: C02, I21, I25

1. INTRODUCTION

1.1. Introduction to the Problem and Explanation of its Importance

Successful operation of the production sector, in the Russian economy in particular, depends on the availability of various resources. And human resources undoubtedly have strategic importance. The quality of human resources and their effective use are necessary for economic success. This is especially true in times of economic crisis due to the transition of the Russian economy to the path of reform and innovation. According to the data of the American scientists, 15-20% increase in the national income accounts for education, 20-40% increase is due to improvement of scientific knowledge and its application, the leading role in this process is played by higher educational institutions. The results of sociological research conducted in many countries (Al-Mubarak and Busler, 2012), which show a significant decrease in the quality of education are really frightening: Almost 45% of employers cannot select the graduates of universities to hire employees who have basic knowledge and

skills for junior positions, and 70 % of employers think the reason is the bad quality of education. There is obvious lack of creative thinking, initiative, and ability to work well for achieving the result (Khasaev and Ashmarina, 2014). Therefore, increased competition in the field of higher education can be considered as economic competition (Maiburov, 2003). In this regard immediate prospects for development should include reorganization of educational management in Russia (Sadovnichiy, 2004).

According to some authors, education covers a specific sector of the economy that meets needs of the community in educational services. Education as an industry “is a system of educational institutions, organizations and enterprises engaged mainly in educational activities aimed at meeting diverse needs of the population in educational services and training of skilled workers” (Schetinin et al., 1998). This makes it possible to consider it a complex economic system.

Besides educational systems must be able to plan their innovative development, which can be based on the knowledge management

system, which, at first, would provide an opportunity to transform society intellectual resources in new products and services; secondly, would create conditions for generating new knowledge and its adaptive implementation in order to increase the added value of a product (service); thirdly, it would increase the level of PPP knowledge and creativity as a resource providing quick adaptation to changing environment (Pogorelova, 2010).

Today in the center of the educational system is a higher educational institution which is viewed from the standpoint of economy as a structural unit of an industry. It is part of the market economy and, in particular, an element of the regional economy (Novikov, 2001).

Higher educational institutions carry out their activities through the provision of educational services in the face of fierce competition for consumer rating, among which are the people who want to get education at various levels to improve their skills. At the same time educational services mean, as a rule, the system of knowledge and skills that are used in order to meet diverse educational needs of individuals, society and the state (Belyakov, 2002). Educational services in the market become a commodity and at the same time the process of highly qualified personnel production.

Recently, there has been a complication of the organizational structure of higher education institutions and the processes occurring in them are characterized by versatility (Balyhin and Balyhin, 2014). For example, in the textbook “Strategic Management of a Higher Educational Institution,” 2004 a classical regional university is viewed as a complex set of quasi-autonomous systems with multiple functions - training, educational, research, communication, commercial, industrial, publishing functions, etc. In the works of Kovalevskiy management of a higher educational institution as part of a regional university complex is described (Kovalevskiy, 2004). All this presupposes the existence of a multi-level management system with strict coordination in order to achieve high values of the criteria for universities in the framework of completing the procedures of certification and accreditation.

1.2. Problem Formulation

Now there is an increasing demand for the national system of higher education (taking into account the implementation of the terms of the Bologna Agreement within the framework of a single European educational space), the efficiency of which with the multiplier effect is dependent on the efficiency of its subsystems, which are interrelated. In addition each of the subsystems has a number of properties, including adaptability, focus, multi-criteria and so on. The efficiency of the entire system will depend on flexibility that should manifest itself in everything: In the choice of the profile of training offered by institutions of higher education to entrants; in the use of the most effective forms of organization of the educational process; in the application of modern methods and means of knowledge transfer; in the individual approach to students, etc. Only in this way and in close coordination with the features of modern economic development high quality of specialists can be achieved.

In our opinion an institution (in its tasks and activities) is very close to enterprise on the assumption of the recent trends such as:

Higher educational institution entrepreneurship; an institution of higher education as a whole or a part may be sold, leased, etc.; a higher educational institution has all types of property intended for its activities, including land, buildings, facilities and so on. In addition, changing tax system in relation to a higher educational institution enhances the approach to it as an industrial company in this interpretation. Therefore, there should be developed scientific approaches and methods (including mathematical ones) contributing to the optimum management of a higher educational institution.

2. METHODS

A wide range of works is dedicated to the problems of higher educational institutions management. Thus, the general problems of management are reflected in (Strategic Management of a Higher Educational Institution, 2004; Modern Education Management: Social and Economic Aspects, 1998). A number of authors focus on accounting and economic problems of education (Vasilyev et al., 2001; Molchanov, 2001), the use of models and mechanisms of dynamically active systems using various mathematical approaches (The Experience of the Development and Use of Models of Higher Educational Institutions Management, 1986; Fedotov, 1995), the introduction of new information technologies and so on.

A lot of authors note that today it is necessary to have clear and effective management of the educational process at a higher educational institution as part of a new economic mechanism of institution management. For example, the works of Atkinson et al., 1969; Itelson, 1964; Kagan and Sychenikov, 1987 consider the various aspects of the organization of studies at a higher educational institution from the standpoint of achieving optimization criteria and at the same time using a variety of economic and mathematical methods and models.

The works of Archangelskiy are of particular interest because they give detailed, versatile material for scientific organization of educational process at a higher educational institution, concerning modeling techniques, their challenges and opportunities; scientific experiment and its features; measuring instruments and indicators; physiological and psychological concepts of learning theory; interconnection of educational process and scientific research. As S.I. Archangelskiy noted (Archangelskiy, 1980), to make an educational system really rational, it is necessary to lay in its organization the means of forecasting optimal ways of its functioning and development, taking into account complex and contradictory situations and their possible solutions. Educational process in the opinion of Archangelskiy is a “big complicated system, expressed in an infinite variety of conditions, behaviors, attitudes, relationships of its components” (Archangelskiy, 1976). In addition, efficiency of an educational process is closely connected with the introduction of new flexible technologies, both of teaching and organization.

We think that complexity of the internal organization of educational institutions, their diverse ties with the external environment having high dynamics ask for the formation of a scientific approach to

higher educational institutions as organizational and economic systems. The problems solved by higher educational institutions are multi-criteria problems. Their quality solution is possible by means of the application of the decision-making (DM) theory and methods (Larichev, 1979), as well as multi-criteria optimization methods (Emelyanov and Larichev, 1986). Considering a higher education institution as an aggregate of a number of sub-systems with structural association of functionally related activities more than twenty directions can be named, the implementation of which involves certain types of methods and models of management. In order to simplify the analysis and subsequent optimization of higher educational institutions it seems appropriate to divide the organizational and economic system in two major groups of subsystems: Economic, business, organizational and educational (the first subsystem essentially performs the production function for the benefit of the second). Therefore there is a need to develop models of multi-criteria optimization separately for each of the selected subsystems.

3. RESULTS

Production tasks to be solved by higher educational institutions are very complex and there is desire to achieve several and often many local purposes. Mathematical objective mappings are criteria. Therefore, the tasks are multi-criteria tasks.

3.1. General Approaches to Solving Tasks at Higher Educational Institutions

Currently, the decisions at high educational institutions are made in one of three ways: (Shepel, 2011) intuitive, i.e., a decision prompted by previous experiences (insight); the results of field tests, treated methods of applied statistics; the results of economic and mathematical modeling. Numerous studies have found that intuitive DM process gives a big mistake, and it is not always possible to arrange full-scale tests. The most acceptable DM is with the use of economic and mathematical modeling. In this case, it seems appropriate to use the following procedure: (a) Formulation of the problem statement; (b) selection of quality criteria; (c) description of the factors that limit the possibility of achieving a purpose(s); (d) drawing up possible solutions of this problem; (e) construction (selection) of a mathematical model and implementation of it with the help of calculations; (f) DM.

The process of DM is a complicated interactive cyclical procedure. In fact, the result of practically every stage of research can influence a task statement and change it. In particular practical result testing is a stimulus to change a task statement and search for new decisions if the result was negative.

3.2. Modeling of the Process of Economic Subsystem Management

Let us solve some administrative task, the result of which depends on the actions taken by the DM, the conditions in which the operation takes place and the properties of objects involved in it the DM is fully aware of. (Shepel 2011) X denotes management (strategy) of the DM. We believe the strategy of the DM is designated by the symbol X and is an n -dimensional vector, i.e.,:

$$X = (x_1, x_2, \dots, x_n) - (x_j), j \in \overline{1, n}. \quad (1)$$

On components x_j of the strategy X a number of restrictions are imposed

$$g_i = g_i(C_i, X) \geq b_i, i \in \overline{1, m}, \quad (2)$$

Where g_i are some functions; C_i are some fixed values; b_i are fixed scalar values. The tolerance range Ω_X of strategy X are defined by conditions (2).

The efficiency (successfulness) of DM is defined by k criteria e_1, e_2, \dots, e_k which characterize some local goals of the operation and have different coefficients of relative importance $\lambda_1, \lambda_2, \dots, \lambda_k$.

We suppose that the purpose of the DM is to increase the possible values of all local efficiency criteria. Selecting Management X from the range of its allowable values Ω_X is a means to achieve objectives of the operation. Obviously, due to the choice of a single X control it is not possible to achieve all the objectives of the local criteria simultaneously. We need some additional compromise in achieving local criteria of the operation.

So, the DM has a task to find optimal management (strategy) \overline{X} satisfying two conditions: (1) Management \overline{X} must be feasible, in other words, it must belong to many possible values Ω_X ; (2) management must be the best in terms of additional compromise taken in the task with the account of the vector of importance of local goals Λ . Then the optimal strategy \overline{X} must satisfy:

$$\overline{E} = E(\overline{X}) = \underset{X \in \Omega_X}{opt} [E(X), \Lambda], \quad (3)$$

Where opt means the operator of optimization, symbols \overline{X} and \overline{E} mean optimal value of management X and the corresponding optimal vector of efficiency E .

Thus, the task (3) is a multi-criteria determinate static DM task (DMT). To obtain practical results it is advisable to transform the task in a one-criterion DMT. This transformation can be made with the help of the five-stage procedure.

3.2.1. The stages of the procedure of multi-criteria task transformation in a one-criterion task

3.2.1.1. The first stage

Determine DMT. A lot of different tasks are reduced to a general statement of multi-criteria DMT (2). Four types of tasks are the most common:

- Type I - problems, in which the optimal strategy is to be found on the set of local criteria (objectives, quality), each of which must be taken into account. Typically, each local criterion has a different unit of measurement
- Type II - problems where the optimal solution is to be found on a set of objects. Each object has an individual criteria and it is necessary to assess functioning of the whole set of objects by the vector criterion. A special feature of this type of tasks is the fact that all local criteria have the same dimension
- Type III - problems of this type arise in the case when the optimal solution must be sought for in different operating

conditions. The quality of work of the developed (researched) system or device depends on these conditions. Local criteria are consistent with the conditions of each option, and the vector criterion assesses the quality of the system functioning according to the whole set of conditions, and this vector criterion is to be optimized. The dimension of the local criteria in the tasks of such type is the same

- Type IV - if the researched (developed) system operates on many stages, then the problem of this type arises. Local criteria assess the quality of the system, depending on management strategy on the appropriate stages. Obviously the dimension of the local criteria is the same. Vector criterion drawn from the local criteria on many stages must be optimized.

3.2.1.2. The second stage

Determine the area of contradiction. In multi-criteria tasks there is a contradiction between some of the criteria. Therefore, in the range of feasible decisions two disjoint sub-domains are identified: Sub-domain of consensus Ω_X^c and sub-domain of contradiction Ω_X^n . Quality of decisions in the sub-domain of consensus can be improved by all criteria simultaneously as there is no contradiction between the criteria. In the sub-domain of contradiction it is impossible to improve decisions simultaneously by all of the criteria because improvement of some criteria leads to deterioration in the quality of others. Consequently only in the sub-domain of contradiction we can find the optimal decision $\bar{X} \in \Omega_X^n$.

So, definition of the optimality principle reduces multi-criteria management task to a one-criterion task.

3.2.1.3. The third stage

It is very useful to bring the local criteria E to one scale of measurement and possibly make dimensionless. This procedure is often called normalization of criteria, and it is typical for the problems of Type I.

3.2.1.4. The fourth stage

It is necessary to determine the scheme of compromise and the principle of optimality corresponding to it since a subsequent search for the optimal strategy in the field of contradiction can be conducted only on the basis of the compromise scheme. Determining the compromise scheme is equivalent to the identification of the optimization operator meaning opt:

$$\underset{X \in \Omega_X}{opt} E(X) = \underset{X \in \Omega_X^c}{opt} E(X) = \max \phi[E(X)], \quad (4)$$

Where $\phi(E)$ is some scalar function of the vector of local criteria E.

3.2.1.5. The fifth stage

Locate the local criteria in order of importance to DM, i.e., carry out the operation of ranking criteria. In practice, this operation represents an adjustment of the chosen scheme of compromise and it is necessary for the subsequent task of finding an optimal management strategy.

So, definition of the principle of optimality transform multi-criteria management task into an equivalent one-criterion problem. Further

problems may arise, but computational ones and in this article they will not be discussed.

3.3. Modeling the Process of Organizational and Educational Subsystem Management

In its turn, organizational and educational subsystem hierarchically also consists of two interrelated subsystems: Managers and managed people. Managers at various levels are the rector, the dean's office staff and so on. One of the most important components of the managed subsystem is the process of education. For high school theoretical and methodological aspects of improving effectiveness of an educational process in the field of organizational management must be constantly improved. The former organization of training in higher education institutions today requires innovative modernization to improve effectiveness of highly qualified personnel training. Therefore, due to the need for effective functioning of the new economic management mechanism of higher educational institution there should be effective direct management of the educational process. The student is influenced not only by external management of the system (via the dean, teacher, etc.) but also internal (let's call it self-government or self-regulation), characteristic of every single individual and corresponding to subjective attitudes and desires. (Speshilova, 2006) Therefore, for effective implementation of management impacts, and therefore educational process organization itself a wide range of criteria must be taken into account.

In case of purposeful management of an educational process, the output parameters of the system are characterized by results development of competencies stated in the working curriculum, assimilation of educational information, transformation of it into knowledge and ability to apply it in practice. Successful work to improve the quality of knowledge of students is largely determined by the level of qualification of entrants as well as the degree of individual discipline material mastering the during the entire course. This becomes especially important in the situation of our redistribution in the direction of increasing self-learning (Speshilova, 2014).

Then, considering students as members of a managed educational system, its state can be at any time determined by the vector (q), with coordinates that characterize the initial level of knowledge, skills and abilities of each individual student. In addition it is an open system in terms of the elements it includes:

$$q = (q_1, q_2, \dots, q_k, \dots), \quad (5)$$

Where $q \in Q$ (the field of the system's states).

As the system changes over time, its behavior can be described by a sequence of states:

$$q(t) = (q_1(t), q_2(t), \dots, q_k(t), \dots). \quad (6)$$

As the direct impact occurs under the direct control of the (u) during classroom work (by the means of diagnosis of the initial state, education planning, direct organization of educational

process, motivation and control) and is time-dependent (a pair of functions $p = (q(t), u(t))$ is called process, the model of the discrete managed system is as follows:

$$q(t+1) = \mu(t, q(t), u(t)), \quad (7)$$

Where, $t = 0, 1, \dots, T - 1$.

As at the initial moment $t = 0$ the state $q(0) = q_0$ is known, then $q(1) = \mu'(0, q_0, u(0))$. In T steps we are going to get the last value $q(T)$. For the tasks of optimization of multistage processes in discrete systems (Kovalenko, 1990) functional has the form:

$$Y(\bar{p}) = \sum_{t=0}^T \mu^0(t, q(t), u(t)) + Y(q(t)). \quad (8)$$

Then we have to determine such optimal process $\bar{p} = (\bar{q}(t), \bar{u}(t))$ due to which $Y(\bar{p}) \rightarrow \max Y(p)$. This is one of the possible models, a general view of which can be transformed depending on the characteristics of the educational process, which is a part of the set of university management models.

4. DISCUSSIONS

The results of the work do not cover all aspects of the problem. Further research is extremely important, both theoretical and practical in the framework of the modernization of higher education in Russia. It should be noted that the vector of further studies may lie in the development of the system of interconnected models that implement management actions coordination between subsystems and their testing in practice. The paper proposes a DM procedure for the criteria approach. It will be useful to develop a DM procedure for non-criteria approach.

5. CONCLUSION

Reviewing the activities of higher educational institutions as production activities, it is possible to represent them as complex organizational and economic systems, aimed at the provision of educational services. Such systems consist of subsystems, which can be grouped into two consolidated independent subsystems: Economic and organizational -educational. Both subsystems are closely interrelated. For DM in higher education institutions there should be used a criteria procedure with economic and mathematical modeling. And in the process of modeling of economic subsystem management it is advisable to apply the procedures of transformation of a multi-criteria problem into a one-criterion problem.

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