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# Volatility Spillovers among the Cryptocurrency Time Series

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**ABSTRACT**

This paper uses different multivariate GARCH models to model conditional correlations and analyze the volatility spillovers between cryptocurrency time series. The dynamic conditional correlation GARCH model is found to fit the data the best. Our empirical results are fourfold. First, on average, a $1 long position in BitShares (BTS) can be hedged for 15% with a short position in MonaCoin (MONA), while a $1 long position in MONA can be hedged for 14% with a short position in Ripple (XRP). Second, the average weight for the BTS/MONA portfolio is 0.48, indicating that for a $1 portfolio, 48% should be invested in BTS and 52% invested in MONA. Third, the average weight for the BTS/XRP portfolio indicates that 27% should be invested in BTS and 73% invested in XRP. Finally, the average weight for the MONA/XRP portfolio indicates that 33% should be invested in MONA and 67% invested in XRP.

**Keywords:** Cryptocurrencies, Multivariate GARCH, Volatility Spillover, Hedging, Portfolio Designs
**JEL Classifications:** C5, C22, C32, G1

## 1. INTRODUCTION

Cryptocurrencies have received much attention by the media and investors alike, which can be attributed to their innovative features, transparency, simplicity and increasing popularity (Urquhart, 2016; 2017). A cryptocurrency is defined as a digital asset designed to work as a medium of exchange that uses cryptography in order to secure its transactions, to control the creation of additional units, and to verify the transfer of assets. Besides, cryptocurrencies are a kind of digital currencies, alternative currencies and virtual currencies. Furthermore, bitcoin, created in 2009, was the first type of cryptocurrency (for more details on bitcoin, Selgin, 2015; Baeck and Elbeck, 2015; Yermack, 2015; Pieters and Vivanco, 2017; Katsiampa, 2017). Since then, numerous other cryptocurrencies have been created.

The literature on cryptocurrencies was initially dominated by studies on the safety, ethical and legal aspects of Bitcoin. Otherwise, recent literature has examined cryptocurrencies from an economic viewpoint. However, little is known about the behaviour of price returns and volatilities of the cryptocurrencies. Undoubtedly, cryptocurrencies evolved from a niche existence to a new asset class for which price time series are increasingly available by now and can be used for empirical analyses (Brauneis and Mestel, 2018). One particularly interesting aspect is whether the highly volatile prices of cryptocurrencies evolve randomly over time or show some predictability. As argued by Cheah and Fry (2015), if bitcoin were a true unit or account, or a form of store of value, it would not display such volatility expressed by bubbles and crashes. According to Dwyer (2015), the average monthly volatility of bitcoin is higher than that of gold or a set of foreign currencies. Besides, he finds that the lowest monthly volatility for bitcoin are less than the highest monthly volatility for gold and currencies. Furthermore, Brière et al. (2015) show that bitcoin offers significant diversification benefits for investors, while Urquhart (2016) shows that bitcoin returns do not follow a random walk. Using an asymmetric GARCH model, Dyhrberg (2016a) shows that bitcoin may be useful in risk management and ideal for risk averse investors in anticipation of negative shocks to the market. Moreover, Dyhrberg (2016a,b) show that Bitcoin has similar hedging capabilities as gold and the dollar, and as such can be employed for risk management. In addition, Balcilar et al. (2017) show that bitcoin volume can predict returns except in bear and bull market regimes and that volume cannot predict the volatility of bitcoin returns.

This empirical research aims to explore if cryptocurrencies behave like a well-known financial asset by analyzing their price returns and volatilities. This analysis will therefore suggest the economic abilities of cryptocurrencies in risk management, portfolio analysis and hedging capabilities. To date, however, very little is known about the volatility dynamics of cryptocurrency prices and the possible correlations, dynamic relationships, and volatility spillover effects between those cryptocurrencies. To the best of our knowledge, this is the first study that aims to fill this void. In this paper, multivariate GARCH models are used to model dynamic correlations and the volatility spillovers between cryptocurrencies. Four multivariate GARCH models (BEKK, diagonal BEKK, constant conditional correlation, and dynamic conditional correlation [DCC]) are compared and contrasted. It is found that the VARMA-GARCH DCC model fits the cryptocurrencies data best and this model is then used to construct hedge ratios and optimum portfolio weights.

The remainder of the paper is organized as follows. Section 2 provides a description of the data and summary statistics. Section 3 discusses the empirical methodology used in this study. Section 4 discusses the empirical results. Section 5 provides the economic implications of the results for designing optimal portfolios and formulating optimal hedging strategies. Section 6 gives some concluding comments.

## 2. DATA DESCRIPTION AND SUMMARY STATISTICS

In this paper, cryptocurrency data covers the period from August 01, 2014 to February 27, 2018. Data, which are sourced from the website www.coinmarketcap.com, comprise open, high, low and close prices, as well as dollar volume and market capitalization on a daily basis. These data are volume weighted averages from a large number of different exchanges.

For each cryptocurrency's data series, continuously compounded daily returns are calculated as $r_t=100\times ln(p_t/p_{t-1})$ where $p_t$ is the daily closing price. Table 1 reports the summary statistics for the three cryptocurrency returns series. The standard deviation is larger than the mean value. Besides, Student *t* statistics indicate that the mean is not statistically significant for both BTS and MONA series. However, Student *t* statistics indicate that the mean is statistically significant at the 10% level for XRP series. Furthermore, each series displays a large amount of both skewness and kurtosis and the returns are not normally distributed.

Descriptive properties of discrete daily returns for three of biggest cryptocurrencies are depicted in Table 1. The mean return is positive and quite small, whereas the corresponding standard deviation of the returns is substantially higher. Moreover, extreme changes in cryptocurrency prices are substantial. Indeed, we observe maximum day to-day losses exceeding 50%. Besides, the return series exhibit positive skewness for all cryptocurrencies. This implies large positive price changes to be more likely than large negative changes. Furthermore, kurtosis is significantly higher than 3 for all cryptocurrencies, implying fat tailed distributions. In addition, the normality tests are performed. The results of the Jarque-Bera test (Jarque and Bera, 1987) show strong departure from normality.

**Table 1: Summary statistics for daily returns**

| Variables | BTS | MONA | XRP |
|---|---|---|---|
| Mean | 0.2400 | 0.1563 | 0.3961 |
| Median | −0.2922 | −0.2405 | −0.2334 |
| Maximum | 51.9989 | 85.2212 | 102.7356 |
| Minimum | −39.1702 | −84.7036 | −61.6273 |
| Variance | 66.2402 | 90.2327 | 53.6046 |
| SD | 8.1388 | 9.4991 | 7.3215 |
| Skewness | 1.0777*** | 0.5239*** | 3.0714*** |
| | 0.0000 | 0.0000 | 0.0000 |
| Kurtosis (excess) | 7.1939*** | 18.2294*** | 42.0584*** |
| | 0.0000 | 0.0000 | 0.0000 |
| Jarque-Bera | 3068.9741*** | 18142.9269*** | 98311.5815 |
| | 0.0000 | 0.0000 | 0.0000 |
| *t*-statistic | 1.0657 | 0.5948 | 1.9551* |
| (*Mean=0*) | 0.2868 | 0.5521 | 0.0508 |
| Ljung-BOX | 60.9217*** | 18.2428 | 68.2013*** |
| *Q*(20) test | 0.000 | 0.5714 | 0.0000 |
| Ljung-BOX | 256.548*** | 196.187*** | 198.995*** |
| $Q^2$ (20) test | 0.0000 | 0.0000 | 0.0000 |
| ADF unit root | −34.0906*** | −35.8436*** | −22.6293*** |
| test | 0.0000 | 0.0000 | 0.0000 |
| PP unit root test | −34.3833*** | −35.844*** | −36.9909*** |
| | 0.0000 | 0.0000 | 0.0000 |
| KPSS unit root test | 0.2892 | 0.0733 | 0.2819 |
| ARCH (1)-LM | 53.346*** | 91.652*** | 68.244*** |
| test | 0.0000 | 0.0000 | 0.0000 |
| ARCH (10)-LM | 13.365*** | 24.220*** | 15.408*** |
| test | 0.0000 | 0.0000 | 0.0000 |
| Observations | 1306 | 1306 | 1306 |

The Jarque-Bera test corresponds to the test statistic for the null hypothesis of normality in the distribution of sample returns. The Ljung-Box statistics, $Q(n)$ and $Q^2(n)$, check for serial correlation of the return series and the squared returns up to the nth order, respectively.*,** and ***Significance levels of 10%, 5%, and 1%, respectively. MacKinnon's (1991) 1% critical value is−3.4352 for both the ADF and PP tests. The critical value for the KPSS test is 0.739 at the 1% significance level. Numbers in parentheses are P values

We also examine the null hypothesis of a white-noise process for sample returns using the Ljung-Box test (Ljung and Box, 1978) for both return and squared return series. As shown in Table 1, both return and squared return series provide the rejection of the null hypothesis of no serial correlation at the 1% significance level. This evidence indicates significant evidence of serial dependence in the cryptocurrency return and squared returns series. Therefore, the cryptocurrency return series has statistically significant first order autocorrelation, which can be removed by fitting an autoregressive AR(1) model to this series.

Otherwise, the Ljung-Box statistic for up to twentieth order serial correlation of squared return series is highly significant, suggesting the presence of strong nonlinear dependence in the

**Figure 1:** Time series plots of BitShares (BTS), MonaCoin (MONA) and Ripple (XRP)

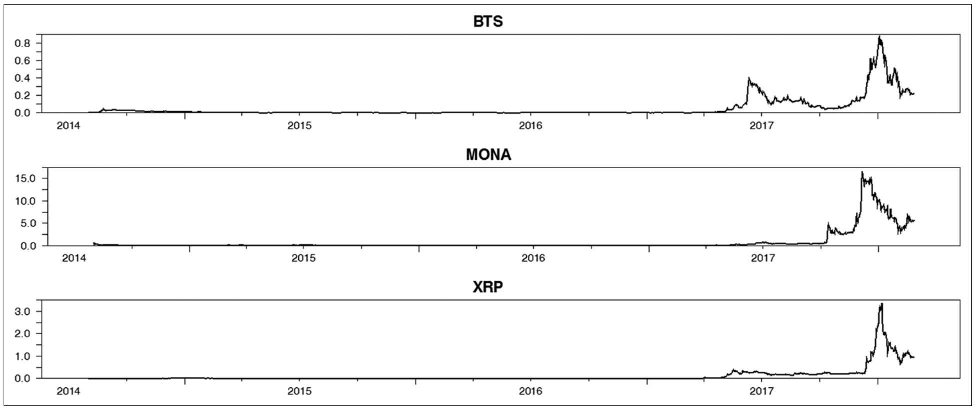
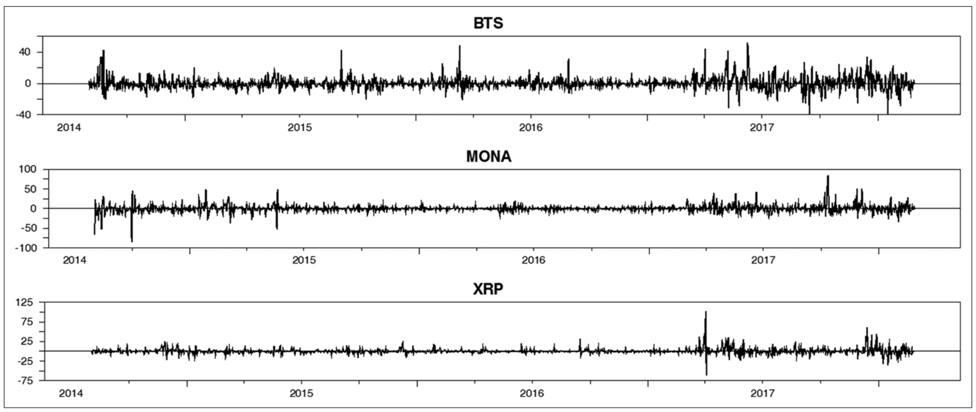



**Figure 2:** Daily returns of BTS, MONA, and XRP




data. Since nonlinear dependence and heavy-tailed unconditional distributions are characteristic of conditionally heteroskedastic data, the Lagrange Multiplier (LM) test (Engle, 1982) can be used to formally test the presence of conditional heteroskedasticity and the evidence of ARCH effects. The LM test for a first-order linear ARCH effect (Table 2) suggests that cryptocurrency return series exhibit ARCH effects, implying that nonlinearities must enter through the variance of the processes (Hsieh, 1989). Such behavior can be captured by incorporating autoregressive conditional heteroskedasticity (ARCH) or generalized autoregressive conditional heteroskedasticity (GARCH) structures in the model. This result is similar to that reported by Dyhrberg (2016a) and Katsiampa (2017) (Figure 1).

Table 1 also depicts the results of three types of unit root tests for the sample crytocurrency return series: Augmented Dickey-Fuller (ADF), Phillips-Perron (PP), and Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) tests. On the one hand, large negative values for the ADF and PP tests support the rejection of the null hypothesis of a unit root at the 1% significance level. On the other hand, the statistics of the KPSS test indicates that all cryptocurrencies returns series are insignificant to reject the null hypothesis of stationarity, implying that they are stationary processes, i.e. $I(0)$.

As shown in Table 2, unconditional correlations show that there is a positive correlation between all pairs of crypto currency return series. Figures 2 and 3 plots the time series graph of the squared daily returns and show how volatility has changed across time. Notice that all three graphs show pronounced volatility clustering.

**Table 2: Correlations between daily returns**

| | BTS | MONA | XRP |
|---|---|---|---|
| BTS | 1 | 0.1211 | 0.3641 |
| MONA | 0.1211 | 1 | 0.0704 |
| XRP | 0.3641 | 0.0704 | 1 |

**Table 3: Correlations between squared daily returns**

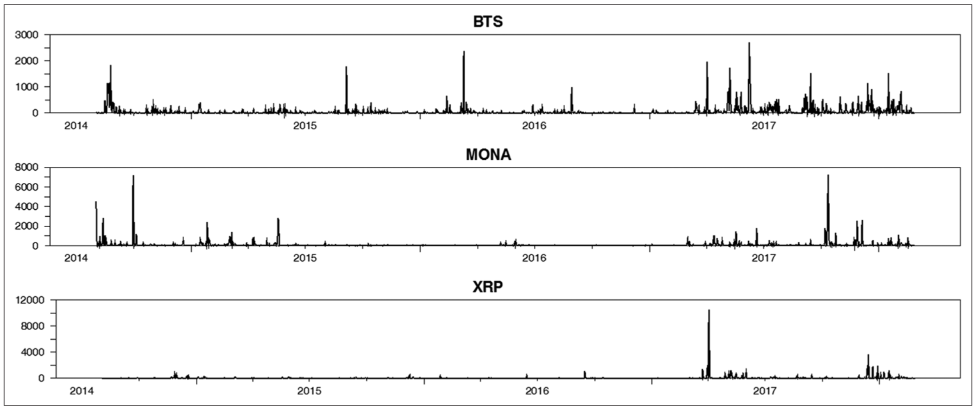
| | BTS | MONA | XRP |
|---|---|---|---|
| BTS | 1 | 0.0209 | 0.3317 |
| MONA | 0.0209 | 1 | 0.0003 |
| XRP | 0.3317 | 0.0003 | 1 |

In addition, the cryptocurrencies series show some big spikes in volatility in different periods. Otherwise, the unconditional correlations between the squared daily returns show a similar pattern as for the unconditional correlations between the returns (Table 3). Volatility clustering and cross-correlations in volatility are shown from the information presented in both Figure 3 and Table 3.

## 3. EMPIRICAL METHODOLOGY

Multivariate GARCH (MGARCH) models have been found to be very useful in studying volatility spillover effects in financial markets. In this paper, five multivariate GARCH models could be used to model the volatility dynamics as well as the volatility spillover effects between the cryptocurrencies prices. These include the VEC (Bollerslev et al., 1988), BEKK (Engle and

**Figure 3:** Squared daily returns of BTS, MONA, and XRP




Kroner, 1995), diagonal BEKK (Engle and Kroner, 1995), constant conditional correlation (Bollerslev, 1990), and DCC (Engle, 2002) models, respectively. While, popular, MGARCH models do have limitations. First, VEC model has a large number of free parameters, which makes it impractical for models with more than two variables (Bauwens et al., 2006). Second, diagonal VEC model lacks correlation between the variance terms (Bauwens et al., 2006). Third, BEKK model can have a poorly behaved likelihood function, making estimation difficult, especially for models with more than two variables (Bauwens et al., 2006).

In this paper, the BEKK model is used as a benchmark. The other models are computationally simpler and can be estimated in two steps. Univariate GARCH models are used to estimate the variances in the first step. In the second step, correlations are modeled based on the standardized residuals from step one.

Two components characterize the econometric specification used in this paper. A vector autoregression (VAR) with one lag is used to model the returns[1]. This allows for autocorrelations and cross-autocorrelations in the cryptocurrencies returns. The time-varying variances and covariances are modeled using a multivariate GARCH model. For the VEC, diagonal BEKK, constant conditional correlation and DCC models, the conditional variance is assumed to be VARMA-GARCH(1,1) process (Ling and McAleer, 2003)[2].

$$r_{i,t} = m_{i0} + \sum_{j=1}^{n} m_{ij} r_{j,t-1} + \varepsilon_{i,t} \tag{1}$$

$$\varepsilon_{i,t} | I_{i,t-1} \sim ST(0, h_{i,t}) \tag{2}$$

$$\varepsilon_{i,t} = z_{i,t} h_{i,t}^{1/2} \tag{3}$$

$$z_{i,t} \sim ST(0,1,v) \tag{4}$$

$$h_{i,t} = c_{ii} + \sum_{j=1}^{n} \alpha_{ij} \varepsilon_{j,t-1}^{2} + \sum_{j=1}^{n} \beta_{ij} h_{j,t-1} \tag{5}$$

Where $r_{i,t}$ denote the return for series $i$ ($\forall i=1,\dots,n$), $\varepsilon_{i,t}$ is the random error term with conditional variance $h_{i,t}$, the innovations $\{z_{it}\}$ follow student-$t$ distribution[3], and $I_{i,t-1}$ indicates the market information available at time ($t$–1). The relationship between the error term $\varepsilon_{i,t}$ and the conditional variance $h_{i,t}$ is specified by Eq. 3. A $GARCH$(1,1) process with $VARMA$ terms (Ling and McAleer, 2003) is specified through Eq. 5. Ling and McAleer (2003) model the conditional variances by allowing large shocks to one variable to affect the variances of the other variables. This is a suitable specification that allows for volatility spillovers.

For the VEC model, we follow Bollerslev et al. (1988) in specifying the multivariate extension of the $GARCH(p,q)$ model. To do so, we consider a system of $n$ regression equations; then the general form of a VEC model is given by the following expression:

$$h_t = vech(H_t) = C + \sum_{j=1}^{p} A_i vech\left(\varepsilon_{t-i}\varepsilon_{t-i}^{'}\right) + \sum_{j=1}^{p} B_j vech\left(H_{t-j}\right) \tag{6}$$

Where $C$ is an $\frac{n(n+1)}{2} \times 1$ vector; $A_i$ and $B_i$ are $\frac{n(n+1)}{2} \times \frac{n(n+1)}{2}$ matrices; and $vech(\bullet)$ is the column stacking operator of the lower portion of a symmetric matrix.

In the Eq. 6, $H_t = E\left(\varepsilon_{t-i}\varepsilon_{t-i}^{'} \mid I_{t-1}\right)$ is the $n \times n$ conditional variance matrix associated with the error vector $\varepsilon_t^{'}$, and $vech(H_t)$ denotes the $\frac{n(n+1)}{2} \times 1$ vector of all the unique elements of $H_t$ obtained by stacking the lower triangle of $H_t$ (Engle and Kroner, 1995).

In the VEC model, each element of the $H_t$ matrix depends on the lagged squared residuals and past variances of all variables in the model as in Eq. 6. The VEC model is very flexible, but it requires restrictive conditions for $H_t$ to be positive definite for all $t$, and the number of estimated parameters is large. For instance, the simplest bivariate model requires the estimation of twenty-one parameters.

1 In applied research, different criterion functions select different lag lengths for the VAR models. AIC chooses 2 lags, while both BIC and HQ choose 0 lag. A Preliminary regression analysis showed very little differences between a VAR with one lag compared to a VAR with two lags. Therefore, in the interest of parsimony, a VAR with one lag is chosen.

2 Recent examples of the VARMA-GARCH approach include Hammoudeh et al. (2009).

3 The student-t distribution is estimated with the parameter (v) which represents the number of degrees of freedom (df) and measures the degree of leptokurtosis displayed by the density (Fiorentini et al. (2003) for multivariate student-t density function). This distribution allows for modeling the excess leptokurtosis, which is not captured by the ARCH process (Filis et al., 2011; Chkili et al., 2012).

According to Bauwens et al. (2006), it is difficult to guarantee the positivity of $H_t$ in the VEC representation without imposing strong restrictions on the parameters. This is why Engle and Kroner (1995) propose a new parametrization for $H_t$ that easily imposes its positivity, i.e. the Baba, Engle, Kraft and Kroner (BEKK) model. The *BEKK(1,1,K)-GARCH* model is described as follows:

$$H_t = C^{*'}C^* + \sum_{k=1}^{K}\sum_{i=1}^{q} A_{ik}^{*'}\varepsilon_{t-i}\varepsilon_{t-i}^{'}A_{ik}^{*} + \sum_{k=1}^{K}\sum_{j=1}^{p} + \sum_{k=1}^{K}\sum_{j=1}^{p} B_{jk}^{*'}H_{t-j}B_{jk}^{*} \quad (7)$$

Where C* $A_k^*$ and $B_k^*$ are $n \times n$ matrices, but C* is upper triangular. The BEKK model is a special case of the VEC model. If C*' C* is positive definite, so is the $H_t$ matrix. For the bivariate case, the BEKK model requires the estimation of eleven parameters.

The generality of the BEKK process is determined by the summation limit *K*. Following Bauwens et al. (2006), the parameters of the BEKK model do not represent directly the impact of the different lagged terms on the elements of $H_t$, like in the VEC model. Note that the BEKK model is a special case of the VEC model. For instance, to avoid observationally equivalent structures, Engle and Kroner (1995) provide sufficient conditions to identify BEKK models with *K=1*. These conditions are that $A_{k,11}^*$, $B_{k,11}^*$ and the diagonal elements of *C** are restricted to be positive. The number of parameters in the *BEKK*(1,1,1) model is $\frac{n(5n+1)}{2}$. To reduce this number, and consequently to reduce the generality, a diagonal BEKK model is imposed (Engle and Kroner, 1995; Bauwens et al., 2006), i.e. $A_k^*$ and $B_k^*$ in Eq. 7 are diagonal matrices.

The constant conditional correlation (CCC) model has been proposed by Bollerslev (1990). This model is characterized by time-varying conditional variances and covariances, but conditional correlations are constant. The variances and covariances can be modeled separately using univariate models to allow different specifications. Based on these conditional variances, the conditional correlation matrix can subsequently be modeled. Assuming constant conditional correlations implies that the conditional covariances are proportional to the product of the corresponding conditional standard deviations, and this reduces the number of parameters to be estimated. The CCC model is defined as follows:

$$H_t = D_t R D_t = \left(\rho_{ij}\sqrt{h_{ii,t}h_{jj,t}}\right) \quad (8)$$

Where $D_t = diag\left(h_{11}^t,\ldots,h_{nn}^t\right)$, $h_{ii,t}$ can be any univariate GARCH process, and $R=(\rho_{ij})$is the constant correlation matrix, with $\rho_{ii}=1\forall i$, and $H_t$ is positive definite if all $n$ conditional variances are well defined and $R$ is positive definite.

The DCC model has been proposed by Engle (2002). It combines the flexibility of univariate GARCH models with parsimonious parametric models for the correlations. The DCC model is estimated in two steps based on the likelihood function. The GARCH parameters are estimated in the first step, while the correlations are estimated in the second step.

$$H_t = D_t R_t D_t \quad (9)$$

In Eq. 9, $H_t$ is the $n \times n$ conditional covariance matrix, $R_T$ is the conditional correlation matrix, and $D_t$ is a diagonal matrix with time varying standard deviations on the diagonal.

$$D_t = diag\left(h_{11,t}^{1/2},\ldots,h_{nn,t}^{1/2}\right) \quad (10)$$

$$R_t = D_t^{-1}H_tD_t^{-1} = diag\left(q_{11,t}^{-1/2},\ldots,q_{nn,t}^{-1/2}\right)Q_t diag\left(q_{11,t}^{-1/2},\ldots,q_{nn,t}^{-1/2}\right) \quad (11)$$

$$Q_t = \left(1-\theta_1-\theta_2\right)\bar{Q} + \theta_1 u_{t-1}u_{t-1}^{'} + \theta_2 Q_{t-1} \quad (12)$$

Where $Q_t=(q_{ij,t})$ is a $n \times n$ symmetric positive definite matrix, and $\bar{Q}$ is the $n \times n$ unconditional correlation matrix of the standardized residuals $u_{i,t}$. The parameters $\theta_1$ and $\theta_2$ are non-negative with a sum of less than unity. The correlation estimator is given by,

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t}q_{jj,t}}} \quad (13)$$

Relaxing the constraint of constant correlations is a very significant step forward, but it creates the difficulty that the time dependent conditional correlation has to be positive definite. The DCC model nests the *CCC* model as a special case since $R_t=R$ and $R_{ij}=\rho_{ij}$. Besides, $\rho_{ij}=0$ for all *i* and *j* in the diagonal MGARCH model. The diagonal case is very restrictive since it assumes that the DCCs between variables are all zero ($h_{ij}=0,\forall i\neq j$). An unconditional covariance matrix is computed by using the standardized residuals from the MGARCH diagonal model.

The MGARCH models are estimated by either the maximum likelihood estimation (MLE) or the Quasi- MLE (QMLE) using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm[4]. *T* statistics are computed using a robust estimate of the covariance matrix. For instance, the estimate of the DCC model is carried out using a two-step maximum likelihood of the probability density function of a bivariate Student-*t* distribution (Engle, 2002):

$$l_t(\grave{E}) = Log\left\{\left[\Gamma\left(\frac{\upsilon+\text{ȡ}}{2}\right)\upsilon\right]/\left[(\upsilon\pi)\Gamma\left(\frac{}{2}\right)(\tilde{o}-2)\right]\right\} - \frac{1}{2}Log\left(|H_t|\right) - \frac{1}{2}(\upsilon+2)Log\left[1+\left(\varepsilon_t^{'}H_t^{-1}\varepsilon_t\right)/(\upsilon-2)\right] \quad (14)$$

Where $\Gamma(.)$ denotes the Gama function, υ is the degree of freedom for the Student-*t* distribution, $H_t$ is a conditional variance-covariance matrix. Θ is a parameter vector with all of the coefficients of the DCC model.

4 All computations are carried out using WinRats 8.0.

## 4. EMPIRICAL RESULTS

In this section, we report the empirical results obtained from estimating multivariate GARCH models. Note that the BEKK model is used as the benchmark and compared to three restricted correlation models (diagonal BEKK, CCC, and DCC). Besides, the BEKK model is the most computationally intensive of the models studied.

### 4.1. Regression Results

Turning first to the VAR for the returns, one of the strongest effects is that a one period lag of XRP positively affects current period BTS (Table 4). The estimated coefficient of XRP in the BTS equation ($m_{13}$) is positive, of the same order of magnitude, and statistically significant at the 1% level for both CCC and DCC MGARCH models. This result is important in establishing a positive relationship between current period BTS returns and last period XRP returns. In other words, current period BTS returns are influenced by last period XRP returns. Besides, according to the statistical significance of the coefficient $m_{12}$ in the case of both CCC and DCC MGARCH models, current period BTS returns are positively influenced by last period MONA returns.

In addition, one period lag of XRP negatively affects current period MONA (Table 4). The estimated coefficient of XRP in the MONA equation ($m_{23}$) is negative, of the same order of magnitude, and statistically significant at the 1% level for both CCC and DCC MGARCH models. This finding indicates evidence of a negative association between current period MONA returns and last period XRP returns. In other words, current period MONA returns are influenced by last period XRP returns. Furthermore, according to the statistical significance of both coefficients $m_{31}$ and $m_{32}$, there is a significant negative linkage between XRP and BTS and between XRP and MONA.

Otherwise, the own conditional mean effects ($m_{ii}$) are clearly important in explaining conditional mean (Table 4). The coefficient $m_{11}$ refers to the mean term in the BTS equation, while $m_{22}$ refers to the mean term in the MONA equation and $m_{33}$ refers to the mean term in the XRP equations. Note that the estimated $m_{11}$ and $m_{22}$ coefficients on the own conditional mean effects are negative and statistically significant at the 1% level. However, the estimated $m_{33}$ coefficient is positive and statistically different from zero in the case of both CCC and DCC MGARCH models.

The own conditional GARCH effects ($\beta_{ii}$) measure the long-term persistence and are important in explaining the conditional volatility. In each of the considered MGARCH models, the estimated coefficients on the own conditional volatility effects are statistically significant at the 1% level. The coefficient $\beta_{11}$ stands for the GARCH term in the BTS equation, while $\beta_{22}$ and $\beta_{33}$ refer to the GARCH terms in the MONA and XRP equations, respectively. For a particular crypto currency $i$, the estimated coefficients for $\beta_{ii}$ are almost similar across the different MGARCH models. BTS shows the most amount of long-term persistence followed by XRP and MONA.

The own conditional ARCH effects ($\alpha_{ii}$) measure the short-term persistence, and are also important in explaining the conditional volatility. As shown in Table 4, for each $i$, the estimated $\alpha_{ii}$ values are smaller than their respective estimated $\beta_{ii}$ values in the case of both BEKK and CCC MGARCH models. This indicates that own volatility long-run (GARCH) persistence is larger than short-run (ARCH) persistence. However, the reverse effect is observed for both Diagonal-BEKK and DCC MGARCH models, stipulating that own volatility long-run persistence is smaller than short-run persistence.

Looking across the overall regression results of MGARCH models, the strongest evidence for volatility spillovers effects is found from the estimates of the DCC-MGARCH model. For short-term persistence, there is evidence of significant volatility spillovers between BTS and MONA (the terms $\alpha_{12}$ and $\alpha_{21}$), between BTS and XRP (the terms $\alpha_{13}$ and $\alpha_{31}$), and between MONA and XRP (the terms $\alpha_{23}$ and $\alpha_{32}$). Furthermore, there is evidence of statistically significant long-term persistence volatility spillovers between BTS and MONA (the terms $\beta_{12}$ and $\beta_{21}$), between BTS and XRP (the terms $\beta_{13}$ and $\beta_{31}$), and between MONA and XRP (the terms $\beta_{23}$ and $\beta_{32}$).

For the CCC-MGARCH model, the correlation between BTS and MONA ($\rho_{21}$), BTS and XRP ($\rho_{31}$) and MONA and XRP ($\rho_{32}$) are each positive and statistically significant at the 1% level. The highest correlation is between BTS and XRP and the second highest correlation is between BTS and MONA.

For the DCC-MGARCH model, the estimated coefficients on $\theta_1$ and $\theta_2$ are each positive and statistically significant at the 1% level. Note that these estimated coefficients sum to a value which is less than unity ($\theta_1+\theta_2<1$), meaning that the DCCs are mean reverting.

Otherwise, both the AIC and BIC criteria[5] show that the DCC-MGARCH model is the best model. Besides, the diagnostic tests for the standardized residuals and standardized residuals squared show no evidence of serial correlation in the squared standardized residuals at the 1% level in the case of DCC-MGARCH model (Table 5). However, the DCC-MGARCH model shows more evidence of autocorrelation in the standardized residuals. It is also worth noting that the AIC and SIC rank the CCC-MGARCH model as the second best. Based on the information criterion and residual diagnostic tests, the DCC-MGARCH model is chosen as the best of the models considered. Finally, the DCC-MGARCH model will be used to construct DCCs, optimal hedge ratios and portfolio weights.

### 4.2. DCCs Analysis

The time-varying conditional correlations from the DCC-MGARCH model are shown in Figure 4. Note that a pattern of volatility clustering is evident for each crypto currency series. Besides, the DCCs can vary a lot from the constant conditional correlations ($\rho_{21}$ = 0.0436, $\rho_{31}$ = 0.2259, and $\rho_{32}$ = 0.0359). This

5 The following equations are used to estimate the AIC (Akaike, 1974) and BIC (Stone, 1979) of a model:
$AIC$=(2×k)-(2×LogL),
$BIC$=(2×k×LogN)-(2×LogL)
Where $L$ denotes the value of the likelihood, $N$ is the number of recorded measurements, and $k$ is the number of estimated parameters.

**Table 4: MGARCH parameter estimates**

| Variables | BEKK | | | DIAGONAL-BEKK | | | CCC | | | DCC | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Coeff. | T-Stat. | Sig. | Coeff. | T-Stat. | Sig. | Coeff. | T-Stat. | Sig. | Coeff. | T-Stat. | Sig. |
| Mean equation | | | | | | | | | | | | |
| $m_{10}$ | −0.3768*** | −2.5789 | 0.0099 | −0.3949*** | −3.2730 | 0.0011 | −0.4238*** | −243.7590 | 0.0000 | −0.4621*** | −2684.4550 | 0.0000 |
| $m_{11}$ | −0.1016*** | −3.7884 | 0.0002 | −0.0632** | −2.2162 | 0.0267 | −0.0784*** | −98.3024 | 0.0000 | −0.0919*** | −19715.4579 | 0.0000 |
| $m_{12}$ | −0.0019 | −0.0955 | 0.9239 | 0.0051 | 0.4003 | 0.6889 | 0.0096*** | 93.3125 | 0.0000 | 0.0085*** | 626.1706 | 0.0000 |
| $m_{13}$ | 0.0281 | 0.7679 | 0.4426 | 0.0088 | 0.3211 | 0.7481 | 0.0409*** | 8.0972 | 0.0000 | 0.0230*** | 370.4172 | 0.0000 |
| $m_{20}$ | −0.1789 | −1.5209 | 0.1283 | −0.1832 | −1.5784 | 0.1145 | −0.1893 | −1.1088 | 0.2675 | −0.2181*** | −101.8280 | 0.0000 |
| $m_{21}$ | 0.0237 | 1.1317 | 0.2578 | 0.0034 | 0.1946 | 0.8457 | 0.0069 | 0.3351 | 0.7375 | 0.0078 | 0.8911 | 0.3729 |
| $m_{22}$ | −0.1346*** | −3.8433 | 0.0001 | −0.1133*** | −3.8527 | 0.0001 | −0.1302*** | −6.6085 | 0.0000 | −0.1386*** | −768.6280 | 0.0000 |
| $m_{23}$ | −0.0097 | −0.1791 | 0.8578 | −0.0136 | −0.7006 | 0.4836 | −0.0130*** | −20.9495 | 0.0000 | −0.0090*** | −121.5335 | 0.0000 |
| $m_{30}$ | −0.2633*** | −3.6273 | 0.0003 | −0.2666*** | −3.5896 | 0.0003 | −0.2903*** | −2.9845 | 0.0028 | −0.3067*** | −297.7718 | 0.0000 |
| $m_{31}$ | −0.0083 | −0.5486 | 0.5833 | −0.0099 | −0.8185 | 0.4131 | −0.0193 | −1.3039 | 0.1923 | −0.0147*** | −14.7475 | 0.0000 |
| $m_{32}$ | −0.0087 | −0.7868 | 0.4314 | −0.0068 | −0.6804 | 0.4963 | −0.0097 | −1.0024 | 0.3161 | −0.0095*** | −129.0152 | 0.0000 |
| $m_{33}$ | 0.0034 | 0.0960 | 0.9235 | 0.0186 | 0.6134 | 0.5396 | 0.0462*** | 110.9602 | 0.0000 | 0.0143*** | 420.0150 | 0.0000 |
| Variance equation | | | | | | | | | | | | |
| $c_{11}$ | 1.2849*** | 2.8528 | 0.0043 | 3.7368*** | 3.4625 | 0.0005 | 6.4635*** | 23.8897 | 0.0000 | 14.3126*** | 15927.9377 | 0.0000 |
| $c_{21}$ | 0.1779 | 0.1567 | 0.8755 | - | - | - | - | - | - | - | - | - |
| $c_{22}$ | 2.2725*** | 4.0909 | 0.0000 | 5.7710** | 2.1934 | 0.0283 | 4.3289*** | 6.5382 | 0.0000 | 18.0298*** | 46.3226 | 0.0000 |
| $c_{31}$ | −1.0018 | −0.8058 | 0.4204 | - | - | - | - | - | - | - | - | - |
| $c_{32}$ | −0.1546 | −0.1623 | 0.8711 | - | - | - | - | - | - | - | - | - |
| $c_{33}$ | 0.00004 | 0.0001 | 0.9999 | 2.0870 | 1.5102 | 0.1310 | 1.5051*** | 17.5564 | 0.0000 | 6.0706*** | 174.5940 | 0.0000 |
| $\alpha_{11}$ | 0.3601*** | 5.2404 | 0.0000 | 0.2880*** | 4.7807 | 0.0000 | 0.6351*** | 445.1332 | 0.0000 | 1.0332*** | 37118.5702 | 0.0000 |
| $\alpha_{12}$ | 0.0359 | 0.3433 | 0.7314 | 0.0443 | 1.4663 | 0.1426 | 0.1707*** | 5781.7221 | 0.0000 | 0.2237*** | 3846.5880 | 0.0000 |
| $\alpha_{13}$ | −0.0298 | −0.5206 | 0.6027 | 0.1355*** | 2.7173 | 0.0066 | 0.2173*** | 21.5275 | 0.0000 | 0.4681*** | 185.6119 | 0.0000 |
| $\alpha_{21}$ | 0.0117 | 0.1488 | 0.8817 | 0.0247 | 0.2789 | 0.7803 | −0.0424** | −2.0232 | 0.0431 | 0.1468*** | 3.4460 | 0.0006 |
| $\alpha_{22}$ | 0.5889*** | 3.9690 | 0.0001 | 0.6126** | 2.3826 | 0.0172 | 0.9776*** | 63.1603 | 0.0000 | 1.7692*** | 76.5655 | 0.0000 |
| $\alpha_{23}$ | 0.0176 | 0.4271 | 0.6693 | −0.0560 | −0.4071 | 0.6840 | 0.0892 | 1.3271 | 0.1845 | −0.4951*** | −16.7204 | 0.0000 |
| $\alpha_{31}$ | 0.0901 | 1.1224 | 0.2617 | −0.0300 | −0.9566 | 0.3388 | −0.1144*** | −9.0816 | 0.0000 | −0.0491*** | −17.6268 | 0.0000 |
| $\alpha_{32}$ | 0.0692 | 0.8213 | 0.4115 | −0.0105 | −0.3877 | 0.6982 | 0.0074 | 0.2855 | 0.7752 | 0.0190** | 2.0154 | 0.0439 |
| $\alpha_{33}$ | 0.5262 | 1.3361 | 0.1815 | 0.6263** | 2.0924 | 0.0364 | 0.7563*** | 71.5174 | 0.0000 | 1.6219*** | 892.4887 | 0.0000 |
| $\beta_{11}$ | 0.9668*** | 44.3561 | 0.0000 | 0.7432*** | 17.2578 | 0.0000 | 0.4484*** | 50.5320 | 0.0000 | 0.6614*** | 81599.1799 | 0.0000 |
| $\beta_{12}$ | −0.0172 | −0.7941 | 0.4271 | −2.4417** | −2.4736 | 0.0134 | 1.2427*** | 578.0063 | 0.0000 | 0.0174*** | 223.0396 | 0.0000 |
| $\beta_{13}$ | 0.1731*** | 2.8300 | 0.0047 | −1.1172** | −2.5738 | 0.0101 | 1.0737*** | 31.1151 | 0.0000 | −0.0836*** | −2553.0002 | 0.0000 |
| $\beta_{21}$ | 0.0158 | 0.2758 | 0.7827 | 45.5582 | 0.5235 | 0.6007 | 1.5355*** | 41.5847 | 0.0000 | −0.1074** | −2.0121 | 0.0442 |
| $\beta_{22}$ | 0.8457*** | 15.6592 | 0.0000 | 0.5862*** | 4.4787 | 0.0000 | 0.4650*** | 29.0132 | 0.0000 | 0.4774*** | 111.9103 | 0.0000 |
| $\beta_{23}$ | 0.1194*** | 2.9033 | 0.0037 | 158.3244 | 0.9150 | 0.3602 | 3.2497*** | 7.9902 | 0.0000 | 0.5993*** | 8.9624 | 0.0000 |
| $\beta_{31}$ | −0.3243** | −2.0660 | 0.0388 | −0.1105 | −0.0978 | 0.9221 | 0.4652*** | 107.5177 | 0.0000 | 0.1185*** | 44.0442 | 0.0000 |
| $\beta_{32}$ | 0.0672 | 0.3030 | 0.7619 | −0.4981 | −0.0975 | 0.9223 | 0.0040 | 0.1421 | 0.8870 | −0.0465*** | −3.5892 | 0.0003 |
| $\beta_{33}$ | −0.9136*** | −5.5230 | 0.0000 | 0.5833*** | 3.4709 | 0.0005 | 0.5157*** | 35.8198 | 0.0000 | 0.4835*** | 703.4087 | 0.0000 |
| $\rho_{21}$ | - | - | - | - | - | - | 0.0436*** | 39.2686 | 0.0000 | - | - | - |
| $\rho_{31}$ | - | - | - | - | - | - | 0.2259*** | 40.1839 | 0.0000 | - | - | - |
| $\rho_{32}$ | - | - | - | - | - | - | 0.0359*** | 19.8224 | 0.0000 | - | - | - |
| $\theta_1$ | - | - | - | - | - | - | - | - | - | 0.0915*** | 6037.8544 | 0.0000 |
| $\theta_2$ | - | - | - | - | - | - | - | - | - | 0.8287*** | 13380.6392 | 0.0000 |
| $\upsilon$ | 2.9519*** | 19.2916 | 0.0000 | 3.0240*** | 17.9571 | 0.0000 | 2.5781*** | 133.9916 | 0.0000 | 2.3158*** | 124.6599 | 0.0000 |
| *LogL* | −12191.8745 | | | −12163.9857 | | | −12120.2825 | | | −12105.1113 | | |
| AIC | 24457.7490 | | | 24395.9714 | | | 24314.5650 | | | 24282.2226 | | |
| BIC | 24649.1855 | | | 24571.8860 | | | 24506.0015 | | | 24468.4851 | | |

***, ** and *Denote statistical significance at the 1%, 5% and 10% levels, respectively

**Figure 4:** Time-varying conditional from DCC model

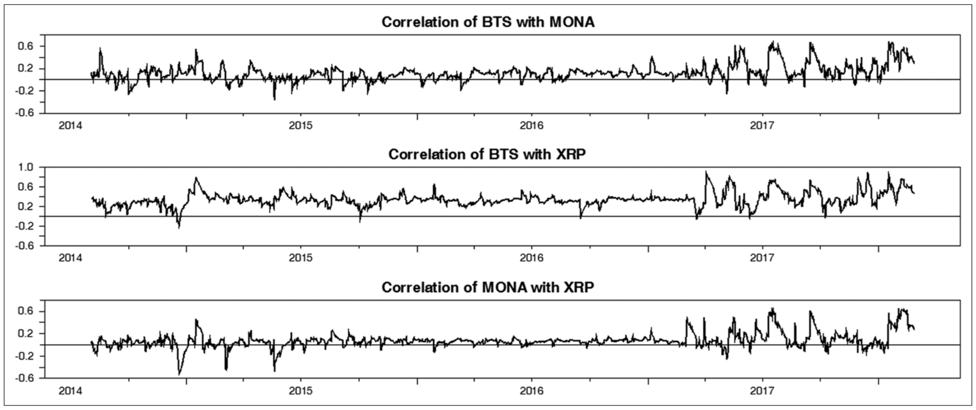
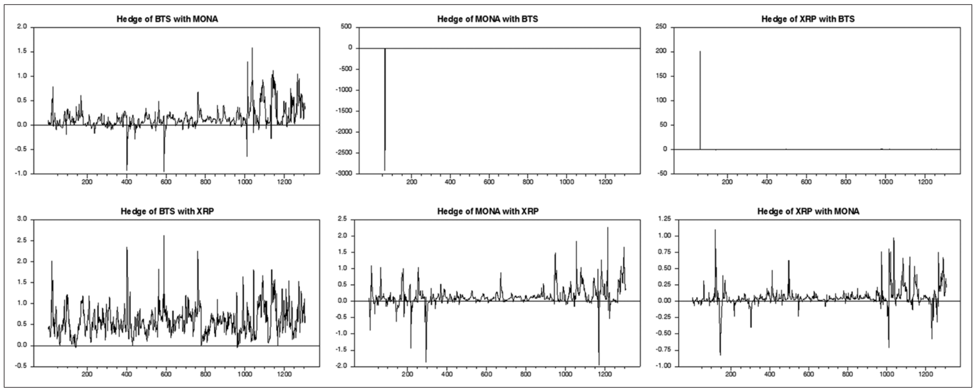



**Figure 5:** Time-varying hedge ratios computed from DCC model




emphasizes the need to calculate DCCs. Notice that the DCCs between BTS and XRP are all positive and generally larger than 0.5. This indicates that there is little scope for portfolio diversification between these two crypto currency time series. The DCCs between BTS and MONA do alternate in sign and cover a range of values between −0.5 and 0.60. These periods of negative correlation provide an opportunity for meaningful portfolio diversification. The time-varying conditional correlations between MONA and XRP show a similar pattern to that of BTS and MONA.

The DCCs between BTS and MONA reached low values around March 2014, October 2014 and October 2016. These DCCs surpass the 0.6 value for the first time in July of 2014. By September of 2016, the DCCs are back above 0.6.

The DCC between BTS and XRP reach low values around June 2014. The DCC between BTS and XRP surpass the 0.6 value for the first time in July of 2014 before lessening somewhat. By September of 2016, the DCCs are back above 0.6.

The DCC between MONA and XRP reach low values around June 2014, August 2014, and October 2014. The DCC between MONA and XRP surpass the 0.4 value for the first time around September of 2016. These findings show that for each pair of series, DCCs reached their highest values in the fall of 2016. Besides, as shown in Figure 4, the time series plots indicate that, for each pair of series, the DCCs provide much more useful information than do the correlations from the CCC-MGARCH model. Notice also that the DCCs were, for each pair of series, much larger than their corresponding values from the constant conditional correlations. This result illustrates that any computations done with the correlations from the CCC-MGARCH model would have been very miss-leading.

## 5. IMPLICATIONS FOR PORTFOLIO DESIGNS AND HEDGING STRATEGIES

In this section, we use the estimates from the best multivariate GARCH model for portfolio design and hedging strategies.

### 5.1. Hedge Ratios

Following Kroner and Sultan (1993), the conditional volatility estimates can be used to construct hedge ratios. A long position in one asset $i$ can be hedged with a short position in a second asset $j$. The hedge ratio between asset $i$ and asset $j$ is given by

$$\beta_{ij,t} = \frac{h_{ij,t}}{h_{jj,t}} \tag{15}$$

For most of the hedge ratios, computed from the DCC model, the graphs (Figure 5) show a considerable variability. For BTS/XRP, BTS/MONA and MONA/XRP many of the hedge ratios, the maximum values were recorded at the end of the sample period. The exceptions are the XRP/MONA, XRP/BTS, MONA/BTS and hedges where the largest values for these hedge ratios were recorded near the beginning of the sample period.

**Table 5: Diagnostic tests for standardized residuals**

| Model | BEKK | | | Diagonal BEKK | | | CCC | | | DCC | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | BTS | MONA | XRP | BTS | MONA | XRP | BTS | MONA | XRP | BTS | MONA | XRP |
| Q (20)_r | 52.6851*** | 39.5667*** | 33.3714** | 42.5231*** | 34.8636** | 30.6297* | 40.0174*** | 33.5935** | 28.1930 | 50.2509*** | 36.1406** | 35.6599** |
| | 0.0001 | 0.0057 | 0.0307 | 0.0024 | 0.0208 | 0.0603 | 0.0050 | 0.0290 | 0.1049 | 0.0002 | 0.0148 | 0.0169 |
| Q (20)_r2 | 8.5305 | 2.2770 | 2.9783 | 7.2426 | 2.1860 | 3.1977 | 7.5307 | 3.9570 | 3.0345 | 7.6134 | 2.2979 | 4.8216 |
| | 0.9877 | 1.0000 | 1.0000 | 0.9958 | 1.0000 | 1.0000 | 0.9946 | 1.0000 | 1.0000 | 0.9941 | 1.0000 | 0.9998 |

***, ** and *Statistical significance at the 1%, 5% and 10% levels, respectively. Numbers in parentheses are *P* values

**Table 6: Hedge ratio (long/short) summary statistics**

| Variables | Mean±SD | Min. | Max. |
|---|---|---|---|
| BTS/MONA | 0.15±0.21 | −0.94 | 1.58 |
| BTS/XRP | 0.59±0.34 | −0.06 | 2.62 |
| MONA/BTS | −2.12±80.82 | −2919.34 | 1.21 |
| MONA/XRP | 0.14±0.28 | −1.96 | 2.27 |
| XRP/BTS | 0.42±5.57 | −0.48 | 2.01 |
| XRP/MONA | 0.06±0.14 | −0.83 | 1.10 |

SD: Standard deviation

**Table 7: Portfolio weights summary statistics**

| Variables | Mean±SD | Min. | Max. |
|---|---|---|---|
| BTS/MONA | 0.48±0.26 | 0.00 | 1.00 |
| BTS/XRP | 0.27±0.27 | 0.00 | 1.00 |
| MONA/XRP | 0.33±0.25 | 0.00 | 1.00 |

SD: Standard deviation

The average value of the hedge ratio between BTS and MONA is 0.15 while the average value of the hedge ratio between BTS and XRP is 0.59 (Table 6). The average value of the hedge ratio between MONA and XRP is 0.14. These findings are important and show that a \$1 long position in BTS can be hedged for 15% with a short position in MONA. A \$1 long position in MONA can be hedged for 14% with a short position in XRP. As shown from the previous DCC analysis, it is not, however, useful to hedge BTS with a short position in XRP. The cheapest hedge is long XRP and short MONA. The most expensive hedge is long BTS and short XRP. Notice that all the hedge ratios record maximum values in excess of unity.

## 5.2. Portfolio Weights

Following Kroner and Ng (1998), the conditional volatilities from MGARCH models can be used to construct optimal portfolio weights.

$$w_{ij,t} = \frac{h_{jj,t} - h_{ij,t}}{h_{ii,t} - 2h_{ij,t} + h_{jj,t}} \quad (16)$$

$$w_{ij,t} = \begin{cases} 0 \;\; if \; w_{ij,t} < 0 \\ w_{ij,t} \;\; if \; 0 \le w_{ij,t} \le 1 \\ 1 \;\; if \; w_{ij,t} > 1 \end{cases} \quad (17)$$

Where $w_{ij,t}$ denotes the portfolio weights between two assets. It represents the weight of the first asset in a one dollar portfolio of two assets (asset *i*, asset *j*) at time *t*. $h_{ij,t}$ stands for the conditional covariance between assets *i* and *j*, while $h_{jj,t}$ denotes the conditional variance of asset *j*. In addition, the weight of the second asset is 1-$w_{ij,t}$.

Table 7 reports the summary statistics for portfolio weights computed from the DCC-MGARCH model. The average weight for the BTS/MONA portfolio is 0.48. This result indicates that for a \$1 portfolio, 48% should be invested in BTS and 52% invested in MONA. Besides, the average weight for the BTS/XRP portfolio indicates that 27% should be invested in BTS and 73% invested in XRP. Furthermore, the average weight for the MONA/XRP portfolio indicates that 33% should be invested in MONA and 67% invested in XRP.

## 6. CONCLUSIONS

Since the amount of money invested in the cryptocurrency sectors grows, it is important to have a better understanding of the volatility dynamics of the cryptocurrency prices. This study examines own volatility, shocks and inter-shock and volatility transmissions in three biggest cryptocurrencies. It uses multivariate GARCH models to investigate correlations and the volatility spillovers between cryptocurrency prices. Empirical findings show that the DCC MGARCH model is preferred over the other models, although the CCC-MGARCH is a close second in model choice to the DCC-MGARCH model. For each pair of cryptocurrency series, the DCCs vary considerably from their respective constant conditional correlations.

The conditional volatilities from the DCC-MGARCH model can be used to estimate dynamic hedge ratios. On average, a \$1 long position in BTS can be hedged for 15% with a short position in MONA. On average, a \$1 long position in MONA can be hedged for 14% with a short position in XRP. It is not, however, useful to hedge an investment in BTS with a short position in XRP. The cheapest hedge is long XRP and short MONA, while the most expensive hedge is long BTS and short XRP.

Finally, the conditional variances and covariances from the DCC model can be used to construct optimal two cryptocurrency portfolios. The average weight for the BTS/MONA portfolio is 0.48, indicating that for a \$1 portfolio, 48% should be invested in BTS and 52% invested in MONA. The average weight for the BTS/XRP portfolio indicates that 27% should be invested in BTS and 73% invested in XRP. Furthermore, the average weight for the MONA/XRP portfolio indicates that 33% should be invested in MONA and 67% invested in XRP.

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