# Methodology of Studying Polygons Using the Computer System "Mathematics" 

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#### Abstract

The urgency of the problem stated in the article is due to the fact that the traditional method of teaching mathematics to students of physics and mathematics generally considers mathematical problems that do not require large computing and constructions, while solving complex problems requires more time, which is usually done at the expense of time that is to be spent on other branches of mathematics, so there is a need to study some sections of mathematics with the help of information technology. The purpose of the article is to develop methods of using information technologies in teaching elementary mathematics to students of physics and mathematics department. The leading method in the study of this problem is a design method aimed at selection and structuring of the contents of the computer tutorial (CT) under the heading "Polygons" in the system "Mathematics." The designed CT for students of physics and mathematics includes a classification of polygons based on their historical information, method of construction, basic properties, the use of polygons in life, in construction, etc. The developed CT is aimed at improving students' knowledge of elementary geometry, it promotes the formation of the students' abilities to create computer textbooks (workshops), which they can use in their professional activity thus endowing them with the skills necessary to modern highly qualified teachers.


Keywords: Polygon, Educational Software, Information Technology, System "Mathematics", Computer Tutorial JEL Classifications: A20, C01, A29

## 1. INTRODUCTION

The role of information technology in the process of studying mathematical disciplines in higher educational institutions increases due to the fact that it acts as an effective didactic tool, with which you can generate students' individual educational trajectories. Such a trajectory is the result of selecting a personally significant learning content, its complexity, types of tasks, their qualitative content, the rate of study, etc. (Ganeeva, 2014; Nasibullov et al., 2015).

By learning software (LS) we mean computer (i.e. realized in a particular computing environment) textbooks and problem books, exercise equipment, laboratory courses, reference books, encyclopedias, test, control and training programs, and some other educational software (Anisimova and Krasnova, 2015; Golitsyna, 2013; Zaripova et al., 2014; Yachina et al., 2015).

A computer tutorial (CT) is a LS for basic training in a specific discipline, the content of which is relatively complete and which is presented in the form of a textbook (or book).

While maintaining the traditional forms of teaching one can devote part of time to computerized learning. It will be of great educational value as it provides visual representation of the teaching material and makes control easier (as a result the quality of teaching and learning results is increased) (Valeeva and Shakirova, 2015). It is known that many of the mathematical issues are difficult to explain at the lectures because of the large computational effort required to illustrate the material. In this case, "Mathematics" can be of great help (Manturov, 2004).

As a platform of our CT we will use the computer system "Mathematics." The use of "Mathematics" in the teaching process can be traced in the works (Squire et al., 2012; Bytev and

Kniehl, 2014; Tam, 2008; Kapustina et al., 2014; Manturov, 2004; Crandall, 1991; Maeder, 1991; and others).

The system Mathematica, created about 25 years ago, has an extremely wide range of tools that transform complex mathematical algorithms into programs. Among them: All the socalled elementary functions and a large number of non-elementary; algebraic and logical operations. Almost all the algorithms contained in the higher mathematics course are laid in the memory of the computer system "Mathematics." This means, among other things, that the majority of the exercises from the course of higher mathematics can be solved with just a single command. Indeed, it includes all the exercises of linear algebra (including such trivial things as the reduction of quadratic forms to the canonical form, bringing a linear operator to Jordan form); all the exercises of analysis, theory of differential equations (both ordinary and partial derivative). With the help of "Mathematics," you can calculate integrals (definite and indefinite), solve differential equations (numerically and analytically). "Mathematics" knows about integrals and differential equations more than any thick book of reference. Furthermore, "Mathematics" does not only give the final answer, but may describe intermediate calculations (e.g. decomposition of proper rational functions into the sum of elementary fractions that is required when integrating rational functions) (Manturov, 2004).
"Mathematics" has a powerful graphics package. You can use it to plot very complex functions of one and two variables. This yields often delightfully beautiful pictures.

Widely respected both for its technical capabilities and for the elegant simplicity of use, the system "Mathematics" provides a solid, integrated, and constantly expanding system, covering a wide range and depths of technical computing, which is now with the advent of Mathematics Online, always at your fingertips, available through any web browser, as well as through native applications on all modern desktop computer systems.

A CT, created with the use of "Mathematics," retains all the features of the existing traditional textboo k (one document contains combinations of texts, tables, graphics, animation, organization of a dialogue with the user, hypertext links, etc.). "Mathematics" allows performing all calculations inside a CT, work in the calculation mode, like with a calculator (numerical and symbolic) and in programming mode like with a powerful programming system, allowing to build and analyze complex mathematical models (Ganeeva, 2013).

## 2. MATERIALS AND METHODS

### 2.1. Research Methods

The study used the following methods: Analysis of scientific literature on the psychological and pedagogical, philosophical, mathematical, methodical and special aspects related to the field of research; analysis of documents and literary sources (regulations, concepts, programs); interview, teacher observation, questioning students and teachers, pedagogical experiment, followed by mathematical processing of the results, design of a CT.

### 2.2. Research Experimental Base

Experimental research work was carried out on the base of Elabuga Institute of Kazan Federal University at the Faculty of Physics and Mathematics.

### 2.3. Research Stages

- At the first stage - The preparatory: The current state of the investigated problem in pedagogical theory and practice was analyzed; a technique of studying elementary mathematics using information technology was developed;
- At the second stage - The main stage: Methods of studying elementary mathematics using information technology were developed and implemented; experimental work to verify the effectiveness of this model was conducted;
- At the third stage - The final stage: Systematization, interpretation and synthesis of the research results were carried out; theoretical conclusions were refined; processing and registration of the results of the study were performed.


## 3. RESULTS

### 3.1. CT Content

The system "Mathematics" allows creating a CT and presenting the material on polygons in full. Each case contains historical information of a polygon, the method of construction, basic properties, the use of polygons in life, construction, etc.

The content of CT is organized in the form of hyperlinks and includes the classification of polygons. Types of flat polygons are divided into two groups: Convex and non-convex. The first group includes: Regular and semiregular (equiangular-semiregular, equilateral-semiregular, arbitrary convex). The second group includes: Stellate (regular, semiregular, arbitrary) and arbitrary non-convex.

The pace of work and sequence of studying various forms of presentation of computer training material are determined by the user. These facts are important, as they have a significant impact on the consolidation of the acquired knowledge and skills and thus improve the efficiency of learning. CTs activate self-education in the learning process, changing its style and nature, enhance the imaginative perception of the material, and increase the level of information.

Let us consider the concept of stellate polygon from the textbook "Elementary Geometry" by Hadamard (1948).

Dividing the circle into $n$ equal parts, let us connect one of the dividing points with the point separated from it by $p$ divisions, this last point with the point distant from it at $p$ divisions, etc. until we arrive again in one of the dividing points that are already passed. If the point $P$ is the first of those points, to which we will come for the second time, this point cannot be different from the point at which we started; indeed, if the point $P$ were the end of the part $N P$, that has already been passed, we would have come to the point $N$ for the second time earlier than to the point $P$.

The Figure 1 thus constructed, represents a stellate regular polygon. For example, Figure 1 shows a regular stellate pentagon,
obtained by dividing a circle into five equal parts and connecting these points through two divisions $(n=5 ; p=2)$.

Figure 1 was built in the system "Mathematics" as follows:
$\left.\mathrm{k} 1=\operatorname{ListLinePlot[Table[\{ Sin}\left[2 \mathrm{Pi}^{*} \mathrm{n} / 5\right], \operatorname{Cos}\left[2 \mathrm{Pi}^{*} \mathrm{n} / 5\right]\right\},\{\mathrm{n}, 0$, $10,2\}$ ], AspectRatio->Automatic, Axes->False, PlotStyle$>\{$ Black,Thickness[0.012]\}];
k2=Graphics[\{Thick,Circle[\{0,0\},1]\}];
Show[\{k1,k2\},PlotRange-> $\{-1,1\}]$
We can assume that the numbers $n$ and $p$ are coprime. In fact, if these numbers had the greatest common divisor $d$, everything would happen as if we had shared the circle into the $n / d$ equal parts and had connected the dividing points after every $p / d$ division.

If $n$ and $p$ are coprime numbers, the stellate polygon will have exactly $n$ sides. Indeed, since each side contains $p$ sections, then constructing $k$ sides we will pass $k p$ divisions. We will return to the starting point when we go through an integer number of circles, i.e. when $k p$ is a number divisible by $n$. Arithmetic proves that this circumstance occurs for some values of $k$, smaller than $n$; if the number $p$ is not the coprime number $n$, then this circumstance occurs for the first time when $k$ is equal to $n$.

Thus, the stellate polygon with $n$ sides is formed by dividing a circle into $n$ equal parts and connecting the dividing point after in every $p$ division, where $p$ is any integer, coprime to $n$ and less than $n$. However, every stellate polygon is obtained in two different ways. Indeed, if its side is the chord of the arc containing $p$ divisions, it will also be a chord of an arc containing $n-p$ divisions. For example, the stellate polygon shown in Figure 1 is obtained by connecting the dividing points after every two divisions or every three divisions. For this reason, it is necessary to drop away half the value of $p$, if we want to get every polygon in the proper sense of the word.
E.g. Supposing $n=15$. Numbers $<15$ and coprime with 15 are, except 1 and 14 , the numbers $2,4,7,8,11$ and 13 . There is no need to consider the value of $p=8,11,13$, which correspond to the same polygons that are obtained at values of $p=7,4,2$.

Figure 1: The stellate regular pentagon


Thus, there is one regular quindecagon in the strict sense and three stellate quindecagons (Figure 2).
b1 $=$ ListLinePlot[Table[\{Sin[2Pi*n/15], $\left.\operatorname{Cos}\left[2 * \mathrm{Pi}^{*} \mathrm{n} / 15\right]\right\}$, \{n,16 ],Axes->False,

AspectRatio->Automatic,PlotStyle-> \{Black,Thickness[0.015]\}];
$\mathrm{b} 2=\mathrm{ListLinePlot}\left[\operatorname{Table}\left[\left\{\operatorname{Sin}\left[2 \mathrm{Pi}^{*} \mathrm{n} / 15\right], \operatorname{Cos}\left[2 * \mathrm{Pi}_{\mathrm{n}} / 15\right]\right\}\right.\right.$, $\{\mathrm{n}, 0,0,2\}]$,Axes->False,

AspectRatio->Automatic,PlotStyle-> \{Black,Thickness[0.015]\}];
b4=ListLinePlot[Table[\{Sin[2Pi*n/15], $\left.\operatorname{Cos}\left[2 * \mathrm{Pi}^{*} \mathrm{n} / 15\right]\right\}$, $\{n, 0,0,4\}]$,Axes->False,

AspectRatio->Automatic,PlotStyle-> \{Black,Thickness[0.015]\}];
b7=ListLinePlot[Table[\{Sin[2Pi*n/15], Cos[2*Pi*n/15]\}, $\{n, 0,05,7\}]$,Axes->False,

AspectRatio->Automatic,PlotStyle-> \{Black,Thickness[0.015]\}];
GraphicsArray[\{b1,b2,b4,b7\}]
Besides the three stellate quindecagons we can obtain stellate polygons by overlaying and rotating similar regular or stellate polygons.

Let us consider a regular quindecagon and connect the vertices in every three divisions. The greatest common divisor of 3 and 15 is three, i.e. such a stellate quindecagon is obtained from three regular pentagons because 15: $3=5$ (Figure 3).
b3 $=$ ListLinePlot[Table[\{Sin[2Pi*n/15], $\left.\operatorname{Cos}\left[2 * \mathrm{Pi}^{*} \mathrm{n} / 15\right]\right\}$, $\{\mathrm{n}, 0,5,3\}]$,Axes->False,

AspectRatio->Automatic,PlotStyle-> \{Black,Thickness[0.015]\}];
b31=ListLinePlot[Table[\{Sin[2Pi*(n+1)/15], $\left.\operatorname{Cos}\left[2 * \mathrm{Pi}^{*}(\mathrm{n}+1) / 15\right]\right\}$ , \{n, $0,15,3\}]$,Axes->False,AspectRatio->Automatic,PlotStyle$>\{$ Black,Thickness[0.015]\}];
b32 $=$ ListLinePlot[Table[\{Sin[2Pi*(n+2)/15], Cos $\left.\left.\left[2 * \mathrm{Pi}^{*}(\mathrm{n}+2) / 1\right]\right\},\{\mathrm{n}, 0,15,3\}\right]$, Axes->False, AspectRatio$>$ Automatic, PlotStyle-> \{Black,Thickness[0.015]\}];
$\mathrm{bS} 3=$ Show $[\mathrm{b} 3, \mathrm{~b} 31, \mathrm{~b} 32$, PlotRange- $>\{-1,1\}]$;
GraphicsArray[\{b1,b3,b31,b32,bS3\}]

Figure 2: One regular quindecagon and three stellate quindecagons


Similarly, let us consider the regular quindecagon and make connections in every five divisions. The biggest common divisor of 15 and 5 is five, that is, such a stellate quindecagon is obtained from five regular triangles, as 15: $5=3$ (Figure 4).
b5 $=\operatorname{ListLinePlot[Table[\{ Sin[2Pi*n/15],~Cos[2*Pi*n/15]\} ,~}$ $\{0,15,5\}]$,Axes->False, AspectRatio->Automatic, PlotStyle$>\{$ Black,Thickness[0.015]\}];
b51=ListLinePlot[Table[ $\left\{\operatorname{Sin}\left[2 \mathrm{Pi}^{*}(\mathrm{n}+1) / 15\right], \operatorname{Cos}\left[2 * \mathrm{Pi}^{*}(\mathrm{n}+1) / 1\right]\right.$ $\},\{n, 0,15,5\}]$,Axes->False,AspectRatio->Automatic, PlotStyle>\{Black,Thickness[0.015]\}];
b52=ListLinePlot[Table[\{Sin[2Pi*(n+2)/15], $\operatorname{Cos}\left[2 * \mathrm{Pi}^{*}(\mathrm{n}+2) / 1\right.$ ] \}, \{n, $0,15,5\}$ ],Axes->False,AspectRatio->Automatic,PlotStyle$>\{$ Black,Thickness[0.015]\}];
b53=ListLinePlot[Table[\{Sin[2Pi*(n+3)/15], $\operatorname{Cos}\left[2^{*} \mathrm{Pi}^{*}(\mathrm{n}+3) /\right.$ $1]\},\{n, 0,15,5\}]$,Axes $->$ False,AspectRatio->Automatic,PlotStyle>\{Black,Thickness[0.015]\}];
b54=ListLinePlot[Table[\{Sin[2Pi*(n+4)/15], $\left.\operatorname{Cos}\left[2 * \mathrm{Pi}^{*}(\mathrm{n}+4) / 1\right]\right\}$, $\{n, 0,15,5\}]$,Axes->False,AspectRatio->Automatic,PlotStyle$>\{$ Black,Thickness[0.015]\}];
bS5=Show[b5,b51,b52,b53,b54,PlotRange-> $\{-1,1\}]$;
GraphicsArray[\{b1,b5,b51,b52,b53,b54,bS5\}]

Similarly, let us consider the regular quindecagon and make connections in six divisions. The biggest common divisor of 15 and 6 is equal to three, i.e. such stellate quindecagon is obtained from three right stellate pentagons (Figure 5).
$\mathrm{b} 6=\mathrm{ListLinePlot}\left[\operatorname{Table}\left[\left\{\operatorname{Sin}\left[2 \mathrm{Pi}^{*} \mathrm{n} / 15\right], \operatorname{Cos}\left[2 * \mathrm{Pi}^{*} \mathrm{n} / 15\right]\right\}\right.\right.$, $\{$ , $0,90,6\}$ ],Axes->False,AspectRatio->Automatic, PlotStyle$>\{$ Black,Thickness[0.015] \}];
b61 $=$ ListLinePlot[Table[\{Sin[2Pi*(n+1)/15], $\operatorname{Cos}\left[2 * \mathrm{Pi}^{*}(\mathrm{n}+1) / 1\right.$ ]\}, \{n,0,90,6\}],Axes->False,AspectRatio->Automatic,PlotStyle>\{Black,Thickness[0.015]\}];
b62 $=$ ListLinePlot[Table[ $\left\{\operatorname{Sin}\left[2 \mathrm{Pi}^{*}(\mathrm{n}+3) / 15\right], \operatorname{Cos}\left[2 * \mathrm{Pi}^{*}(\mathrm{n}+3) / 1\right.\right.$ ] $\},\{n, 0,90,6\}]$,Axes->False,AspectRatio->Automatic,PlotStyle$>$ \{Black,Thickness[0.015]\}];
b63 $=$ ListLinePlot[Table[ $\left\{\operatorname{Sin}\left[2 \mathrm{Pi}^{*}(\mathrm{n}+5) / 15\right], \operatorname{Cos}\left[2 * \mathrm{Pi}^{*}(\mathrm{n}+5) / 1\right.\right.$ $]\},\{n, 0,90,6\}]$,Axes- $>$ False, AspectRatio->Automatic, PlotStyle$>\{$ Black,Thickness[0.015] \}];
bS6=Show[b6,b61,b62,b63,PlotRange-> $\{-1,1\}] ;$
GraphicsArray[\{b1,b6,b61,b62,b63,bS6\}]

Thus, Figure 6 shows regular stellate quindecagons derived from regular quindecagon by connecting vertices in two, three, four, five, six and seven divisions.

GraphicsArray[\{b1,b2,bS3,b4,bS5,bS6,b7\}]
Thus, the classification of regular stellate quindecagons was made. On the example of these constructions in the CT other types of computer-built polygons were produced.

The main purpose of the pedagogical experiment was to test the assumptions of the hypothesis:

- Method of studying elementary mathematics by students of physics and mathematics in higher educational institution

Figure 3: Regular quindecagon and stellate quindecagon (Type 4)


Figure 4: The regular quindecagon and stellate quindecagon (Type 5)


Figure 5: The regular quindecagon and stellate quindecagon (Type 6)


Figure 6: The regular stellate quindecagons

can be intensified and put on a qualitatively new level with the introduction of new information technologies based on the use of the powerful features of the computer system "Mathematics";

- The use of a computer system "Mathematics" in teaching mathematics creates students' positive motivation for studying; it increases the students' activity and their interest in the subject;
- For the effective organization of training it is necessary to develop new LS created on the basis of the system "Mathematics."

The effectiveness of computer-assisted teaching of mathematical disciplines at the university (for example, a course of elementary mathematics) was determined during the formative experiment.

The immediate object of the formative experiment were changes in the students' volume, the nature and quality of knowledge and skills, taking place under the influence of pedagogical impact of computer books, created in the system "Mathematics."

During the preparatory and ascertaining stages of the pedagogical experiment the problems relating to the research state and the objective possibilities to use the system "Mathematics" in teaching elementary mathematics were being solved.

The following is a brief description of the above stages of the experiment.

### 3.2. The Preparatory Stage

Objective:

- Analysis of the psychological and pedagogical aspects of the study;
- Compiling a bibliography of research;
- Selection and justification of the goals and objectives of the study;
- Study of the experience of teachers on the issue of the introduction of new information technologies and the use of the system "Mathematics" in teaching mathematics.

Methods used are analytical, including:

- Analysis of reference literature, psycho-pedagogical and methodological literature on the issues of the study;
- Study and understanding of modern forms and methods used in the teaching activities of the university teachers;
- Study of the students' experience in the use of LS in some disciplines at the university, or the use of them in selfeducation;
- Study of the students' opinions on the use of computer technology in education;
- Study the teachers' experience in the organization of
students' independent work with the use of new information technologies and the use of the system "Mathematics" in teaching mathematics;
- Study the experience of physics and mathematics' students with the system "Mathematics";
- Study of international experience in the application of the system "Mathematics" for the applied, scientific and educational purposes.

The results:

- It was found out that abroad there are not only a large educational, reference and specialized periodicals about the system "Mathematics," but also samples of didactic literature and computerized tutorials on mathematical disciplines with their application; a very different situation is observed in Russia: There is little reference and educational literature about the system, little attention is paid to it in a special periodical literature, and there is absolutely no didactic and methodical literature on the use of the system "Mathematics" as a means of new information technologies of training in the university educational process;
- In the course of this study, it was found out that the students do not use software for self-education due to the lack of educational software products relevant to the curriculum;
- It was found out that teachers of informatics adequately acquaint students of physics and mathematics with the system "Mathematics," but teachers of mathematics make little use of it in practice;
- It was found out that the most successful methodical approaches to the use of the system "Mathematics" are carried out with adapting the specially designed LS to the environment "Mathematics" (CTs, fragments of material from training courses in the form of electronic files), which is done by the teachers themselves in accordance with their specific academic disciplines and points of view on the teaching methods.

In order to study the students' opinions on the use of new information technologies of training in higher educational institutions the following survey of students of physics and mathematics was held. This survey was conducted prior to the formative stage of the experiment. The purpose of the survey was to get an answer the question "Should CT, created in the system 'Mathematics,' be used in education?"'

### 3.3. Ascertaining Stage

Objective: To establish the characteristics of the practical application of CT, created in the system "Mathematics" as a means of information technology in teaching mathematics, including:

- Defining the nature of the relationship between students' knowledge about the system "Mathematics" and its application in practice
- Determining the ways, means and nature of the use of CT created by the system "Mathematics," in teaching mathematical disciplines
- Determining the ways, means and nature of the use of CT created by the system "Mathematics," in the students' independent work on mathematical disciplines.

Methods: Questionnaires, interviews, observations, studying Students' and teachers' work.

Results: The relationship between students' knowledge obtained in the course of studies with the use of a CT that was created in the system "Mathematics," and its application in practice while using the system; besides there were identified some difficulties in working with CTs; the ways and methods of using the CT created in the system "Mathematics" in the educational process of the university were mapped.

### 3.4. The Formative Stage

During 2012-2014 we carried out a pilot study with students of physics and mathematics.

Third-year students of the faculty of physics and mathematics of Elabuga Institute of Kazan (Volga region) Federal University participated in the formative stage. Students of the specialty "Mathematics-Physics" were the control group and the students of the specialty "Mathematics-Informatics" were the experimental group.

For the validation of hypotheses there were conducted monitoring activities in the experimental and control groups to verify the acquired knowledge and practical skills in solving problems

After studying the chapter "Polygons" in elementary mathematics there was carried out a testing of the students of the experimental group that used information technology and of the control group that was taught according to traditional methods. The experimental group consisted of 13 students; the control group of 17 students. Students were given 20 groups of questions.

In the experimental group students who received a total score of at least 13 were considered successfully passing the theoretical material in the form of the test. To measure the degree of mastery of the material on the basis of the data obtained, information Statistics Mann-Whitney test was used (Meyer, 1997).

As the null hypothesis we accept the proposition that there is no difference between the groups.

Processing of the experimental data gives the following results of ranks for the experimental and control groups: $R_{1}=153.5 ; R_{2}=311.5$ (Table 1).

To calculate the statistics we use formulas:
$U_{1}=n_{1} \cdot n_{2}+\frac{n_{1}\left(n_{1}+1\right)}{2}-R_{1} ; \quad U_{2}=n_{1} \cdot n_{2}+\frac{n_{2}\left(n_{2}+1\right)}{2}-R_{2} ;$
$U_{1}=13 \cdot 17 \_91-153.5=158.5$;
$U_{2}=13 \cdot 17+91+153-311.5=62.5$.
As a screening statistics of $U$-criterion Mann-Whitney we take the minimal values of $U_{1}$ and $U_{2}$.

In our case statistics of $U=62.5$. The null hypothesis is rejected if the numerical value of $U$ is smaller than the critical value, which at $n_{1}=13$ and $n_{2}=17$ at significance level $\alpha=0.05$ takes the value $U_{\text {krit }}=63$. As $U<U_{k r i t}$, $(62.5<63)$, the null hypothesis should be refuted. Therefore, judging by the quality of the test, the difference between the experimental and control groups is statistically significant at the 0.05 level or probability of 0.95 .

To test the students' skills in solving problems in the section "Polygons" formed in the classroom during the experiment a control work was carried out. It was solved by the 13 students of the experimental group (where classes were taught with the use of the experimental method), and by the 17 students of the control group (in which classes were conducted according to traditional methods).

Table 1: Results of the final test in the experimental and control groups (the maximum possible score - 20)

| No. | Experimental group |  |  | Control group |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Test results | Ranks | Average value | Test results | Ranks | Average value |
| 1 | 13 | 10 | 10 | 9 | 14 | 14 |
| 2 | 12 | 11 | 11 | 17 | 3 | 3 |
| 3 | 15 | 5, 6, 7 | 6 | 6 | 18,19 | 18.5 |
| 4 | 5 | 20, 21, 22 | 21 | 7 | 17 | 17 |
| 5 | 15 | 5, 6, 7 | 6 | 3 | 25, 26, 27, 28, 29 | 27 |
| 6 | 14 | 8, 9 | 8,5 | 15 | 5, 6, 7 | 6 |
| 7 | 16 | 4 | 4 | 3 | 25, 26, 27, 28, 29 | 27 |
| 8 | 18 | 2 | 2 | 2 | 30 | 30 |
| 9 | 4 | 23, 24 | 23,5 | 14 | 8, 9 | 8.5 |
| 10 | 3 | 25, 26, 27, 28, 29 | 27 | 8 | 15, 16 | 15.5 |
| 11 | 10 | 12, 13 | 12,5 | 10 | 12, 13 | 12.5 |
| 12 | 5 | 20, 21, 22 | 21 | 5 | 20, 21, 22 | 21 |
| 13 | 19 | 1 | 1 | 6 | 18, 19 | 18.5 |
| 14 |  |  |  | 3 | 25, 26, 27, 28, 29 | 27 |
| 15 |  |  |  | 4 | 23, 24 | 23.5 |
| 16 |  |  |  | 8 | 15, 16 | 15.5 |
| 17 |  |  |  | 3 | 25, 26, 27, 28, 29 | 27 |
| Total | 149 |  | $R_{1}=153.5$ | 123 |  | $R_{2}=311.5$ |

As the null hypothesis we accepted the proposition that there is no difference between the groups.

The results of the control work are represented in the form of Table 2 and Figure 7.

Each correct solution of the problem scored 5. Maximum total score was 20.

As a result of the control work in the experimental group sample mean $\overline{X_{1}}=14.07692$ and the value of the sample variance $s_{x}^{2}=23.24359$, while in the control group sample mean $\overline{X_{2}}=10.05882$; and the value of the sample variance $s_{x}^{2}=25.30882$.

The results analysis shows that in this case the value of mean in the experimental group is higher than the corresponding values in the control group. Consider the hypothesis of the randomness of mismatch of the averages $\overline{X_{1}}$ and $\overline{X_{2}}$, i.e., let us show that the deviation in the average is significant. Suitable for a statistical test of this hypothesis is Student's t-test (Meyer, 1997). First, on the basis of a comparison of individual values, respectively, we calculate the average estimate of unbiased variance in the "general totality." To do this, the sum of squares of the deviations of the individual values from the respective average must be divided by the total number of degrees of freedom. Then we calculate Student's $t$-criterion and the number of degrees of freedom $v$.
$\sigma_{*}^{2}=24.42372 ; t=2.206735 ; v=13+17-2=28$.
We get: $t_{\text {Ha6 }}=2.206735$, while $t_{\text {ma6 }}=2.0484$, we see that $t_{\text {ha6 }}>t_{\text {ma6 }}$.

Hence, the null hypothesis is rejected. On this basis, it can be concluded that with the probability of 0.95 results of examinations are due to the difference in the education system.

This shows the positive impact of the use of information technology in the classroom, while doing homework, etc. Undoubtedly, the

Figure 7: The results of the control work assignments

good results of the experimental group were achieved through the use of the electronic tutorial created in the system Mathematica.

Thus, the results of experimental work confirmed the consistency of the hypothesis put forward by us.

As a result of the introduction of a CT in the educational process we can trace a deeper understanding of the material by students of physics and mathematics. Students have a desire to work in the system Mathematica, creating fragments by computer textbooks. Thus, not only do we inculcate interest in mathematics, but also teach future teachers to use information technology in the mathematics classroom. The system Mathematica has allowed us to design a CT that can easily create polygons and give their detailed classification.

## 4. DISCUSSIONS

The previous research carried out by Squire et al. (2012), Bytev and Kniehl (2014), Tam (2008), Kapustina et al. (2014), Manturov (2004), Crandall (1991), Maeder (1991) are dedicated to observation of the system Mathematica and contain some recommendations for its use in various fields of knowledge. There are also a number of works on Polygons that disclose the mathematical essence of these geometrical figures in the framework of methodical literature for secondary schools and universities. Our study aims at combining all the accumulated by teachers and scientists experience in studying polygons and by unfolding it from the methodological point of view to implement it in a CT designed in the system Mathematica.

Mathematica is a multifunctional software product, for a full study of which one can spend more than a year. And therefore, when used merely as a base to create a CT, much of its potential remains unclaimed. Nevertheless you must be able to have a perfect command of that part of the capacity of the system that is to be used. Due to the fact that comprehension of the syntax of this computer system does not cause much trouble, the CT can be constructed in such a way that the development of skills necessary to use the system will develop in parallel with the study of the subject.

## 5. CONCLUSION

The CT in the environment Mathematica can be used at lectures, in workshops and for students' independent work. The need to create a CT in the environment Mathematica arises if there is a necessity in complex computation and exact building of various geometrical figures. For this purpose it is required to structure the teaching material to identify the themes that need to be supplied with the design of a CT. The teacher is able to create computer textbooks

Table 2: The results of the control work

|  | Results of control work |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Problem number | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | Score |
| Experimental group | 3.923077 | 3.230769 | 3.769231 | 3.153846 | 14.07692 |
| Control group | 2.941176 | 2.352941 | 2.823529 | 1.941176 | 10.05882 |
| Difference between the mean | 0.9819 | 0.877828 | 0.945701 | 1.21267 | 4.0181 |

in the environment Mathematica as this software possesses all shell elements for their creation.

The computer mathematical system as a means of new information technologies of teaching must be used only in case:

- The computer system will not arouse difficulties in studying the material that is taught,
- The material is not to difficult for the students' independent studies,
- The use of the computer system can be justified (for example, by the need in complex computation in the material under study, the necessity of visualization of the studied mathematical objects, etc.).


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