Study of Speculative Bubbles: The Contribution of Approximate Entropy

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ABSTRACT: The purpose of this article is to provide a survey of the presence of speculative bubbles. The empirical study uses a new technique borrowed from statistical physics suggested by Pincus (1991). We use approximate entropy to test the presence of bubbles in the Tunisian and French markets. To this end, weekly data of primary stock markets of the countries under study (Tunisia, France) were used. Since our aim is to study the crisis, data before and after the crash was also analyzed. The crisis decade has been identified as the sample period for our study. More specifically, we consider weekly data over the 2002 to 2012 period. The obtained results confirm the presence of a speculative bubble during the "subprime" crisis (2008) in the two markets.

Keywords: Speculative bubbles; approximate entropy; financial crashes; speculation.
JEL Classifications: C32; C63

1. Introduction

The issue of the financial markets efficiency has long fuelled a large part of the financial literature. This theory states that in an efficient market consisting of rational agents, prices absorb, fully and instantaneously, all relevant information regarding the fundamentals. This framework allows asset prices to reproduce its fundamental value, given by the discounted sum of expected future cash flows, and thus absorb any sustainable difference generated between prices.

However, the theoretical predictions of the efficient markets hypothesis (EMH) do not conform to the reality of financial markets functioning. Indeed, speculation accidents that marked the economic and financial history, particularly stock markets crashes hitting financial markets across the globe, starting from the "Tulip bulbs (1637)" in the Netherlands until the "subprime crisis (2008)", are clear examples of deviations generated primarily by speculative mania. This sequence of exceptional bubbles which led most observers to question the relevance of the efficiency theory, thus revealed a persistent and sustainable gap between the asset and its fundamental value. This disruption between the real sphere and financial sphere is the main reason which encouraged the development of speculative bubbles theory.

This theory made it possible for the existence of large and sustainable differences between stock prices and their fundamental values. A bubble describes a state of an overvalued market in which asset prices deviate from their fundamental values. Investors admit the possibility of a temporary difference between the share price and the real value. Consequently, they remain alert because the wider the gap grows the more dangerous and threatening a bubble explosion becomes. In response to the least unfavourable signal, investors suddenly reverse their behaviour, which, by a multiplier panic effect, causes the crash, the bubble explodes and price falls sharply, leading to a collapse of the market.

Various techniques have been used to reveal the presence of bubbles. Perhaps the earliest techniques include tests of volatility excess that have been used by Friedman (1953), Baumol (1957), Kohn (1978), and Shiller (1981). Other techniques used were bubbles tests suggested by Hardouvelis (1988), and Rappoport and White (1993).

Stationarity and cointegration tests developed by Diba and Grossman (1988) have also gained importance in the following years. However, these techniques have been criticized for their low predictive power and limitations in the study of speculative bubbles. Lately, time dependence models provided important techniques to study bubbles. Unlike many traditional tests such as autocorrelation, skewness and kurtosis designed to identify bubbles, duration dependence models obtained by McQueen and Thorley (1994) may provide more discriminatory power to test returns' non-linearity due to bubbles.

The first model of rational speculative bubbles was presented by McQueen and Thorley (1994). This process of speculative bubbles allows the bubble to grow and explode every time it persists. Therefore, they assumed that a long series of abnormal returns suggests the presence of a bubble. In other words, within rational bubbles, stock prices should deviate from the random walk.

This would result in a low degree of uncertainty, specific to stock prices' random walk behaviour. As the random walk process is the most complex in terms of predictability, it is natural to use entropy as a measure of uncertainty, and thus reveal the presence of bubbles. Its ability to quantify order probability in financial series was used to examine assumptions of rational speculative bubbles. New research has focused on the approach of entropy for a better understanding of the dynamics of the organization and functioning of financial markets.

The paper presents approximate entropy (APEN) as a measure of statistical regularity to test speculative bubble in stock markets. However, in the presence of a rational bubble, series returns generate repetitive signals, leaving a revealing sign, "tell tale signs". Therefore, a rational bubble episode should be associated with relatively low approximate entropy values .

The application of the concept of entropy to economy is relatively new and offers a variety of models to study uncertainty. Dionísio (2001) wrote that: "The use of entropy as a measure of uncertainty in finance seems to have a lot of potential and a wide range of development, both theoretically and empirically."

The rest of the paper is organized as follows: In Section 2, we describe the theoretical background of bubbles and the concept of entropy. In Section 3, we present the methodology, and detection of bubbles is explained. Section 4 presents the results for the Tunisian and French stock markets. Finally, in Section 5 conclusions are drawn.

2. Approximate Entropy and Speculative Bubbles

In the first model of rational speculative bubbles proposed by McQueen and Thorley (1994), the process enables a bubble to grow and explode every time it persists. Therefore, they assumed that a long series of abnormal returns suggests the presence of a bubble. In other words, in rational bubbles, stock prices should deviate from the random walk. This results in a low degree of uncertainty.

Unlike economists who exclude uncertainty, econo-physicists use this concept in an epistemic manner to study how its various dimensions. Each of these entropies defines a particular statistical framework which can be used as a function of the reality being studied. By providing this diversity, econo-physicists offer a collection of operational instruments for uncertain situation. According to Dionísio (2001), entropy captures uncertainty and confusion of time series, so that a variable is totally disorganized when its entropy is maximized Zunino et al. (2010).

In this context, several authors have recently introduced different versions of entropy to measure informational efficiency of various financial markets. The concept of entropy was used to quantify the efficiency of exchange markets and detect stock market bubbles. Entropy approach is used to measure different aspects of time series. We can find the Renyi entropy, Dionisio (2001), approximate entropy Oh and al. (2007), Tsallis entropy Zunino and al. (2007) and Shannon entropy Risso (2008) 14, permutation entropy Zunino et al. (2009) and Zunino et al. (2010). Pincus (1991) proposed approximate entropy "APEN" to quantify randomness of time series. Approximate entropy calculations are simple and based on the probability that time series models that are similar remain similar over the next additional comparisons. When time series data have a high degree of chance, we will have a high value of "APEN".

Therefore, any time series which contains many repetitive patterns has an "ApEn" value relatively low compared to others that are more random. Although initially developed for measuring irregularities of a complex nonlinear system, it was gradually introduced in the financial literature as a measure of efficiency of equities and foreign exchange markets Pincus and Kalman (2004), and Oh and Kim (2006). Oh and al. (2007) found that the average "APEN" for European markets is 2, for North America is 1.98, which are larger than the "APEN" values of Asian markets which stand at 1.1 (with the exception of Japan), and African markets with 1.52. This is due to the volume of liquidity or trading in the European and North American Exchange markets, which are much larger than those of other foreign exchange markets.

Pincus and Kalman (2004) suggested that approximate entropy algorithm can be adapted to the analysis of financial time series, as it applies to very short sequences and can be used as a marker of market stability, with a rapid increase possibly scoring significant changes in a financial variable.

Alvarez-Ramirez J. (2011), Alvarez-Ramirez J. et al. (2012) used entropy methods to monitor movement of crude oil prices in developed and emerging markets. They found that developed exchange markets are more efficient than emerging exchange markets, and that financial crises significantly promote inefficiency of emerging exchange markets like China, Hong Kong, South Korea and the African market. Wang and al. (2012) found the same results. Known for its randomness in measuring time series, approximate entropy method was adopted to evaluate the relationship between efficiency and predictability of prices Eoma and al. (2008). They found that Hurst exponent (as a measure of efficiency) and "ApEn" value is negatively correlated. However, predictability is positively correlated with Hurst exponent. Then it has been used to identify speculative bubbles Bhadri (2012).

Approximate entropy (APEN) was used to quantify probability of "order" (repetition) in financial time series. Bhadri (2012) proposed a first step to test speculative bubble in a developing market like India. Particularly in India, there have been countless speculations about the existence of bubbles in the stock market. In addition to these speculative activities, the Indian stock market has also seen various scams and crises leading to irrational prices behaviour during the last decade. This is a unique opportunity to test the power of approximate entropy to identify irregular market movements in the Tunisian market which resembles the Indian market. We will use this ability to track repetitive signals in time series to identify speculative bubble in the data.

Most empirical studies examining bubbles have mainly focused on developed countries. Ironically, it is the market efficiency hypothesis which is more likely to be violated in the case of developing economies than their developed counterparts because of poor legal and information systems. Therefore, the study of speculative bubbles has taken on a renewed focus in the context of developing economies such as Tunisia. Despite significant progress, these markets (in terms of structure, systems and regulations) have been seeing several speculation episodes in recent times. However, most existing studies have addressed the question of whether prices evolution in many developing markets is consistent with bubbles characteristics. We will examine the existence of rational speculative bubble and empirically test its properties through markets such as Tunisia and France.

3. Methodology
3.1 The Empirical Model
In a simple rational speculation bubble model, Shiller (1978), Blanchard and Watson (1982) argued that market price of a share may deviate from its fundamental value under a rational bubble episode, as the bubble grows at a specific rate.

$$E_t(R_{t+1}) = \frac{P_{t+1} - P_t}{P_t} \qquad (1)$$

• Where P_t is price at time t.

• E_t denotes expectation given information set t, in a situation where the market is efficient, the expected return of a stock is equal to its required rate of return, let:

$$E_t(R_{t+1}) = (R_t + 1) \qquad (2)$$

In other words, the equilibrium condition requires that current price must be equal to the expected future price, in terms of the required return rate, that is to say:

$$P_t = \frac{E_t(P_{t+1})}{r_{t+1} + 1} \tag{3}$$

Moreover, Shiller (1978) and Blanchard and Watson (1982) and West (1987), among others, have observed that any price is written in the form $P_t = P_t^* + b_t$, or $E_t(b_t + 1) = (1 + r_{t+1})b_t$ is also a solution to the equilibrium condition. Therefore, market price of an asset can deviate from its fundamental value by a factor b rational bubble t, this bubble factor grows at the rate r (t).

As we have already mentioned, first-generation model of rational bubbles proposed by Blanchard and Watson (1982) and McQueen and Thorley (1994) provides a process of speculative bubble that allows the bubble to grow and explode with the bubble to fulfill the following condition:

$$b_{t+1} = \frac{(1 + r_{t+1})b_t}{\pi} - \frac{1 - \pi}{\pi} a_0 \quad \text{with a probability } \pi \tag{4}$$

$$= a_0 \quad \text{with a probability } (1 - \pi)$$

In other words, in this process, the bubble grows in the exact amount needed to compensate for the loss due to an accident with a probability (1 - π).

It is important to note that the model proposed by Blanchard and others suggests a bubble with a long "run-up" price followed by a crisis with π> 1/2.

The first model of rational bubbles was introduced by McQueen and Thorley (1994) in which they suggested that bubbles may lead to explosive price changes which grow each time the bubble persists. Therefore, they hypothesized that a long series of positive abnormal returns suggests the presence of a bubble if conditional probability of the execution end is a decreasing function of the duration of the operation.

We present approximate entropy (APEN) as a measure of statistical regularity to test speculative bubbles in the stock markets. However, we try to use for the first time these statistics to test rational speculative bubble in the Tunisian stock market.

To estimate ApEn, we need the market index returns series. Given a sequence containing N returns series, the market index S_i, $U_i = S_{i+1} - S_i$, two input parameters are defined as, m, r to calculate ApEn (S_N,m,r), where m defines the length of the pattern, or comparison size and r denotes similarity criterion, which defines tolerance. The latter is usually set at ± 20% of the standard deviation of the time series. In the case of neural signals r will be between 1.5 and 2.5 uV.

To calculate approximate entropy, ApEn in the series S_N, the first series of vectors of length m, $V(n) = [U(n), U(n+1), \ldots\ldots U(n - m + 1)]^T$ is extracted from the signal S_N of the sample. The distance D (i, j) between two vectors v (i) and v (j) is defined as the maximum difference between scalar components of v (i) and v (j), $N^{m,r}(i)$ i.e, the number of vectors j (with j N-m +1) so that the distance between v vectors (j) and the generic vector v (i) (with i N-m +1) is less than r, D (i, j) r, where r reflects similarity criterion.

Probability $C^{m,r}(i)$ to find a vector which differs from v (i) with a distance r, is defined as follows:

$$C^{m,r}(i) = \frac{N^{m,r}(i)}{(N - m + 1)} \tag{5}$$

And the logarithmic mean of all vectors of the probability $C^{m,r}(i)$ is defined as follows:

$$F^{m,r}(i) = \frac{\sum_{i=1}^{N-m+1} Ln(C^{m,r}(i))}{(N - m + 1)} \tag{6}$$

ApEn is given by:

$$ApEn^{m,r} = F^{m,r} - F^{m+1,r} \tag{7}$$

Therefore, ApEn of the time series S_N measures the log likelihood that executes signals with length m that are close to each other, m +1. Then, the fact of having a greater likelihood of remaining close (high regularity) produces smaller values (ApEn with low regularity produces higher ApEn values). This has been used to test the presence of rational speculative bubbles, "similar Pattern". Subsequently, in the presence of a rational bubble, the return transfers repetitive signals both before and after the crisis, leaving "tell tale" revealing signs over returns series. Therefore, a rational bubble episode should be associated with relatively low ApEn values .

3.2 Sample

We try to present descriptive statistics in order to analyze the bubble. We investigate the speculative bubble during the 2008 stock market crash for the French Tunisian markets.

To this end, the weekly data of primary stock market in the respective countries (Tunisia, France) were used. Since the aim is to study the crisis period, data before and after the crash was also analyzed. Usually, the crisis decade has been identified as the sample period for our study. More precisely, we consider weekly data for the period 2002 to 2012. As argued by McQueen and Thorley (1994), the choice of our weekly data rather than daily data is based on the fact that conclusions on bubbles may be contaminated by the signal-to-noise ratio in daily returns.

In several studies, ApEn is estimated at a dimension m = 2 as a similarity measure and r = 20% of the standard deviation of the time series, similar to the work of (Pincus et al 2004). To test the robustness of our results, several values of m and r are generated. However, as our conclusion remains invariant for different choices of m and r, we declare that ApEn (5, 1%).

4. Results and Interpretations

4.1 Descriptive statistics

Table 1 presents the descriptive statistics of the returns over the past ten years. The descriptive statistics presented suggest some irregularities signs in the data. First, asymmetry is negative for France, suggesting that abnormally high negative returns associated with the crisis and is often considered a "tell-tale" sign, indicative of the bubble of observed returns. Second, consistent with the flat tail because of mixed distribution as the bubble grows, all stock returns are leptokurtic; it is the case of Tunisia. Finally, the rational bubble often suggests a strong positive correlation because the returns are generally positive when the bubble grows.

Table 1. Descriptive statistics of weekly returns of the CAC 40 and Tunindex

	Tunindex (2001-2012)	Cac 40 (2001-2012)
Number of observations	517	582
Mean	0.0012526	-0.0003166
Standard Deviation	0.226795	0.0143178
Skewness	0.0876206	-1.024209
Kurtosis	149.0954	9.610748
Jarque-Bera	4.6e +05	1162
(p-vaue)	0.000	6.e-253

4.2 Stationarity tests

Table 2 illustrates the stationarity test for the weekly returns of the two indices (Tunindex, Cac 40). ADF test of Tunindex weekly returns.

Table 2. Model with constant and trend

ADF Test Statistic	-14.37065			
Test critical values:	1% level		-3.976819	
	5% level		-3.418981	
	10% level		-3.132041	
*MacKinnon critical values for rejecting the hypothesis of a unit root				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.006328	0.015990	0.395768	0.6925
@TREND(1)	**-4.49E-06**	**5.40E-05**	**-0.083194**	**0.9337**
R-squared	0.812487	Mean dependent var		-5.07E-06
Adjusted R-squared	0.809382	S.D. dependent var		0.402550
S.E. of regression	0.175753	Akaike info criterion		-0.621352
Sum squared resid	14.91943	Schwarz criterion		-0.544551
Log likelihood	161.8526	Hannan-Quinn criter.		-0.591195
F-statistic	261.6035	Durbin-Watson stat		2.035024
Prob(F-statistic)	0.000000			

In this table, the trend is not significantly different from zero. It has an error of 93%. Then, we move to step 2. The constant is not significant (table 3). Tl results in the appropriate model to test stationarity is the one which has neither constant nor trend.

Table 3. Model with constant

ADF Test Statistic	-14.38519			
Test critical values:	1% level		-3.443415	
	5% level		-2.867195	
	10% level		-2.569844	
* MacKinnon critical values for rejecting the hypothesis of a unit root				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	**0.005173**	**0.007924**	**0.652880**	**0.5141**
R-squared	0.812485	Mean dependent var		-5.07E-06
Adjusted R-squared	0.809773	S.D. dependent var		0.402550
S.E. of regression	0.175573	Akaike info criterion		-0.625403
Sum squared resid	14.91964	Schwarz criterion		-0.557135
Log likelihood	161.8491	Hannan-Quinn criter.		-0.598596
F-statistic	299.5891	Durbin-Watson stat		2.035020
Prob(F-statistic)	0.000000			

Table 4. Model without constant or trend

ADF Test Statistic	-14.37888		t-Statistic	Prob.*
Test critical values :	1% level		-2.569671	
	5% level		-1.941468	
	10% level		-1.616267	

The calculated value of the ADF statistic in absolute value is equal to 14.37888 and exceeds the critical values at all levels of significance (Table 4). We reject the unit root null hypothesis. Weekly returns of the Tunindex from 2001 to 2012 are stationary. ADF test of the CAC40 index weekly returns

Table 5. Model with constant and trend

ADF Test Statistic			t-Statistic	Prob.*
Test critical values:	1% level		-3.974381	
	5% level		-3.417793	
	10% level		-3.131338	
* MacKinnon critical values for rejecting the hypothesis of a unit root				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
RENDEMENTS02(-1)	-1.087007	0.042015	-25.87217	0.0000
C	-0.000312	0.001213	-0.257376	0.7970
@TREND(1)	**-4.99E-08**	**3.61E-06**	**-0.013803**	**0.9890**
R-squared	0.543614	Mean dependent var		-4.62E-05
Adjusted R-squared	0.541990	S.D. dependent var		0.021322
S.E. of regression	0.014430	Akaike info criterion		-5.633717
Sum squared resid	0.117023	Schwarz criterion		-5.610689
Log likelihood	1594.525	Hannan-Quinn criter.		-5.624729
F-statistic	334.7068	Durbin-Watson stat		1.876234
Prob(F-statistic)	0.000000			

The trend is not significantly different from zero, since it has an error of 98%, then we move to the second step.

Table 6. Model with constant

ADF Test Statistic	-25.89603			
Test critical values:	1% level		-3.441695	
	5% level		-2.866437	
	10% level		-2.569437	
* MacKinnon critical values for rejecting the hypothesis of a unit root				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
RENDEMENTS02(-1)	-1.087002	0.041976	-25.89603	0.0000
C	**-0.000327**	**0.000607**	**-0.538439**	**0.5905**
R-squared	0.543614	Mean dependent var		-4.62E-05
Adjusted R-squared	0.542803	S.D. dependent var		0.021322
S.E. of regression	0.014417	Akaike info criterion		-5.637256
Sum squared resid	0.117023	Schwarz criterion		-5.621905
Log likelihood	1594.525	Hannan-Quinn criter.		-5.631264
F-statistic	670.6042	Durbin-Watson stat		1.876243
Prob(F-statistic)	0.000000			

The constant is not also significantly different from zero. We will therefore move to the third step.

Table 7. Model without constant or trend

ADF Test Statistic	-25.90685			
Test critical values:	1% level		-2.569061	
	5% level		-1.941385	
	10% level		-1.616323	
* MacKinnon critical values for rejecting the hypothesis of a unit root				

From a purely econometric point of view, returns series of the two indices (CAC40 and Tunindex) are stationary in level. This enables us to accept the null hypothesis of unit root. This

finding can detect bubbles with anticipated growth constraints, but limited in scope, causing a divergence in prices from their fundamentals.

However, these returns characteristics like asymmetry, stationarity and flatness can be attributed to other abnormalities. We need therefore more discriminatory statistics to provide more reliable evidence to detect bubbles. Therefore, we present approximate entropy as a statistical alternative to analyze bubbles in the data.

Next, we analyze "ApEn" trends during the crash. Trends in Table 8 show a significant decline in "ApEn" (5.1%) during the crash period, confirming a "tell tale" sign indicative of a crisis. During the crash, approximate entropy is low because there is is a high probability of repetitive signals.

4.3 Approximate entropy during the crash

To examine entropy trends of a seven-year approximate observation window, we considered data around the crash period. The results presented in Table 8 and figure 1are based on the standardized ApEn (5, 1%). There is significant evidence of a decline in ApEn before and after the "subprime" crisis period of 2008. However, it is important to note that for the CAC 40 and TUNINDEX, analysis based on ApEn highlights speculative activities episode during 2008.

Table 8. Evolution of Approximate entropy (5 1%) during the crisis period (2008)

	T-3 (2005)	T-2 (2006)	T-1 (2007)	CRASH (2008)	T+1 (2009)	T+2 (2010)	T+3 (2011)
Tunindex	0,306021065	0,317860613	0,359688561	0,018768585	0,226076195	0,434369325	0,16380932
Cac 40	0,222111	0,267766	0,472253	0,019386	0,203728	0,353755	0,208573

Figure 1. Trends of approximate entropy (5.1%) during the crash
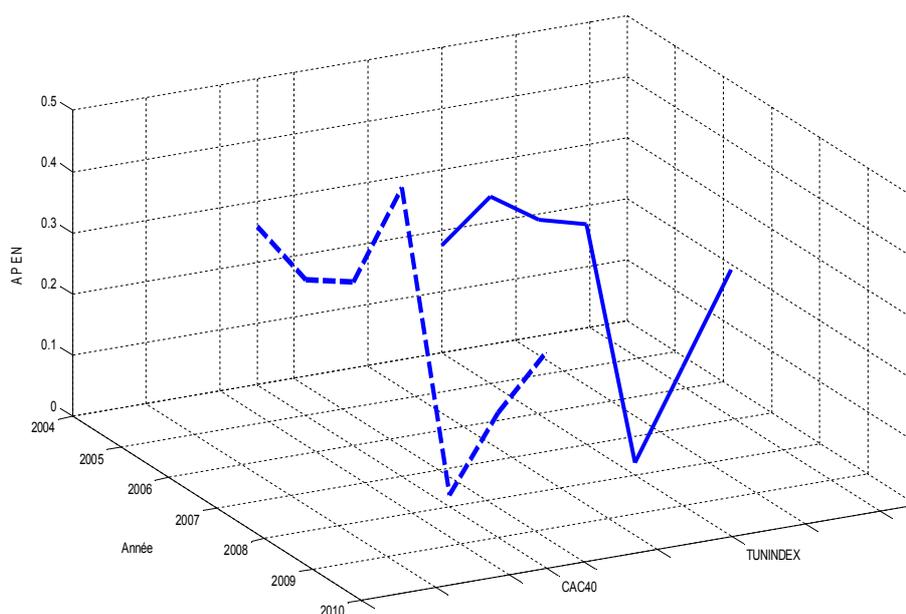


4.4 A case study of a developing market: Tunisia

As we have already mentioned, the study of speculative bubbles has gained a renewed interest in the context of developing economies. Despite a significant development which has made these markets almost on equal footing with the best in the world (in terms of structure, systems and regulations), recently these markets have witnessed several speculative activities episodes. They provide us with a unique opportunity to test ApEn power to identify bubbles episodes and other anomalies such as fraud or changing political scenarios.

There are two episodes of interest showing the Tunisian market collapse, the first episode considered as a potential bubble that took place in 2008 and during which several episodes of extreme movements were observed (table 9 and figure 2).

Table 9. Evolution of approximate entropy (5.1%) for Tunindex

Year	2002	2003	2004	2005	2006
EnAp(5,1%)	0,050601013	0,387110828	0,175505072	0,306021065	0,317860613
2007	**2008**	**2009**	**2010**	**2011**	**2012**
0,359688561	0,019768585	0,226076195	0,434369325	0,163809102	0,213820224

Figure 2. Approximate entropy (5.1%) for Tunindex during 10 years.
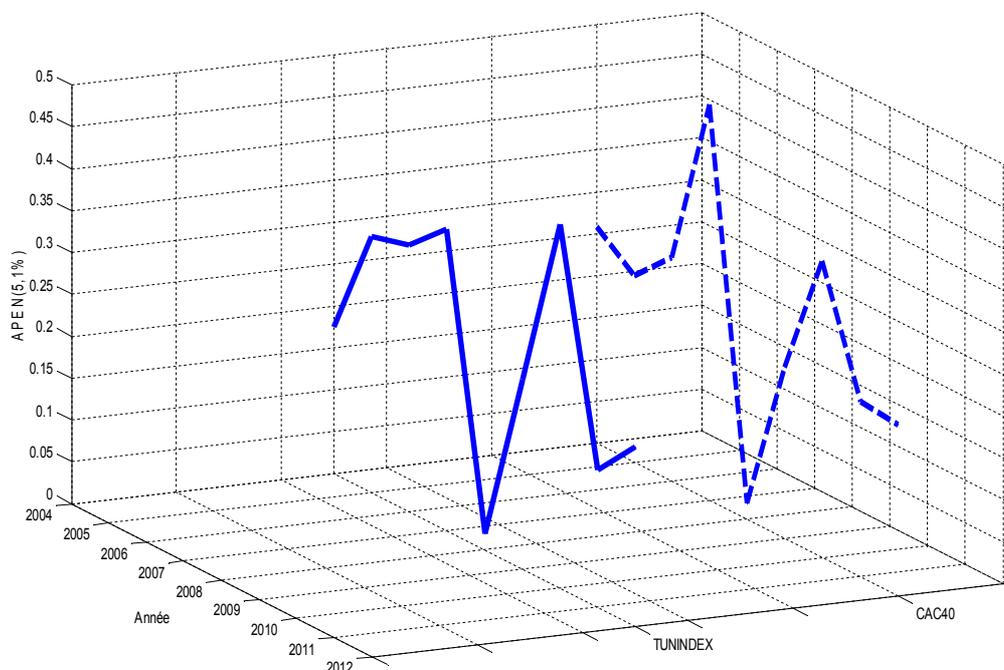
The first incident took place on October 17, 2007, when the Tunindex declined. However, Tunindex recovered sharply after the fall. It had its second biggest fall of 600 points during the month of January 2011 due to the change in the political scenario (fall of the Ben Ali regime). It also dropped dramatically from 4700 points to 4550 within a period of two weeks before elections date.

These two market crash episodes offer a unique opportunity to test ApEn power in explaining that the underlying factors that affect stock market crashes are very different and ApEn should be able to distinguish between these two episodes. As we expected, we should expect a significant lower level of "ApEn" during the period 2007-2008, because these events are often identified by the press as a bubble. Unlike in 2011, the crisis is purely dictated by political events and there may not have been a speculative activity.

5. Conclusion

In this paper, a new approach, that of entropy, has been proposed to detect speculative bubbles in the Tunisian and French financial markets during the crisis. However, in the presence of a rational bubble, returns series presents repetitive signals, leaving "tell tale" signs. Therefore, a rational bubble episode should be associated with relatively low approximate entropy values. Now, it is possible to detect any presence of bubbles in financial markets using this approach. This analysis allowed us to identify the bubble in the first quarter of 2008 as the approximate date of the beginning of the formation of this phenomenon on the Tunisian and French markets. We also note that the curve given by approximate entropy has two falls (2008, 2011). Nevertheless, the fall in (2008) is deeper than the one in (2011), pointing to the presence of bubbles in the Tunisian market during the "subprime" crisis.

Second, we highlighted ApEn to study characteristics of the Tunisian stock market in a crisis period. Furthermore, using the case study of the Tunindex we also showed ApEn's ability to differentiate the speculative bubble of a market of other crises. Although this study focuses primarily on crisis events, it can be extended to provide an early warning system that would allow investors and

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agents to be aware of the realities of price movements on the stock market. Early detection of a bubble evolution, characterized by a sharp drop of "ApEn" in a very short time, may help investors and policy makers to intervene and correct market anomalies, thus preventing market collapse.

Finally, it is important to note that ApEn has some weaknesses due to its dependence on sequence length (m) and standard deviation (r), since returns series have a fairly low standard deviation. Although we faced many problems, we have addressed these shortcomings and we changed the sequence (m) and standard deviation (r) according to our sample. Further research using statistical measures in the same line like approximate entropy (e.g sample entropy ... MSE), which provided a more solid understanding of rational speculative bubbles.

Références

Alvarez-Ramirez, J. (2011). Multiscale entropy analysis of crude oil price dynamics. Energy Economics, 33, 936–947.

Alvarez-Ramirez, J., Rodriguez E., Alvarez J. (2012). Efficiency of crude oil markets: Evidences from informational entropy analysis. Energy Policy, 41, 365-373.

Baumol, W.J. (1957). Speculation, profitability and stability. Review of Econometrics and Statistics, 39, 263-271.

Bhadri, S. (2012). Applying approximate entropy (ApEn) to speculative bubble in the stock market. Munich Personal RePEc Archive 11, 38015.

Blanchard, O.J. Watson, M.W. (1982). Bubbles, rational expectations and financial markets in Watchel, P. (Ed.), Crisis in the Economic and Financial System. Lexington Books,Lexington, Massachusetts.

Diba, B., Grossman, H. (1988). Explosive rational bubbles in stock prices. American Economic Review, 78, 23-50.

Dionísio, A. (2001). Entropy Analysis as Uncertainty Measure and Information Value in Portuguese Stock Market. Evora University.

Eoma, C., Oh, G., Jung, W.S. (2008). Relationship between efficiency and predictability in stock price change. Physica A ,387, 5511–5517.

Friedman, M. (1953). Essay in positive economics. Chicago. University of Chicago Press.

Gulko, L. (1999). The entropic market hypothesis, International Journal of Theoretical and Applied Finance, 2, 293-330.

Hardouvelis, G.H. (1988). Evidence on stock market speculative bubbles: Japan, the United States and Great Britain. Quarterly Review Federal Reserve Bank of New York, 4-6.

Kohn, M. (1978). Competitive Speculation. Econometrica, 46(5), 1046-1061.

McQueen, G., Thorle, S. (1994). Bubbles, stock returns, and duration dependence. Journal of Financial and Qualitative Analysis, 29, 196–7.

Oh, and Kim, (2006). Market Efficiency in Foreign Exchange Markets Division of Business Administration, Pusan National University Korea, 609-735.

Oh, G., Kim, S., Eom, C. (2007). Market efficiency in foreign exchange markets. Physica A, 382, 209-212.

Pincus, S.M. (1991) Approximate entropy as a measure of system complexity. Proceedings of the National Academy of Sciences USA, 88, 2297-2301.

Pincus, S.M., Kalman, R.E. (2004) Irregularity, volatility, risk, and financial market time series. Proceedings of the National Academy of Sciences, 101(38), 13709-13714.

Rappoport, P., White, E. (1993). Was there a bubble in the 1929 stock market? Journal of Economic History, 53, 549-574.

Risso, W.A. (2008). The informational efficiency and the financial crashes. Research in International Business and Finance, 22, 396–408.

Shiller, R.J. (1978). Rational expectations and the dynamic structure of macroeconomic models: A critical review. Journal of Monetary Economics, 5, 1-44.

Shiller, R.J. (1981). Do stock prices move too much to be justified by subsequent changes in dividends. American Economic Review, 71, 421-436.

Wang, G.J., Xie, C.W., Han, F. (2012). Multi-Scale Approximate Entropy Analysis of Foreign Exchange Markets Efficiency. Systems Engineering Procedia, 3, 201–208.

Zunino, L., Zanin, M.., Tabak, B.M., Pérez, D.G., Rosso, O.A. (2010). Complexity-entropy causality plane: A useful approach to quantify the stock market inefficiency. Physica A, 389, 1891-1901.

Zunino, L., Massimiliano, Z., Tabak, B.M., Pérez, D.G., Rosso, O.A. (2009). Forbidden patterns, permutation entropy and stock market inefficiency. Physica A, 387, 6558-6566.

Zunino, L., Tabak, B.M., Pérez, D.G., Garavaglia, M., Rosso, O.A. (2007) Inefficiency in Latin-American market Indices. The European Physical Journal, 60, 111-121.