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Debt Level and the Firm Levered Cost of Capital

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ABSTRACT

The cost of capital is one of the most relevant variables in the firm's valuation models. The well-known models to estimate the cost of capital are based on a defined debt level. Therefore, they can be used only if the debt level is known and constant in the valuation period; consequently, the debt level cannot be defined based on its effects on the cost of capital. The firm levered cost of capital (FLCC) proposed is a theoretical model structured on the linkage between debt level and the cost of debt through the definition of a non-linear function able to consider debt benefits and costs. Based on FLCC, the firm's debt level can be defined every time based on its effects on the cost of capital, and then on the firm's value. In this sense, the FLCC can be considered a theoretical model with a normative function.

Keywords: Capital Structure, Leverage, Cost of Capital, Firm Value, Discounted Cash Flow
JEL Classifications: E22, G32

1. INTRODUCTION

The cost of the capital has a central role in the valuation models of the firm and especially for ones based on the financial approach. Despite a wide experiences approach in both academic and practice, it should not surprise that the method to estimate the cost of capital is still under an intensive discussion.

The theories of the Modigliani and Miller (MM) (1958; 1963; 1977) are considered the starting point of the modern theory of the capital structure.

The MM's basic idea (1958) is that the firm's value is function of its business operating activities only, and the capital structure defines the way by which this value is distributed between the investors both in equity and debt. Thus, the capital structure choices are irrelevant both on the firm's value and on its cost of capital (Propositions I and II).

It is worth noting that the MM's propositions are complied with the law of the conservation of investment value (Williams, 1938): In the absence of corporate taxes, the firm's value is function to the operating cash-flows only; the way by which these cash-flows are distributed among different investors is irrelevant in the firm's value perspective. Therefore, the firm's asset value cannot be influenced by the composition of the capital structure in term of equity and debt amounts. Based on this arguments, are derived the rules of the cost of capital. If the value of assets is independent by the capital structure choices, the discount rate of the operating cash-flows cannot be influenced by the relationship between equity and debt.

In a second time (MM, 1963), by introducing the corporate taxes and considering the positive effects arising from the tax savings due to the interest on debt deducibility and does not considering the negative effects of debt, MM argue that the firm levered value is equal to the firm unlevered value plus the value of tax shields.

The most widely models to estimate the cost of capital is derived from the MM's approach. Among them, two are the most well-known models: The weighted average cost of capital (WACC) and Milles and Ezzel (ME) (1980) cost of capital model.

The WACC is probably the most well-known model to estimate the cost of capital. In the textbook it is usually used to discount the expected operating cash flows of the firm.

The WACC can be considered closely derivation of the MM's propositions. Thus, it is independent to the firm's capital structure and it measures only the return required by the market on the referring to the risk class of the firm. Indeed, the firm's value is function of the business and not the way by which it is financed. The relationship between the cost of capital and the leverage is assumed linear for each debt level; also, the debt level stock is assumed constant.

In the WACC's approach, the cost of equity cannot be estimated until the definition of the leverage ratio that, in turn, requires the estimation of the firm levered value that it is the aim of the analysis in which the WACC is used as discount rate. Consequently, the WACC can be used only if the debt level is an exogenous variable and, thus, it is defined and well known (Harris and Prigle, 1985).

The Miles and Ezzell (ME) (1980) cost of capital model, as well as the WACC, it is derived from the MM's approach. Thus, the market value of any levered cash flows stream is equals to the market value of the unlevered cash flows plus the market value of the tax shields. The ME's model uses the unlevered cost of capital to discount the expected unlevered cash flows and for the first period only, the cost of debt to discount tax savings.

The ME's model tries to overcome the limits of the WACC by assuming a constant debt level as function of the firm's value. Thus, the debt level changes on the basis of the changes of the firm levered value. The main limit of the model is that if the leverage ratio is held constant, the amount of the tax savings at any time different from the first (in which the debt is known) depends upon the firm levered value that depends upon the value of the tax shields that, in turn, depend upon the debt level.

Finally, it is worth noting that the adjusted present value (APV) approach to firm's valuation (Myers, 1974) is characterized by the same problems of the WACC and ME's model. The APV's model valuates the tax benefits separately to the firm unlevered value. Thus, the firm levered value is equal to the sum of its unlevered value and the value of the tax shields. But also in this case, the debt level must be known in order to estimate the value of the tax shields. It requires an explicit valuation of the tax benefits that, in turn, requires the knowledge of the debt level. Consequently, the APV's model is neutral to the firm's financing policy and it can be used only if the debt level is an exogenous variable and, thus, it is defined. In this sense, also the APV's model can be considered as normative implication of the MM's approach.

The main problem of these models, as well as all other cost of capital models based on the MM's approach, is that they require a constant and well-known debt level. Then, they can be used only if the debt level is an exogenous variable, and as such defined.

The firm levered cost of capital (FLCC) proposed in this paper, is a theoretical model that tries to overcome the limit of the WACC and the ME's model, by defining the relationship, based on a exponential function, between the debt level, the cost of debt and the FLCC. This relationship is defined on the basis of the trade-off approach by considering the effects, both positive (mainly with regard to the tax shields) and negative (mainly with regard to the default risk), of the debt level on the cost of debt and thus on the levered cost of capital. Consequently, it can be used to estimate the effects of the debt level changes on the levered cost of capital and thus on the firm levered value.

The paper is structured as following: The Part II defines the relationship between the MM's Propositions and the WACC and the ME's model; the Part III defines analytically the FLCC; the Part IV presents the conclusion in which are highlighted the FLCC application and limits.

2. THE MM'S PROPOSITIONS AND THE FIRM COST OF CAPITAL

To well analysed the FLCC proposed, it is useful clarify the relationship between the MM's Propositions, the WACC and ME's model.

The Propositions I and II (1958, 1963), are based on strong and restrictive assumptions. Among these, a key role is assumed by the assumption based on which firms can be divided into "equivalent return classes": The return on the stocks issued by any firm in the same class is proportional to the return on the shares issued by any other firm in the same class; the returns on the shares as perfectly correlated between them. Thus, the shares within the same class differ between them of a scale-factor at most. Consequently, if it is considered the ratio between the return and the expected return, the probability distribution of the ratio is the same for all shares within the same class. Then, only two are the relevant properties of a share: First, the referenced class; second, its expected return. This assumption allows classifying the firms in homogeneous classes of stock.

In equilibrium condition the price of every stocks in any given class must be proportional to its expected return. This "proportionality factor" for any class is the same for all firms within the class. Formally:

$$p_j = \frac{1}{p_k} x_j \rightarrow p_k = \frac{x_i}{p_j} \tag{1}$$

Where: $1/p_k$, is the proportionality factor for any k^{th} class. Specifically, p_k is a constant for all j-firms in the same k-class and it is one for each of the k-classes; p_j, is the price per share of the j^{th} firm in the k^{th} class; x_j, is the expected return per share of the j^{th} firm in the k^{th} class.

Therefore in the same class, the price per share of the j^{th} firm (p_j) is proportional to its expected return (x_j) based on the proportionality factor $\left(\frac{1}{p_k}\right)$ that is the same for all firm within the same k^{th} class. Based on the equation, p_k, can be interpreted as the price that an investor has to pay for a dollar's worth of expected return in the k^{th} class. In equivalent terms, p_k can be regarded as the market rate of capitalization for the expected value of uncertain streams of the kind generated by the k^{th} class of firms. In other words, p_k can be interpreted as the expected rate of return of any share in the k^{th} class.

Based on this strong assumption, and assuming no corporate taxes, the Proposition I (1958) argues that: The market value of any firm is independent of its capital structure and is given by capitalizing its expected return at the rate (p_k) appropriate to its class. Equivalently, the average cost of capital to any firm is completely independent of its capital structure and is equal to the capitalization rate of a pure equity stream of its class. Formally:

$$W_j \equiv \left(E_j + D_j\right) = \frac{X_j}{p_k} \quad \rightarrow \; p_k = \frac{X_j}{\left(E_j + D_j\right)} \equiv$$

$$\frac{X_j}{W_j} \quad \Rightarrow W_j \equiv W_L = W_U \tag{2}$$

Where: j, refers to the j^th^ firm; W_j, is the market value of the j-firm; W_L, is the firm levered value; W_U, is the firm unlevered value; X_j, is the expected return of the j-firm's assets and thus the expected return of income; D_j, is the market value of the debt of the j-firm; E_j, is the marker value of the equity of the j-firm; p_k, is the expected rate of return of the stock of the k^th^ class of the j-firm included. It is constant for all firms within the same k^th^ class and it is one for each k^th^ class; X_j/W_j, is the "average cost of capital." It is constant for all firms in the same k^th^ class, and thus it is independent from the capital structure of the firm.

The Proposition II (1958), as direct derivation of the Proposition I, and argues that: The expected yield of a share of stock is equal to the appropriate capitalization rate (pk) for a pure equity stream in the class, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between p_k and r. Formally:

$$i_j = p_k + \left(p_k - r_D\right)\frac{D_j}{E_j} \tag{3}$$

Where: i_j, is the expected rate of return (or expected yield) on the j^th^ firm's stocks (the after-tax yield on equity capital) belonging to the k^th^ class; r_D, is the rate return of debt.

In the same paper (1958), MM introduced the corporate taxes. The conclusions they reached in this paper will be reviewed and modified in the subsequent paper (1963). In this context are considered only the "correction" (1963).

By introducing the corporate taxes, the Proposition I (1963) argues that: With corporate tax rate, the value of levered firm is equal to sum of unlevered firm plus the value of the tax shields due to the interest on debt deducibility.

Therefore, the value of levered firm size $\bar{X}$ with a permanent level of debt (D_L) in the capital structure, is equal to:

$$W_L = \frac{\left(1-t\right)\bar{X}}{p_t} + \frac{tR}{r} = W_U + tD_L \quad \leftrightarrow \; W_L = W_U + W_{TS} \tag{4}$$

Where: W_L, is the firm levered value; W_U, is the firm unlevered value; W_{TS}, is the value of the tax shields; $\bar{X}$, is the expected value of the operating income; R, is the amount of interest on debt; t, is the marginal corporate income tax rate (assumed equal to the average); p_t, is the rate at which the market capitalizes the expected net returns of an unlevered firm of size $\bar{X}$ within k^th^ class.; r, is the rate at which the market capitalizes the sure streams generated by tax savings on interest payments; D_L is the permanent level of debt in the capital structure.

It is worth to note that $r<p_t$ by construction, because the extra after tax earnings (tR) is a sure income while the expected after tax earnings $\left(\left(1-t\right)\bar{X}\right)$ is uncertain.

Based on the redefinition of the Proposition I, the redefined Proposition II (1963) argues that with corporate taxes other than zero, the cost of equity is positively correlated to the degree of leverage.

Formally:

$$i_E = p_t + \left(1-t\right)\left(p_t - r\right)\frac{D}{W_U} \tag{5}$$

Where i_E, is the after-tax yield on equity capital.

The equation shows an increase in the after tax yield on equity capital (i_E) as leverage increases which is smaller than the original version of the Proposition II by a factor of (1−t). However, the linear increasing relation of this equation is fundamentally different from the original: While in the original version the cost of equity is completely independent from the leverage, in this case it is dependent from the leverage.

The WACC is the most widely model to estimate the firm cost of capital. The WACC is equal to the sum of the cost of equity levered (K_{EL}) and the cost of debt (K_D) weighted based on the weights of equity $\left(\frac{E}{E+D}\right)$ and debt $\left(\frac{D}{E+D}\right)$ respectively in the capital structure.

By assuming no corporate taxes, and by denoting the WACC with K_A, it gets:

$$WACC \equiv K_A = K_{EL}\left(\frac{E}{E+D}\right) + K_D\left(\frac{D}{E+D}\right) \tag{6}$$

And by introducing the corporate taxes (t_c), the WACC can be rewritten as following:

$$WACC \equiv K_A = K_{EL}\left(\frac{E}{E+D}\right) + K_D\left(\frac{D}{E+D}\right)\left(1-t_c\right) \tag{7}$$

The WACC is a strictly derived by the MM's Propositions. The value of the firm is function of the value of its assets only. The firm's value is completely independent from the firm's capital structure and it is function of the value of its assets estimates based on their expected operating cash flows discounted at a risk rate own of the k^th^ class within which the firm is placed. Thus, this risk rate is completely independent from the capital structure.

Therefore, the value of the assets, and thus the value of the firm, is constant in respect to the capital structure. Consequently, the WACC measures the expected return from the investors based on the risk assumed in the capital markets as defined by the risk rate of the k-class of risk in which the firm in considered, as argued by the MM's approach. Then, K_A is constant because it is independent to the firm's capital structure and it measures the expected return of investors for expected operating cash flows based on the risk k-class of the firm.

It is worth noting, that if the firm is financed by equity only, the WACC is equal to the cost of equity of unlevered firm and thus the unlevered cost of equity (K_{EU}) so that WACC≡K_A≡K_{EU} (Massari and Zanetti, 2008).

By solving the equations [6] and [7] for the cost of equity, it gets:

$$K_{EL}=K_A+(K_A-K_D)\left(\frac{D}{E}\right) \quad (8)$$

And

$$K^t_{EL}=K^t_A+\left[K^t_A-K^t_D(1-t_c)\right]\frac{D}{E} \quad (9)$$

The equations [8] and [9], define the levered cost of equity of the firm as direct derivation of the MM's Propositions I and II.

A relevant problem concerns the rate used to discount the expected tax savings. Usually, the tax savings are discounted to the cost of debt as in the MM's Propositions. The main reason is that the risk of the expected tax savings is strictly related to the risk of debt. Tax savings arising from debt. Therefore, the firm does not realize tax savings if it is unable to face debt obligations. Consequently, it is possible to discount the expected tax savings by the cost of debt (K_D). Several studies are placed in a critical way and highlight the need to use the unlevered cost of equity (K_{EU}) to discount the expected tax savings (Miles and Ezzell, 1980; Taggart, 1991; Kaplan and Ruback, 1995; Ruback, 2002). The main argument is that the debt level is known in the first year only. Thus, only in this case, it is possible to use the cost of debt to discount the expected tax savings. In each year different from the first, the debt level is unknown because the firm levered value is unknown; it is function of the tax savings that, in turn, are function of the debt level that, unfortunately, is unknown. Consequently, the risk of the expected tax savings is similar to the risk on firm's assets value, and thus they must be discounted to the unlevered cost of equity (K_{EU}).

The Miles and Ezzell (ME) (1980) cost of capital model tried to solve some problems about WACC. As well as, the WACC also ME's model is based on the MM's approach.

ME's model used the unlevered cost of capital to discount the expected unlevered cash flows and, differently from MM, the cost of debt to discount expected tax savings for the first year only.

The ME's cost of capital is based on a different assumption of the WACC. It assumes a defined leverage target of the firm must be kept constantly over time through the continuous rebalancing of the debt level as function of the firm value. The leverage ratio (L) is defined based on the firm levered value (W_L), as following:

$$D_{i-1}=\overline{\overline{L}}.W_{L_{i-1}} \quad \rightarrow \quad \overline{\overline{L}}=\frac{D_{i-1}}{W_{L_{i-1}}} \quad (10)$$

By assuming a constant leverage ratio $\left(\overline{\overline{L}}\right)$, the debt level must be changed on the basis of the firm levered value changes, as following:

$$\overline{\overline{L}} \rightarrow \begin{cases} \text{if } W_L\uparrow \rightarrow D\downarrow \\ \text{if } W_L\downarrow \rightarrow D\uparrow \end{cases} \quad (11)$$

Based on this assumption and a strictly argumentation, ME's model defines the cost of capital of the firm (K) as following:

$$K=K_{EU}-K_D t_c \overline{\overline{L}}\left(\frac{1+K_{EU}}{1+K_D}\right) \quad (12)$$

The equation [12] shows that if the cost of unlevered equity of the firm (K_{EU}) is independent from the amount and the time of the expected unlevered cash flows realization, than the rate to discount the levered cash flows is independent from these. Thus, the cost of capital (K) is function to the unlevered cost of equity (K_{EU}) (and thus the unleverd cost of capital (K_U) is that K_{EU}≡K_U), the cost of debt (K_D), the corporate tax rate (t_c), and the constant leverage ratio $\left(\overline{\overline{L}}\right)$.

The ME's model is strictly function of the MM's approach. Indeed, from the equation [12] it is possible always to derive the WACC.

The WACC, as well as the ME cost of capital, are derived from MM's Propositions approach and thus both are characterized by strong assumptions about debt level. They can be used only if the debt level is constant and well known, and thus they are not useful to evaluate the leverage effects on the cost of capital.

In the WACC perspectives, the cost of equity cannot be estimated until the definition of the leverage ratio that, in turn, it requires the definition of the firm levered value that is the aim of the analysis in which the WACC is used. Then the WACC can be used only if the debt level is an exogenous variable and, thus, it is defined (Harris and Prigle, 1985).

In the ME's cost of capital, the leverage is assumed constant as function of the firm levered value. The firm maintains a constant leverage ratio by adjusting the debt level in the capital structure with the aim to realize the firm levered value defined. But if leverage ratio is held constant, the amount of the tax savings at any time different from the first (in which the debt is known) depends upon the firm levered value that depends upon the value of the tax savings that, in turn, depends upon the debt level.

3. THE FLCC

The FLCC proposed is a theoretical model that tries to overcome the limits of the WACC and the ME's cost of capital.

The FLCC is defined on the basis of the estimation of the cost of equity and the cost of debt of the firm. Thus, it is function of the capital structure of the firm and it measures the expected return of firm's investors in equity and in debt.

The FLCC is based on the relationship between the debt level and the cost of debt. This relationship is defined on the basis of the trade-off approach by considering the effects, both positive (mainly with regard to the tax shields) and negative (mainly with regard to the default risk), of the debt level on the cost of debt and thus on the levered cost of capital. Specifically, it is estimated on the basis of an exponential function between risk and debt level: The increase of leverage, on the one hand, increases the benefits due to the tax shields by reducing the FLCC but, on the other hand, it increases the default risk probability. Consequently, on the basis of these two effects, FLCC draws a curve with a minimum point identifies the debt level that minimized the cost of capital.

Therefore, the FLCC can be used to define the debt level of the firm by measuring its effects on the cost of capital: By changing debt level, changes the cost of debt and, thus, the cost of capital along the curve.

In this context, the general baseline assumptions are the following:
1. The capital structure is based on two capital sources: Equity and debt. Hybrid forms are not considered. Also, it is assumed that firm uses a single class of equity and debt;
2. The cost of capital measures the cost of sources invested in the firm. Thus, it can be interpreted as the expected return of investors: The expected return by investors in equity is the cost of equity for the firm, as well as, the expected return by investors in debt is the cost of debt for the firm;
3. The investor is diversified;
4. It is considered a one-period time with no events between the start and the end of the period. In each time firm defines its debt level;
5. There is a single class of zero coupon risky debt of maturity;
6. The cost of debt is defined based on the debt level at the start of the period;
7. The investors personal taxes on debt and equity are not considered;
8. The share of debt paid at the end of the year is based on the debt level from the start of the year.

Based on the portfolio theory (Markowitz, 1952) the cost of equity can be estimated by using the capital asset pricing model (Sharpe, 1964; Lintner, 1965; Mossin, 1966), as following:

$$K_E = R_F + \beta\left(R_M - R_F\right) \;\leftrightarrow$$
$$K_E = R_F + \frac{\sigma\left(R_M;R_F\right)}{\sigma^2\left(R_M\right)}\left(R_M - R_F\right) \quad (13)$$

Where: R_F, is the free-risk rate; R_M, is the expected return in market investment; (R_M-R_F), is the premium for the market's investment; β, is the beta coefficient of the firm and it is equal to the ratio between the covariance of the return in market investment and the return of the investment in the firm, and the variance of the return in market investment.

The cost of debt K_D represents the expected return of investors in debt of the firm. It can be divided in three parts (Elton et al., 2007):
1. The first, is the financial cost of the time and it can be approximate by the risk-free rate. It can be approximated to the return of riskless government bond and thus it measures the return on default-free bonds;
2. The second, is the market risk-premium due to the higher volatility of the corporate bonds than government bonds;
3. The third, is the default risk of corporate bonds due to the risk of the bondholder's loss in firm insolvency case.

Based on these three main parts, the cost of debt can be defined as following:

$$K_D = R_F + \left(YC_M - R_F\right) + \rho e^{\delta} \quad (14)$$

Where:
- R_F: Is the risk-free rate;
- YC_M: Is the expected medium market yield of corporate bonds. Therefore, the difference (YC_M-R_F) measures the market risk premium of corporate bonds due to their higher risk than government bonds;
- δ: Is a composite index that measures the firm's capabilities to face debt obligations based on its fundamentals;
- e^{δ}: Is the exponential function that measures the default risk of the firm based on its fundamentals. The exponential function is used for two main reasons: First, it highlights the negative effects of debt when debt increases; second, it can be approximated with a quadratic form that is compatible with mean-variance approach. It is expressed on based 100 ($e^{\delta} = e^{\delta}/100$);
- ρ: Is a discretionary variable. It walks between zero and one ($0 \le \rho \le 1$) and it defines the part of default risk that investor wants to assume.

Therefore, the equation [14] can be divided in three main parts:
- R_F: Is the risk-free rate and it measures the financial cost of time. It can be approximated to the return of riskless government bond and thus it measures the return on default-free bonds;
- (YC_M-R_F): Is the market risk-premium of corporate bonds due to their higher risk than government bonds;
- ρe^{δ}: Is the default premium and it measures the specific risk of default of the company due to its incapability to face debt obligations.

The second and third parts of the Equation [14] measure the spread and thus the difference in interest rates between corporate bonds and government bonds due to the higher risk of the first than the second:

$$\text{Spread} = (YC_M - R_F) + \rho e^{\delta}$$

The composite index (δ) that measures the firm's capabilities to face debt obligations based on its fundamentals only (for its analysis see De Luca, 2017). It can be defined as following:

$$\delta = \gamma_1 + \gamma_2 + \gamma_3 \quad (15)$$

Where: γ_1, γ_2 and γ_3 are the coefficient based on the firm's fundamentals. They can be defined as a relevance factors based on the firm's characteristics.

The coefficient γ_1 is equal to the ratio between cash flows out linked to the debt commitment (CDC) and the current operating cash flows (CFO), as following:

$$\gamma_1 = \frac{CDC}{CFO_{(A)}} \quad 0 \le \gamma_1 \le 1 \quad (16)$$

Where: CDC is equal to the interest on debt to be paid accrued to debt level at the end of previous period ($DK_{D(-1)}$) plus the share of debt capital (principal) to be reimbursed in the period based on the debt level at the start of the period (αD). Thus, the CDC can be rewritten as following:

$$CDC = \alpha D + DK_{D(-1)} = D(\alpha + K_{D(-1)}) \quad (17)$$

The $CFO_{(A)}$ is the current operating cash flows (realized in the period) and it is equal to the EBITDA plus the changes in the net capex and in the net working capital (equal to the changes in inventories plus the receivables minus the payables) minus the use of the Fund.

By using the equation [17], the equation [16] can be rewritten as following:

$$\gamma_1 = \frac{D\left(\alpha + K_{D(-1)}\right)}{CFO_{(A)}} \quad 0 \le \gamma_1 \le 1 \quad (18)$$

Therefore, the lower the distance between CDC and $CFO_{(A)}$, the higher the default risk due to firm's inability to face debt obligations.

If the firm is financed by equity only, the coefficient is equal to zero. Otherwise, if the firm is financed by equity and debt, the limit of coefficient is equal to one and consequently the CDC is equal to $CFO_{(A)}$. When the value of the coefficient is greater than one, the firm is unable to face debt obligations and, thus it can be considered in default.

The coefficient γ_2, is equal to the ratio between the financial debt book value (D) and liquidable assets (LA), as following:

$$\gamma_2 = \frac{D}{LA} \quad 0 \le \gamma_2 < \varepsilon \quad (19)$$

The LA is equal to LA: Tangible, intangible and financial assets, credit, inventors. In order to be considered as liquidable, the asset has to have two characteristics: First, it is characterized by a market value; second, it must be marketable in the short-time.

If the amount of the LA in place is greater than the amount of debt, they can be easily sold on the market and proceeds to used in order to satisfy bondholders; otherwise, if the debt amount is greater than LA in place, even if they sold them on the market the bondholders' expectations cannot be satisfy completely.

Therefore, this coefficient measures the coverage of debt obligations in insolvency case and, thus it is a measure of bondholders' guaranty. Much more are the LA in place, higher is the bondholder's guaranty in insolvency case. Otherwise, greater is debt then the LA, greater is the bondholders' risk.

If the firm is all-equity financed, the coefficient is equal to zero. Otherwise, if the firm is financed by equity and debt, the coefficient can vary between zero and a value (ε) defined by the bondholders on the basis of their risk aversion. In any case, one is the coefficient value that can be considered as alert point (ε = 1).

The coefficient γ_3, is equal to the ratio between cash flow to equity net current debt ($CFNE_{(A)}$) and cash flows to equity net debt expected ($CFNE_{(E)}$) with regard the same time, as following:

$$\gamma_3 = \left(1 - \frac{CFNE_{(A)}}{CFNE_{(E)}}\right) \quad 0 \le \gamma_3 < \varepsilon \quad (20)$$

The cash flows to equity is equal the operating cash flows (CFO) plus the variances in equity (only for its increase or decrease due to cash and cash-equivalent) and net financial position (equal to financial debts minus cash and cash-equivalent) and minus the net interest paid on debt. It is possible that the dividends are paid by debt. In this case the increase in debt is not used to increase the investments but to pay dividends. The firm reduces the capabilities to increase the future CFO and then the future CFE. Therefore, in this context are considered the cash flows to equity net of debt increases (CFNE).

The positive amount of the CFNE indicates the dividend amount for the shareholders; otherwise its negative amount indicates the increase in equity to cover the firm's needs.

Smaller is the difference between CFNE current and expected, greater is shareholders' satisfaction then higher is the firm's capability to raise capital on favourable conditions in financial markets. Therefore, the coefficient γ_3 can be considered a proxy of the discipline effects of debt on the management (Jensen and Meckling, 1976; Jensen, 1986). High debt level allows management to invest capital in positive net present value projects that increase the CFO and the CFNE with maximization of the equity value.

In this context, it is reasonable to assume: First, CFNE expected ($CFNE_{(E)}$) is the max value for the CFNE; second, CFNE current ($CFNE_{(A)}$) cannot be higher than $CFNE_{(E)}$. Therefore, $\gamma_3 = 0$ is the max value of the coefficient and represent the best condition. In this case the firm maximize the equity value because the shareholders' expectations are realized.

Otherwise, lower is the $CFNE_{(A)}$ respect to the $CFNE_{(E)}$, higher is the coefficient value. In absence of the $CFNE_{(A)}$ the coefficient is equal $\gamma_3 = 1$. Note that the coefficient value is higher than 1 ($\gamma_3 > 1$) if the $CFNE_{(A)}$ is negative.

Since the CFNE is equal to the CFO minus the CDC, the equation [20] can be rewritten as following:

$$\gamma_3 = \left(1 - \frac{CFO_{(A)} - D\left(\alpha + K_{D(-1)}\right)}{CFO_{(E)} - D\left(\alpha + K_{D(-1)}\right)}\right) \rightarrow$$

$$\gamma_3 = \left(\frac{CFO_{(E)} - CFO_{(A)}}{CFO_{(E)} - D\left(\alpha + K_{D(-1)}\right)}\right) \quad (21)$$

On the basis of the equations [18], [19] and [21], the equation [15] can be rewritten as following:

$$\delta = \frac{D\left(\alpha + K_{D(-1)}\right)}{CFO_{(A)}} + \frac{D}{LA} + \frac{CFO_{(E)} - CFO_{(A)}}{CFO_{(E)} - D\left(\alpha + K_{D(-1)}\right)} \quad (22)$$

By denoting the differences between $CFO_{(E)}$ and $CFO_{(A)}$ as $\Delta^{CFO}_{(E-A)}$, the equation [22] can be rewritten as following:

$$\delta = D\left[\frac{\left(\alpha + K_{D(-1)}\right)}{CFO_{(A)}} + \frac{1}{LA}\right] + \left[\frac{\Delta^{CFO}_{(E-A)}}{CFO_{(E)} - D\left(\alpha + K_{D(-1)}\right)}\right] \quad (23)$$

Substituting the equation [23], the general equation [14] can be rewritten as following:

$$K_D = R_F + \left(YC_M - R_F\right) + \rho e^{\left\{D\left[\frac{\left(\alpha + K_{D(-1)}\right)}{CFO_{(A)}} + \frac{1}{LA}\right] + \left[\frac{\Delta^{CFO}_{(E-A)}}{CFO_{(E)} - D\left(\alpha + K_{D(-1)}\right)}\right]\right\}} \quad (24)$$

Defined the cost of debt and the cost of equity, it is possible define the FLCC (K_L) as following:

$$K_L = K_E - L[K_E - K_D\,(1-t_c)] \quad (25)$$

Where:

$$L = \frac{D}{E+D} \quad \rightarrow \quad 1 - L = \frac{E}{E+D} \quad (26)$$

And thus:

$$K_L = K_E - \left(\frac{D}{E+D}\right)\left[K_E - K_D\left(1-t_c\right)\right]$$

$$K_L = K_E - \left(\frac{D}{E+D}\right)K_E + K_D\left(\frac{D}{E+D}\right)\left(1-t_c\right)$$

$$K_L = K_E\left(1 - \frac{D}{E+D}\right) + K_D\left(\frac{D}{E+D}\right)\left(1-t_c\right)$$

$$K_L = K_E\left(\frac{E+D-D}{E+D}\right) + K_D\left(\frac{D}{E+D}\right)\left(1-t_c\right)$$

$$K_L = K_E\left(\frac{E}{E+D}\right) + K_D\left(\frac{D}{E+D}\right)\left(1-t_c\right)$$

It is relevant to note that explicating the equation [26] in the equation [25], it can be rewritten as following:

$$K_L = K_E\left(\frac{E}{E+D}\right) + K_D\left(\frac{D}{E+D}\right)\left(1-t_c\right) \quad (27)$$

The equation [27] shows that the FLCC (K_L) is equal to the sum of the cost of equity and the cost of debt net to the tax shields weighted for the value of equity and debt in the capital structure respectively.

The structure of the FLCC is only formally similar to the WACC. They are different by construction.

Substituting equations [13] and [24], in the equation [25] the FLCC is equal to:

$$K_L = R_F + \beta\left(R_M - R_F\right) - L\left\{R_F + \beta\left(R_M - R_F\right) - \left[R_F + \left(YC_M - R_F\right) + \rho e^{\left[D\left[\frac{\left(\alpha + K_{D(-1)}\right)}{CFO_{(A)}} + \frac{1}{LA}\right] + \left[\frac{\Delta^{CFO}_{(E-A)}}{CFO_{(E)} - D\left(\alpha + K_{D(-1)}\right)}\right]\right]}\right]\left(1-t_c\right)\right\} \quad (28)$$

By placing:

- $A = K_E = R_F + \beta(R_M - R_F)$
- $F = R_F = (YC_M - R_F)$
- $H = \frac{\alpha + K_{D(-1)}}{CFO_{(A)}} + \frac{1}{LA}$
- $I = \Delta^{CFO}_{(E-A)} = CFO_{(E)} - CFO_{(A)}$
- $M = CFO_{(E)}$
- $N = \alpha + K_{D(-1)}$

The equation [28] can be simplified as following:

$$K_L = A - L\left\{A - \left[F + \rho e^{\left[DH + \frac{I}{M - DN}\right]}\right]\left(1 - t_c\right)\right\}$$

And thus

$$K_L = A - \left(\frac{D}{E+D}\right)\left\{A - \left[\left(F + \rho e^{\left(DH + \frac{I}{M-DN}\right)}\right)\left(1 - t_c\right)\right]\right\} \quad (28)$$

The function is continuously and differentiable at least twice by construction. In this contest the main question is to demonstrate that the function has a minimum point. If is there, it defines the minimum level of the FLCC. Thus, based on the effects of the debt level changes on the change of the FLCC, the firm can choose the debt level that minimizes the FLCC.

In order to calculate the K_L derivative, it can be useful to explicit the eqaution [29] as following:

$$K_L = A - \left(\frac{D}{E+D}\right)A + \left(\frac{D}{E+D}\right)\left[\left(F + \rho e^{\left(DH + \frac{I}{M-DN}\right)}\right)\left(1 - t_c\right)\right] \quad (29)$$

And thus

$$K_L = A - \left(\frac{D}{E+D}\right)A + \left(\frac{D}{E+D}\right)\left[F + \rho e^{\left(DH + \frac{I}{M-DN}\right)} - Ft_c - t_c\rho e^{\left(DH + \frac{I}{M-DN}\right)}\right]$$

$$K_L = A - \left(\frac{D}{E+D}\right)A + \left(\frac{D}{E+D}\right)F + \left(\frac{D}{E+D}\right)\rho e^{\left(DH + \frac{I}{M-DN}\right)} - \left(\frac{D}{E+D}\right)Ft_c - \left(\frac{D}{E+D}\right)t_c\rho e^{\left(DH + \frac{I}{M-DN}\right)}$$

It is simple calculate the derivative of each element as following:

- $\frac{\partial}{\partial D}[A] = 0; \frac{\partial}{\partial D}[H] = 0$
- $\frac{\partial}{\partial D}\left[\frac{D}{E+D}\right] = \frac{D}{E+D} = \frac{E}{(E+D)^2}$

- $\frac{\partial}{\partial D}\left[\left(\frac{D}{E+D}\right)A\right]=\left(\frac{E+D-D}{(E+D)^2}\right)A=\left(\frac{E}{(E+D)^2}\right)A$
- $\frac{\partial}{\partial D}\left(\frac{D}{E+D}\right)F=\left(\frac{E+D-D}{(E+D)^2}\right)F=\left(\frac{E}{(E+D)^2}\right)F$
- $\frac{\partial}{\partial D}\left[\left(\frac{D}{E+D}\right)Ft_c\right]=\left(\frac{E+D-D}{(E+D)^2}\right)Ft_c=\left(\frac{E}{(E+D)^2}\right)Ft_c$
- $\frac{\partial}{\partial D}\rho e^{\left(DH+\frac{I}{M-DN}\right)}=\rho e^{\left(DH+\frac{I}{M-DN}\right)}\left(H+\frac{IN}{(M-DN)^2}\right)$

  $\frac{\partial}{\partial D}\left[\left(\frac{D}{E+D}\right)\rho e^{\left(DH+\frac{I}{M-DN}\right)}\right]=\left(\frac{E}{(E+D)^2}\right)\rho e^{\left(DH+\frac{I}{M-DN}\right)}+$
- $\left(\frac{D}{E+D}\right)\rho e^{\left(DH+\frac{I}{M-DN}\right)}\left(H+\frac{IN}{(M-DN)^2}\right)$
- $\frac{\partial}{\partial D}\left[\left(\frac{D}{E+D}\right)t_c\rho e^{\left(DH+\frac{I}{M-DN}\right)}\right]=\left(\frac{E}{(E+D)^2}\right)t_c\rho e^{\left(DH+\frac{I}{M-DN}\right)}+$

  $\left(\frac{D}{E+D}\right)t_c\rho e^{\left(DH+\frac{I}{M-DN}\right)}\left(H+\frac{IN}{(M-DN)^2}\right)$

To simplify calculations, denote:

- $\tau=DH+\frac{I}{M-DN}$
- $S=H+\frac{IN}{(M-DN)^2}$

The K_L derivate can be rewritten as following:

$$\frac{\partial K_L}{\partial D}=-\left(\frac{E}{(E+D)^2}\right)A+\left(\frac{E}{(E+D)^2}\right)F-\left(\frac{E}{(E+D)^2}\right)Ft_c+\left(\frac{E}{(E+D)^2}\right)\rho e^{\tau}+\left(\frac{D}{E+D}\right)\rho e^{\tau}S-\left[\left(\frac{E}{(E+D)^2}\right)t_c\rho e^{\tau}+\left(\frac{D}{E+D}\right)t_c\rho e^{\tau}S\right]$$

And then:

$$\frac{\partial K_L}{\partial D}=-\left(\frac{E}{(E+D)^2}\right)A+\left(\frac{E}{(E+D)^2}\right)F-\left(\frac{E}{(E+D)^2}\right)Ft_c+\left(\frac{E}{(E+D)^2}\right)\rho e^{\tau}+\left(\frac{D}{E+D}\right)\rho e^{\tau}S-\left(\frac{E}{(E+D)^2}\right)t_c\rho e^{\tau}-\left(\frac{D}{E+D}\right)t_c\rho e^{\tau}S$$

$$\frac{\partial K_L}{\partial D}=\left[\left(\frac{E}{(E+D)^2}\right)\left(F(1-t_c)-A+\rho e^{\tau}(1-t_c)\right)\right]+\left[\left(\frac{D}{E+D}\right)\rho e^{\tau}S(1-t_c)\right]$$

$$\frac{\partial K_L}{\partial D}=\left[\left(\frac{E}{(E+D)^2}\right)\left(\left((1-t_c)\left(F+\rho e^{\tau}\right)\right)-A\right)\right]+\left[\left(\frac{D}{E+D}\right)\rho e^{\tau}S(1-t_c)\right]$$

And by re-substituting τ and S with respectively equations, the derivative is equal to:

$$\frac{\partial K_L}{\partial D}=\left[\frac{E}{(E+D)^2}\right]\left\{\left[\left(F+\rho e^{\left(DH+\frac{I}{M-DN}\right)}\right)(1-t_c)\right]-A\right\}+\left(\frac{D}{E+D}\right)\rho e^{\left(DH+\frac{I}{M-DN}\right)}\left(H+\frac{IN}{(M-DN)^2}\right)(1-t_c) \quad (30)$$

It is stationary point. In order to have a point of minimum, the curve must be convex. In this case, the second derivative must be positive. In order to demonstrate the convexity of the curve without the determination of the second derivative (that is very complex) it can be used the intermediate value theorem.

Assume that the first derivative is a function defined and continued for each debt level between 0 (all-equity financed) and 1 (all-debt financed), so that:

$$\forall D\in[0;1] \quad (31)$$

By using the intermediate value theorem, if the first derivative is negative in D = 0 and it is positive in D=1, there is a debt level D* between 0 and 1 in which the first derivative is equal to zero. Formally:

$$\left\{\frac{\partial K_L}{\partial D}(D{=}0)<0 \ \text{ and } \ \frac{\partial K_L}{\partial D}(D{=}1)>0\right\}$$
$$\Rightarrow\left\{\exists D^*\in(0,1):\frac{\partial K_L}{\partial D}\left(D^*\right)=0\right\} \quad (32)$$

Furthermore, if the first derivative is negative on the left of D* and it is positive on the right, there is a minimum point necessarily (D*).

Assume that the value of the capital structure is equal to 1, as following:

$$E+D=1 \quad (33)$$

Based on the equation [33], for D = 0 (E = 1) the firm is all-equity financed; otherwise, for D = 1 (E = 0) the firm is all-deb financed.

The value of the first derivative can be study for D = 0 and D = 1. These are the two points that define the function fields.

The value of the first derivative for D = 0 is always negative, as following:

$$D{=}0 \ \rightarrow \ \frac{\partial K_L}{\partial D}=\left(\frac{1}{E}\right)\left[F(1-t_c)-A\right] \quad (34)$$

The equity risk is greater than the debt risk by definition, and thus:

$$A = R_F+\beta_E(R_M-R_F) > F = R_F+(YC_M-R_F) \quad (35)$$

Therefore, the value of the first derivative is always negative.

Differently, the value of the first derivative for D = 1 is always positive as following:

$$D = 1 \;\rightarrow\; \frac{\partial K_L}{\partial D} = \rho e^{\left(H+\frac{I}{M-N}\right)}\left(H+\frac{IN}{(M-N)^2}\right)(1-t_c) \quad (36)$$

All terms are positive:

- $0<\rho\leq 1 \rightarrow \rho>0 \forall D$. Theoretically $\rho \epsilon (0;1)$. But if the firm is all-debt financed, assuming $\rho = 0$ is equivalent to assume a riskless debt and it is difficult to assume. In this case it is right to assume $\rho = 1$;
- $H = \frac{\alpha + K_{D(-1)}}{CFO_{(A)}} + \frac{1}{LA} > 0 \forall D$ because all element are positive by definition;
- $IN = \Delta^{CFO}_{(E-A)}\left(\alpha + K_{D(-1)}\right) > 0 \forall D$ because the first term is positive based on the assumption about CFO Current and Expected and the second term is always positive by definition;
- $e^{\left(H+\frac{I}{M-N}\right)} > 0 \forall D$ by definition;
- $(M-N)^2 = (CFO_{(E)}-(\alpha+K_{D(-1)}))^2>0 \forall D$ by definition.

Therefore, the value of the first derivative is always positive.

Explicating all variables, the equation [30] can be rewritten, as following:

$$\frac{\partial K_L}{\partial D} = \left(\frac{E}{(E+D)^2}\right)\left\{\left[\begin{array}{l}\left(R_F+\left(YC_M-R_F\right)\right)(1-t_c)- \\ \left(R_F+\beta_E\left(R_M-R_F\right)\right)\end{array}\right] + \rho e^{\left[D\left(\frac{\alpha+K_{D(-1)}}{CFO_{(A)}}+\frac{1}{LA}\right)+\frac{\Delta^{CFO}_{(E-A)}}{CFO_{(E)}-D\left(\alpha+K_{D(-1)}\right)}\right]}(1-t_c)\right\}$$
$$+\left(\frac{D}{E+D}\right)\rho e^{\left[D\left(\frac{\alpha+K_{D(-1)}}{CFO_{(A)}}+\frac{1}{LA}\right)+\frac{\Delta^{CFO}_{(E-A)}}{CFO_{(E)}-D\left(\alpha+K_{D(-1)}\right)}\right]}$$
$$\left[\left(\frac{\alpha+K_{D(-1)}}{CFO_{(A)}}+\frac{1}{LA}\right)+\frac{\Delta^{CFO}_{(E-A)}\left(\alpha+K_{D(-1)}\right)}{\left(CFO_{(E)}-D\left(\alpha+K_{D(-1)}\right)\right)^2}\right](1-t_c) \quad (37)$$

The minimum point can be found by searching the root of the derivate in equation [37] by using a numerical methods.

Therefore, the function of the FLCC, as defined by the equation [37], draws a curve with a minimum point as in Figure 1 where the ordinate is the cost of capital and the abscissa is the leverage.

The curve reflects the combined effects of the stock market rates (exogenous variable) and the firm's specific default risk (endogenous variable). Two are the main movements of the curve:

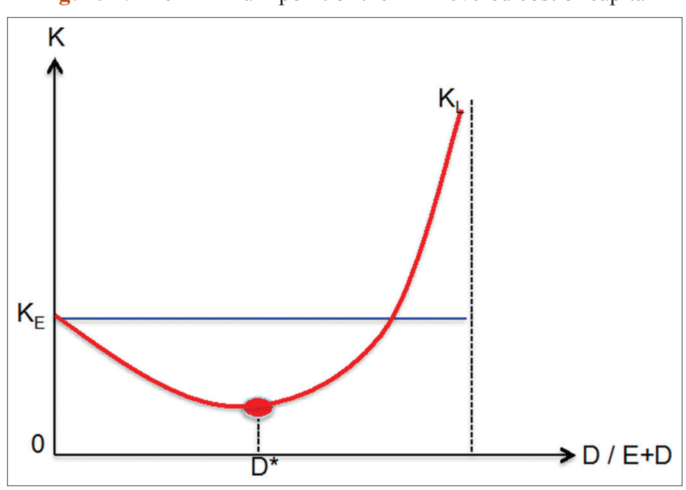
- The first relates the shifting of the D* point between 0 (all-equity financed) and 1 (all-debt financed) along the abscissa. This movement is mainly due to the firm's fundamentals. If the firm's default risk increases, the point D* moves to the left; otherwise, if it reduces, the point D* moves on the right. The left movement reduces the firm's capabilities to face debt while the right movements increased it. It is mainly function of the operating cash flows and their capabilities to face debt obligations;
- The second relates the shifting of the curve upwards and downwards along the ordinate. In this case it is mainly due to the market rates. Generally, isolating the rate movements if: R_M or YC_M increase, the curve shift upwards; R_M or YC_M decrease, the curve shift downwards. The effect of the change R_F is due to the coefficient beta on equity. It is relevant to note that if the distance between R_M and YC_M increases, ceteris paribus, the curve shifts upwards and the point D* moves on the right.

4. CONCLUSION

The FLCC uses the same weighting approach of the WACC, but this similarity is only formal. The two models are deeply different by construction. Three are the main differences.

First, the WACC is defined by market as function of the risk class of the firm and it is used to discount the unlevered cash flows assuming all-equity financed firm. Differently, the FLCC is based on the estimation of the cost of equity and the cost of debt on the basis of the expected return of the investors in equity and debt respectively. Then, the FLCC is directly function of the firm capital structure. It is used to discount unlevered cash flows of the firm because they are the source to satisfy expectations of investors in equity and debt.

Figure 1: The minimum point of the firm levered cost of capital



Second, the WACC is based on the assumption of a linear relationship between the cost of capital and the leverage. Differently, the FLCC is estimated on the basis of non-linear relationship between leverage and the cost of capital. The FLCC is estimated by assuming an exponential function between risk and debt level. The increase of leverage, on the one hand, increases the benefits due to the tax shields by reducing the FLCC but, on the other hand, it increases the default risk probability. Based on these two effects, the FLCC draws a curve with a minimum point that define the debt level that minimizes the cost of capital.

Third, in WACC the debt level is assumed constant and well-known. Differently, in the FLCC the definition of debt level is function of its effects on the cost of capital. By changing debt level, changes the cost of debt and, thus, the cost of capital along the curve. Also, this is the main difference between FLCC and the ME's cost of capital model.

Two are the most relevant implication for financial manager.

First, the FLCC can be used to define the optimal level of debt by considering the changes in the capital markets as well as the changes in the firm's fundamentals. The optimal level of debt can change, by requiring a reduction or increase, due to the changes in capital markets condition even if there are not changes in the company's fundamentals and vice-versa.

Therefore, the debt level must be defined not only by considering the expectations about company's fundamentals but also expectations about capital markets changes. It is possible that in favourable condition of rates in the capital markets, the optimal debt level of the firm can increase even if the its fundamentals are the same or even worse.

Second, the FLCC can be used to estimate the firm levered value by discounting the expected operating cash flows. Since the FLCC draws a curve with a minimum point, and having defined the first derivative, it is possible to define the debt level that minimizes the cost of capital and thus maximize the firm levered value, as following:

$$\min_{D^* \in D} K_L \rightarrow \frac{\partial K_L}{\partial D} = 0 \quad \rightarrow \quad \max_{K_L} W_L = \sum_{t=1}^{\infty} \frac{CFO}{\left(1+K_L\right)^t} \qquad (38)$$

In this sense, the FLCC can be considered a theoretical model with a normative function. It can be used to define in any time the debt level of the firm based on its effects on the levered cost of capital and then on the firm levered value.

Two are the main limits of the FLCC. First, it is a static model. It defines the debt level on the basis of the variables definition in any time but it does not consider their evolution over time. Second, some parameters used in the estimation of the cost of debt with regard to the firm's fundamentals lend themselves to subjective assessments with direct effects on the estimation of the cost of debt.

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