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Default-implied Asset Correlation: Empirical Study for Moroccan Companies

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ABSTRACT

The asset correlation is a key regulatory parameter in the calculation of the capital charge for credit risk under the second Basel agreement. This parameter has been set in a uniform manner for all banking institutions wishing to integrate the Basel framework. However, estimation of the asset correlation has not often been discussed, even though it substantially affects the estimates of the unexpected loss. Importantly, it is essential that financial institutions use the appropriate method and data to calculate the asset correlation in order to compute the unexpected loss accurately. In this work, we developed the theoretical framework for the calculation of the default-implied asset correlation. Using the developed model, we calculated the correlation of the assets that was decreasing according to the probability of default. By comparing our model with the Basel model, we found a significant difference on the asset correlation value and the regulatory capital coefficient. This resulted in a large risk-weighted assets difference between our model and the Basel framework.

Keywords: Default-implied Asset Correlation, Credit Risk Modeling, Asymptotic Single Risk Factor
JEL Classifications: G17, G21, G24, G28, G32, G38

1. INTRODUCTION

Credit risk management is an essential part of any banking system. The Basel Committee (1999), as well as regulator and supervisor of the management of financial risks, obliges the banks belonging to this organization to respect the guidelines in management methodologies.

Credit risk, by definition, is factors that heavily impact the solvency of a bank. Having said that all procedures and decisions must consider management, anticipation and evaluation of this element.

The economy of a country is based primarily on the development of financial ecosystem; it is influenced by the fundamental role of banks. Any failure of the latter could have a dangerous impact on the stability of a country.

Our motivation in this work is to calculate the correlation of assets based on the default history, we proceed by:

- Observing the previous studies of asset correlation estimation;
- Developing the theoretical framework of calculation the default-implied asset correlations;
- Comparing our default-implied asset correlations to the asset correlation parameters in the Basel II Internal Rating Based (IRB) framework, and analyzing the impact on regulatory capital.

Before making any analysis, the asymptotic single risk factor model must be mathematically described, which has been used in the development of economic capital formulas for a financial institution. Great importance will be given to items of Vasicek (1991, 2002) and Gordy (1998).

In our analysis, we use Moroccan bank data (2008-2014) to estimate the default correlations for each rating class, because these data provide historical default data for Moroccan companies. Using this data, we calculate the default rate for each year and rating class, and we estimate default-implied asset correlations.

The structure of the paper is as follows:
- Section 2 presents the literature review, principally the definition of asset correlation and the Merton model used in the analysis.
- Section 3 provides the estimation of default-implied asset correlation.
- Section 4 discusses the results and draws conclusions.

2. LITERATURE REVIEW

2.1. One-factor Merton Model

The one-factor Merton model describes a company's value with a systematic factor (a factor common to the values of several companies) and an idiosyncratic factor (a factor specific to the company's value). The asset value of a company is then the weighted sum of a common (systematic) factor and an individual (idiosyncratic) factor. For example, when the macroeconomic development can be regarded as the systematic factor, the asset value of the company can be explained by the macroeconomic development and the company's individual factor. When these two factors change over time, the asset value of the company also changes.

In the Merton model, the occurrence of default is regarded as the time when the company's value is below a certain threshold at the time of maturity.

In this paper, we classify companies into several rating classes using the probability of default, and calculate the asset correlation for each class. We refer to the model that sets a common systematic factor for all companies as the Single index.

The basic formula of the one-factor Merton model is described by the following formula, where t ($t \geq 0$) is time and the asset value $Z_i(t)$ of company a_i is:

$$Z_i(t)=\sqrt{\rho_i}X(t)+\sqrt{1-\rho_i}\varepsilon_i(t) \tag{1}$$

$-1 \leq \rho\_i \leq 1$; $\quad i=1, 2,\ldots, n$

And n is the number of companies. The random variable of asset value $Z_i(t)$ is provided by two random variables: A common factor $X(t)$ that affects all companies and an idiosyncratic factor $\varepsilon_i(t)$ that affects only company a_i.

$X(t)$ and $\varepsilon_i(t)$ are independent of each other and follow a standard normal distribution.

2.2. Asset Correlation

Correlations between assets show how the value of a borrower's assets depends on the asset value of another borrower. As well as the dependence of a borrower's asset value on the general state of the economy (in the event of an economic crisis by impacting all assets). The correlations of the assets finally determine the form of the risk formulas. They depend on the asset class because different borrowers and/or asset classes have different degrees of dependence on the overall economy.

The Basel Committee has classified the calculation of the correlation into two formulas:
- Correlation for large companies:

$$\rho = 0.12*\frac{1-\exp(-50*PD)}{1-\exp(-50)} +0.24*\left(1-\frac{1-\exp(-50*PD)}{1-\exp(-50)}\right) \tag{2}$$

- Correlation for small and medium-sized companies:

$$\rho = 0.12*\frac{1-\exp(-50*PD)}{1-\exp(-50)}+0.24*\left(1-\frac{1-\exp(-50*PD)}{1-\exp(-50)}\right) -0,04*\left(1-\frac{s-10}{156}\right) \tag{3}$$

In this formula, "s" corresponds to the consolidated annual turnover expressed in millions of dirham (Moroccan currency). Any company, whose turnover is lower to 50 million dirham, is treated as equivalent to this amount.

These correlations, proposed by the Basel Committee, were deduced from the study of the loss profiles of different portfolios of large G10 banks In the Merton model (1974).

The model of BIS for the calculation of regulatory capital has been criticized by some works, mainly by the analysis of Altman, Saunders (2001).

In parallel with the recommendations of the Basel Committee, there was several works which attempted to estimate the correlation of assets using default-data history. The Table 1 gives us a summary of previous studies of asset correlation that use a Merton-type single-factor model with time-series default data.

On the other hand, the work of Stoffberg, Van Vuuren (2015), Bandyopadhyay (2016) have used the correlation of assets by calculating the Risk Weighted Assets without studying the correlation of defaults, Such as.

2.3. Risk Weighted Assets (RWA)

RWA are computed by adjusting each asset class for risk in order to determine a bank's real world exposure to potential losses.

Table 1: Previous studies of asset correlation that use a Merton-type single-factor model with time-series default data

Author/Article	Default data	Asset correlation
Gordy and Heitfiel, 2002	Moody's (1970-1998)	0.06-0.11
Hamerle and Liebig, 2003	S&P (1982-1999)	0.04-0.07
Bluhm and Overbeck, 2003	Moody's (1970-2001)	0.12-0.43
Lopez, 2004	KMV Credit Monitor Database	0.10-0.55
Dietsch and Petey, 2004	Germany, 280,000 companies (1997-2001)	0-0.066
Jakubik, 2006	Monthly default rate in Finland (1988-2004)	0.15-0.017

Regulators then use the risk weighted total to calculate how much loss-absorbing capital a bank needs to sustain it through difficult markets. Under the Basel III rules, banks must have top quality capital equivalent to at least 7% of their RWA or they could face restrictions on their ability to pay bonuses and dividends.

The risk weighting varies accord to each asset's inherent potential for default and what the likely losses would be in case of default - so a loan secured by property is less risky and given a lower multiplier than one that is unsecured.

The formula for calculating RWA is in the form of:

$RWA=K*EAD$

Exposure at default (EAD)[1]: Is seen as an estimation of the extent to which a bank may be exposed to counterparty in the event of, and at the time of, that counterparty's default. EAD is equal to the current amount outstanding in case of fixed exposures like term loans. In our calculation the value of loss given default (LGD) was set at 45%.

K: Is the capital requirement is in the form of:

$$K = \left[\mathrm{LGD} * \mathrm{N} \left[\begin{array}{l} \dfrac{1}{(1-\rho)^{0,5}} * N^{-1}(\mathrm{PD}) \\ + \left(\dfrac{\rho}{1-\rho} \right)^{0,5} * N^{-1}(0,999) \end{array} \right] - \mathrm{PD} * \mathrm{LGD} \right] * 12,5 \quad (4)$$

- PD is the probability that the borrower falls default and LGD is the loss rate in the presence of a fault. For large companies the Basel Committee proposed this formula.

The probability of default corresponds to:

$PD_i=P[A_i<B_i]$

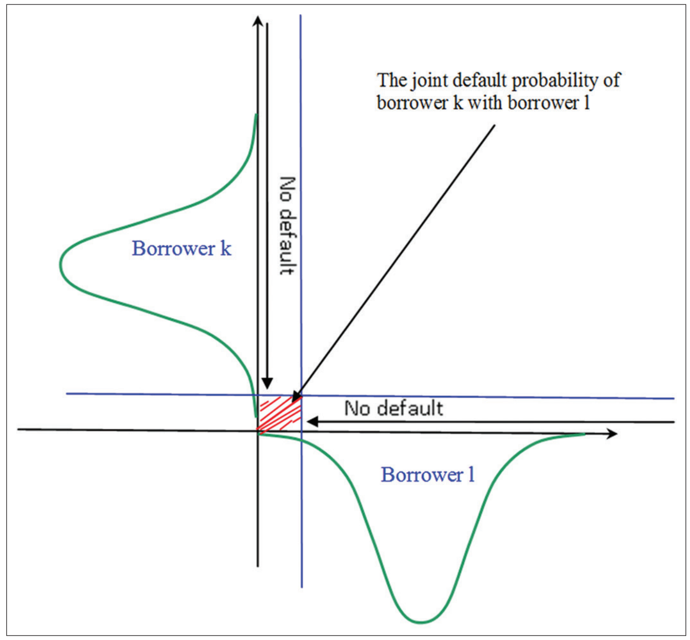
With:
- A_i is the asset value i
- B_i is the value of obligations i
- N is the standard normal cumulative distribution function
- N^{-1} is the inverse of the standard normal cumulative distribution function.

3. METHODOLOGY AND DATA

3.1. Estimate of the Default-implied Asset Correlation

Based on the article of J.Zhang et al. (2008), it is possible to deduce the default correlation of two borrowers by determining their individual default probabilities and their asset correlation. A borrower will be probably in default when its asset value falls under the value of its obligations (i.e., its default position). The joint probability of two borrowers defaulting during the same time period is simply the possibility that both borrowers' asset values falling under their respective default points through that period. This likelihood can be determined from knowing the correlation between the two firms' asset values and the individual probability of each firm defaulting, as depicted in Graph 1.

Graph 1: Estimate of the default-implied asset correlation (default position)



The joint default probability of borrower k with borrower l, represented by JDP_{kl}:

So JDP_{kl}=Pr (Asset value k<default point k; asset value l<default point l) (5.1)

JDP_{kl}=Pr[$A_k \leq B_k$ and $A_l \leq B_l$]=$N(B_k; B_l; \rho_{kl})$ (5.2)

B_k is the obligation of borrower k

The implicit asset values of two obligors at the horizon are jointly normally distributed and their Joint default probability follows a bivariate distribution. The JDP_{kl} can therefore be obtained by using the following expression:

$$JDP_{kl} = \frac{1}{2\pi\sqrt{1-\rho_{kl}^2}} \int_{-\infty}^{B_k} \int_{-\infty}^{B_l} \exp[-\frac{1}{2} \frac{x^2 - 2\rho_{kl}xy + y^2}{1-\rho_{kl}^2}]dxdy \quad (5.3)$$

$N(B_k; B_l; \rho_{kl})$ is the cumulative bivariate standard normal distribution.

ρ_{kl} is the asset correlation between two obligors k and l are their default thresholds.

We calculate JDP_{kl} with the *BIVNOR* function such that:

JDP_{kl}=*BIVNOR* (*NormInver*(PD_k); NormInver (PD_l); ρ_{kl}) (5.4)

BIVNOR is the cumulative bivariate standard normal inverse

1 Draft Supervisory Guidance on Internal Ratings-based Systems for Corporate Credit.

In our work, we calculated the value of this function with VBA programming (see Appendix: Code VBA for the bivariate normal distribution)

The complexity now to estimate default-implied Asset Correlation is to determine the joint probability of default JDP_{kl}. Based on the Lucas and Douglas (1995), the Joint probability of default is determined by firs looking at the number of C-rated companies that default in a particular year and by computing all possible pairs of defaulting C-rated companies in that year. If d number of C-rated companies defaults in a year, the possible pairs are $d(d-1)/2$.

This process is repeated for all years, in which there are one-year default rates, form 2008 to 2014. Then all 8 years of results are summed. That summation is used as a numerator of JDP_{kl}.

The number of total possible combinations of C-rated companies in each year is next computed and summed. That summation is used as a denominator, and the ratio is the calculation of the historic joint probability of default, or JDP_{kl}.

For a class C having n firms and d companies in default after one year, the joint probability is in the form:

$$P(k\ et\ l) = \frac{(d(d-1)/2)}{(n(n-1)/2)} \tag{6.1}$$

$$JDP_{kl} = \frac{d(d-1)}{n(n-1)} \tag{6.2}$$

Such as:

n: Is the size of the class;

d: Is the number of companies in default;

So now to determine the correlation between two assets k and l, we must solve the equation:

$$\frac{d(d-1)}{n(n-1)} = BIVNOR\,(NormInver\ (PD_k);\ NormInver\ (PD_l);\rho_{kl} \tag{6.3}$$

Whose, the unknown is $\rho_{l,k}$ default-implied asset correlation between two assets k and l. The correlation of the class C will be the average of the correlations of the assets, denoted ρ_c.

Borrowers' k and l belong to the C rating class.

To solve this equation, we will use iterations to find the exact value of the asset correlation. For this we have found it useful to use the Binary search algorithm, the huge advantage of this algorithm is that it's complexity depends on the array size logarithmically in worst case. In practice it means, that algorithm will do at most $ln(_n)$ iterations, which is a very small number even for big arrays. It can be proved very easily. Indeed, on every step the size of the searched part is reduced by half. Algorithm stops, when there are no elements to search in.

The instructions of the algorithm are:

1. Set L to 0 and R to$n-1$.
2. If $L > R$, the search terminates as unsuccessful.
3. Set m (the position of the middle element) to the floor (the largest previous integer) of $(L+R)/2$.
4. If $A_m < T$, set L to $m+1$ and go to step 2.
5. If $A_m > T$, set R to$m-1$ and go to step 2.
6. Now $A_m=T$, the search is done; return m.

This iterative procedure keeps track of the search boundaries via two variables. Some implementations may place the comparison for equality at the end of the algorithm, resulting in a faster comparison loop but costing one more iteration on average. In the annex, the programming work leading to the calculation of the default-implied asset correlation.

3.2. Confidence Interval for Default-implied Asset Correlation

The objective of this section is to develop a confidence interval of the estimated default-implied correlation. The difficult is that the correlation coefficient of Pearson is not a normally distributed variable, its distribution is bounded to +1 and −1, whereas the normal distribution is defined on the set of real numbers. The solution is quite simple to implement that one can apply a correction to the values of ρ_c, called Fisher transformation (the same as ANOVA). After the transformation of the value ρ_c into the note ρ_c', the distribution obtained is approximately normal. The transformation is the so-called "hyperbolic arctangent" function whose formula is:

$$\rho_{c'} = \frac{1}{2} ln\left(\frac{1+\rho_{\rho_c}}{1-\rho_c}\right) \tag{7.1}$$

ρ_c is the default-implied asset correlation for the rating class C it's the average correlation between the assets in the class C.

That we take the natural logarithm of the ratio $(1+\rho_c)/(1-\rho_c)$, and dividing the result by 2. We also note the limits to this transformation because it is not defined where $\rho_{l,k}$ is exactly equal to +1 or −1 because.

a. We cannot divide by 0 (or if $\rho_c=1$ then $1-\rho_c=0$)
b. The logarithm function is not defined for the value 0 (or if $\rho_c=-1$, then $1+\rho_c=0$ and the ratio is equal to 0).

Whose $\mathrm{Var}\left(\rho_c'\right) = \frac{1}{n-3}$

So the boundaries of IC are:

$$\text{Lower limit} = \frac{1}{2}\ln\left(\frac{1+\rho_c}{1-\rho_c}\right) - Z_{\left(1-\frac{\alpha}{2}\right)} * \sqrt{\frac{1}{n-3}} \tag{7.2}$$

$$\text{Upper limit} = \frac{1}{2}\ln\left(\frac{1+\rho_c}{1-\rho_c}\right) + Z_{\left(1-\frac{\alpha}{2}\right)} * \sqrt{\frac{1}{n-3}} \tag{7.3}$$

n is the number of possible correlations,

3.3. Data

In our study, we use Moroccan bank data (2008-2014) to estimate the default-implied Asset Correlation for each rating class. Using

Table 2: Database from 2008 to 2014

Company	Company in default	Equity/net debt	Profit margin ratio R	Repayment capacity	Immediate liquidity	Growth in net income (%)	Net income/ financial expenses (%)	Turnover/ financial expenses	Turnover in DH	Default rate (%)
Company 1	0	57.167	0.0470	0.47477	51.7500	12	23	0.651230419	13971000	0.247
Company 2	0	63.018	0.0402	0.10866	8.8680	19	90	0.733891279	19720269	0.247
Company 3	0	NR	0.0926	0.54576	NR	18	88	1.619381978	11007451	0.081
Company 4	0	NR	0.4449	0.70866	NR	25	−38	1.745411599	3062562	0.493
Company 5	0	20.939	0.0919	0.23953	12.6046	10	11	3.771266028	8246704	0.354
Company 6	0	12.123	0.0885	0.18828	2.8733	10	−9	4.217192557	5053760	0.354
Company 7	0	5.736	0.0297	0.43508	3.1088	NR	24	5.362358031	35970014	0.373
Company 8	0	30.410	0.0888	0.48949	20.2798	23	10	5.447182444	3928601	1.040
Company 9	0	4.716	0.0259	0.21854	1.2491	158	−7	5.752430191	33055014	0.543
Company 10	0	0.423	0.4819	0.95630	2.9094	15	16	6.811081694	11437780	0.444
Company 11	0	0.506	0.0039	0.49593	1.2065	19	19	7.008248881	40608546	0.893
Company 12	0	5.065	0.0473	0.25023	1.5770	10	52	8.741721854	4342163	1.508
Company 13	0	1.156	0.0360	0.48585	1.3716	21	55	8.854984894	3211505	1.622
	……	……	……	……	……	8	……	……	……	……
	……	……	……	……	……	……	……	……	……	……
Company 1.959	1	1.038	0.0117	0.00673	0.1078	−10	−30	0.352	3169000	11.935
Company 1.960	1	2.059	0.0013	0.021321	0.0989	−28	−59	0,326436782	4057000	12.498

0 means that the company is not in default. 1 means that the company is in default

Table 3: Rating grid

Rating class	Percentage of class	Cummulated percentage	Average default rate (%)	Percentage of empirical default (%)	Score
A	8	7.6	0.16	0.67	[96; 85.5]
B	5.2	12.8	0.32	0.99	[85.5; 80.25]
C	18	30.7	0.48	1.14	[80.25; 74.5]
D	30.7	61.3	0.80	1.33	[74.5; 66.5]
E	15	76.7	1.42	1.66	[66.5; 62.5]
F	15.4	92.0	2.35	2.66	[62.5; 56]
G	7	99.2	4.24	4.26	[56; 46.5]
H	1.1	100	9.06	9.09	[46.5; 34.75]

this data, we calculate the default rate for each year and rating class, and we estimate default-implied asset correlations. The number of companies included in the study is 1.960 for small and medium-sized enterprises and large enterprises (defaulted and not defaulted companies).

Our database is in the form presented in Table 2.

4. RESULT AND DISCUSSION

The objective of this section is to compare our model developed with Basel II IRB framework in terms of asset correlations values, the coefficient of the capital requirement (K) and RWA amount.

We proceed by the calculation of the default probability, and the determination of the rating classes, and thereafter the estimating of default-implied asset correlations, and finally comparing our default-implied asset correlations to the asset correlation parameters in the Basel II IRB framework, and analyzing the impact on regulatory capital.

4.1. Calculate the Default Rate

Based on a data history over a period of 6 years, we estimated the default rate[2], by using the logistic regression[3]. The rating classes are presented in the Table 3.

It should be noted that we have 8 rating classes with different rates of defaults, presenting the quality of the borrowers. The three classes A, B and C have very good credit quality with a default rate less than 0.5%.

4.2. Calculation of the Default-implied Asset Correlation

The Table 4 shows the results by ratings. Firms rated into 8 rating classes, the default-implied asset correlations range from 8.1% to 25.1%.

2 Based on the previous default prediction research, list of most frequently used financial ratios was assessed, and calculated for each defaulted and healthy company in the sample. The data patterns were analyzed for the total data set and for each of the groups of companies separately (defaulted and healthy group). Two main groups of methods were used to test the posed hypothesis and answer the research questions. The collected data was analyzed by a group of traditional statistical methods represented by logistic regression and multiple discriminant analysis.

3 Logistic regression as a statistical method is suited and usually used for testing hypothesis about relationships between a categorical dependent or an outcome variable and one or more categorical or continuous predictor or independent variables. The dependent variable in logistic regression is binary or dichotomous.

For the rating class A there is only one default, this single default leads to a very high default-implied asset correlation. For the rest of the ratings, we observe that default-implied correlation generally increases with better credit quality.

Our results presented in this analysis based on the population marginal small firms. On average, small firms tend to have higher default probabilities and lower asset correlations.

4.3. Confidence Interval of the Default-implied Asset Correlation

Based on the correlation confidence interval estimation formula developed in the previous section, we have established the confidence intervals for each value of the default-implied asset correlation. The Table 5 shows the bounds of the confidence interval at 95% confidence level.

4.4. Comparison between Default-implied Asset Correlation and Asset Correlations in the Basel II IRB

We calculated the correlation of assets based on the Basel II IRB framework. For corporate borrowers, the asset correlation parameter ρ is given as function of PD:

The Table 6 shows the values of the two correlations: Default implied asset correlation and Basel II IRB asset correlation.

From this table, it should be noted that the two Basel II IRB asset correlations (with and without size adjustment) are different from the default-implied asset.

The two correlations with and without the size adjustment, is plotted in Graph 2, together with the default-implied asset correlation

It should be noted that our model gave asset correlation values decreasing in function of the probability of default observed on each rating class. The greatest correlation is that of Class A, with a value of 25.1%, Pr against the small value is that of the last class of notation, which of 8.1%.

The Tables of Rating classes and default rate are Highlights in the Appendix (see Appendix: Rating classes and Scores and default rate).

We can see that the Basel II correlation function for large corporate borrowers is roughly in line with our empirical estimates from our default data.

As discussed before, smaller firms tend to have smaller asset correlation. The default-implied asset correlations are lower than the two Basel II correlations function with and without size adjustment; this will have a significant impact on the value of regulatory capital, what we will see in the next section.

4.5. Value of RWA using the Basel IRB Framework

The Table 7 shows the RWA value based on Basel IRB framework asset correlation.

Table 4: Default-implied asset correlation

Rating class	Average number of firms	Average default rate (%)	Joint probability of defaults (%)	Default implied asset correlation (%)
A	149	0.16	1.3	25.1
B	101	0.32	2.0	18.5
C	351	0.48	1.4	17.4
D	601	0.80	1.5	15.0
E	301	1.42	2.0	13.9
F	301	2.35	3.0	11.7
G	141	4.24	5.0	9.4
H	22	9.06	13.6	8.1

Table 5: Confidence interval of the default-implied asset correlation

Rating class	Average default rate (%)	Default implied asset correlation (%)	Average number of firms	Confidence interval (%)	
				Lower bound	Upper bound
A	0.16	25.1	149	23.23	26.97
B	0.32	18.5	101	15.70	21.22
C	0.48	17.4	351	16.65	18.24
D	0.80	15.0	601	14.59	15.51
E	1.42	13.9	301	12.98	14.82
F	2.35	11.7	301	10.79	12.63
G	4.24	9.4	141	7.46	11.41
H	9.06	8.1	22	−4.85	21.11

Table 6: Default implied asset correlation and Basel II IRB asset correlation

Rating class	Average default rate (%)	Default implied asset correlation (%)	Basel II IRB asset correlation (large firms) (%)	Basel II IRB asset correlation (small firms) (%)
A	0.16	25.1	23.1	20.0
B	0.32	18.5	22.2	19.2
C	0.48	17.4	21.4	18.4
D	0.80	15.0	20.0	17.0
E	1.42	13.9	17.9	14.9
F	2.35	11.7	15.7	12.7
G	4.24	9.4	13.4	10.4
H	9.06	8.1	12.1	9.1

IRB: Internal Rating Based

To properly understand the impact of risk, it is necessary to observe the coefficient k, which is directly related to the probability of default and the correlation of assets. It should be noted that a significant default probability implies a higher k coefficient. The rating class A has a 50% cost of own funds, for the reason that it has a lower probability of default. Moreover, the last three classes of notation: F, G and H have a coefficient k greater than 150% in line with the highest probability of default, which is very logical then that a company that is more risky requires a higher regulatory fund.

4.6. Calculation of RWA, using the Default-implied Asset Correlation

Using the default-implied asset correlation, we calculated the capital requirement (k) and the RWA, as shown in the Table 8.

Graph 2: Default implied asset correlation versus Basel II IRB asset correlation

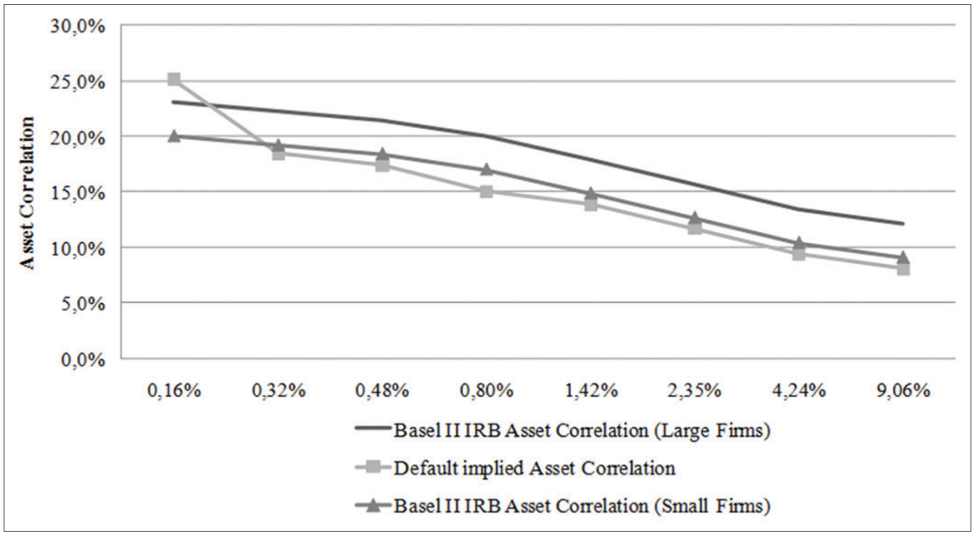


Table 7: RWA value using Basel IRB framework asset correlation

Rating class	Average default rate (%)	Basel II IRB asset correlation (large firms) (%)	Capital requirement (K) (%)	RWA
A	0.16	23.1	52.1	1686
B	0.32	22.2	74.2	20045
C	0.48	21.4	89.3	19059
D	0.80	20.0	109.5	24036
E	1.42	17.9	131.2	20427
F	2.35	15.7	149.0	4279
G	4.24	13.4	172.7	144
H	9.06	12.1	220.3	20

RWA: Risk weighted assets, IRB: Internal Rating Based

Table 8: Risk weighted assets. using the default-implied asset correlation

Rating class	Average default rate (%)	Default implied asset correlation (%)	Capital requirement (K), using implied asset correlation (%)	Risk weighted assets, using implied asset correlation
A	0.16	25.1	59.2	1916
B	0.32	18.5	57.8	15605
C	0.48	17.4	68.6	14643
D	0.80	15.0	78.3	17182
E	1.42	13.9	100.4	15634
F	2.35	11.7	112.6	3234
G	4.24	9.4	128.5	107
H	9.06	8.1	166.4	15
Total				68337

It should be noted that the capital requirement values (k) increase significantly, in line with the default rate and asset correlation estimated, over an interval of 59.2% to 166.4%.

For RWA, the total stood at 68.337 million DHs.

Table 9: Risk weighted assets: Basel II IRB versus implied asset correlation, in million DH

Rating class	Average default rate (%)	Risk weighted assets, using Basel II IRB	Risk weighted assets, using implied asset correlation	Risk weighted assets, using Basel II standard approach
A	0.16	1686	1916	1839
B	0.32	20045	15605	22703
C	0.48	19059	14643	15821
D	0.80	24036	17182	15212
E	1.42	20427	15634	11208
F	2.35	4279	3234	1961
G	4.24	144	107	60
H	9.06	20	15	40
Total		89696	68337	68844

IRB: Internal Rating Based

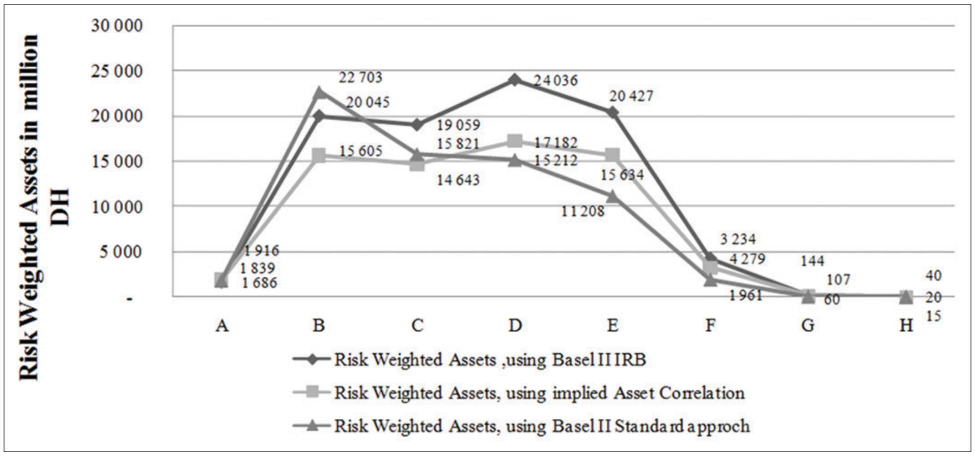
4.7. Comparison the RWA, using the Default-implied Asset Correlation versus Basel IRB Framework Asset Correlation

The Table 9 shows the comparison of the RWA values, calculated by the three methods: Using Basel II IRB framework asset correlation, Basel I standard approach and our method using implied asset correlation.

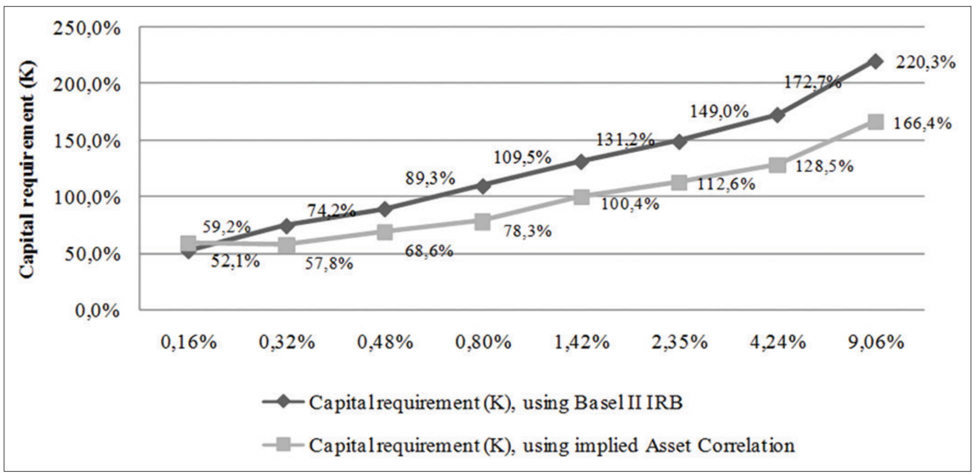
By observing this Table 9, it should be noted that the implied-asset correlation has a great advantage in regulatory minimization, which should be considered on the total RWA of the three methods. The Basel II IRB Framework gives an overestimation of RWA with a difference of more than 20 million DH, which is very expensive for this bank. Also, the standard approach also gives an increase of 0.5 million DH on the RWA.

At the conclusion of this comparison it can be deduced that the implied-asset correlation method gives a great advantage to the bank to converge towards the Basel committee advanced

Graph 3: Risk weighted assets: Basel II IRB versus implied asset correlation, in million DH



Graph 4: Capital requirement (K): Basel II IRB versus implied asset correlation, in million DH



approach and while setting an adequate level of regulatory capital.

This difference of RWA is clearly highlighted in the Graph 3.

This difference of the values RWA is impacted by the difference of the coefficient of the capital requirement (K), which can be seen in the Graph 4.

5. CONCLUSION

This study was carried out in order to better analyze the asset correlation formula in the framework of the Basel approach. Our motivation first to verify the reliability of this formula for a Moroccan bank, knew that this formula was calculated on a very different sample to the Moroccan economic situation. For this, we proposed an implied asset correlation based on the observations of the history of defaults. Above all, several studies have been carried out on the estimation of this correlation and have shown that the development of an implied asset correlation specific to the situation and the environment of the bank, remain more reliable and relevant.

After applying the implied asset correlation formula to the bank's portfolio, we released the following results:

1. With the implied asset correlation, we have asset correlation values decreasing as a function of the probability of defaults;
2. Our model resulted in a significant difference in the calculation of the weighting coefficient for the two Basel standard and IRB approaches;
3. The bank could converge these tools and methods towards the Basel Committee's advanced approaches, if it uses an implied asset correlation to have a minimum amount of regulatory capital.

The limit of this research is conditioned by the observation years of which 6 years have been exploited (from 2008 to 2014) in the calculation of the implied asset correlation, the increase of this number could certainly increase the certainty of the calculation of the this correlation.

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APPENDIX

1. Rating classes

Rating grid					
Rating class	**Percentage of class**	**Cummulated percentage**	**Average default rate (%)**	**Percentage of empirical default (%)**	**Score**
A	8	7.6	0.16	0.67	[96; 85.5]
B	5.2	12.8	0.32	0.99	[85.5; 80.25]
C	18	30.7	0.48	1.14	[80.25; 74.5]
D	30.7	61.3	0.80	1.33	[74.5; 66.5]
E	15	76.7	1.42	1.66	[66.5; 62.5]
F	15.4	92.0	2.35	2.66	[62.5; 56]
G	7	99.2	4.24	4.26	[56; 46.5]
H	1.1	100	9.06	9.09	[46.5; 34.75]

2. Scores and default rate

	Company in default	**Score**	**Default rate (%)**
Company 1	0	75.00	0.247
Company 2	0	75.00	0.247
Company 3	0	67.50	0.081
Company 4	0	57.50	0.493
Company 5	0	73.00	0.354
Company 6	0	73.00	0.354
Company 7	0	72.00	0.373
Company 8	0	66.50	1.040
Company 9	0	69.50	0.543
Company 10	0	72.50	0.444
Company 11	0	66.50	0.893
Company 12	0	64.00	1.508
Company 13	0	63.75	1.622
	……	……	……
	……	……	……
Company 1.959	1	43.5	11.935
Company 1.960	1	42.50	12.498

3. Code VBA for the bivariate normal distribution

```
Function BIVAR(x1 As Double, x2 As Double, rho As Double)
Dim sgn1 As Double, sgn2 As Double, delta As Double
Dim rho1 As Double, rho2 As Double, q1 As Double, q2 As Double
9020 If x1 * x2 * rho > 0 Then GoTo 9100
9030 If x1 <= 0 And x2 <= 0 And rho <= 0 Then BIVAR = altbivar(x1, x2, rho): GoTo 9500
9040 If x1 <= 0 And x2 >= 0 And rho >= 0 Then BIVAR = cumnorm(x1) - altbivar(x1, -x2, -rho): GoTo 9500
9050 If x1 >= 0 And x2 <= 0 And rho >= 0 Then BIVAR = cumnorm(x2) - altbivar(-x1, x2, -rho): GoTo 9500
9060 If x1 >= 0 And x2 >= 0 And rho <= 0 Then BIVAR = cumnorm(x1) + cumnorm(x2) - 1 + altbivar(-x1, -x2, rho)
: GoTo 9500
9100 If x1 < 0 Then sgn1 = -1 Else sgn1 = 1
9110 If x2 < 0 Then sgn2 = -1 Else sgn2 = 1
9120 rho1 = (rho * x1 - x2) * sgn1 / (Sqr(x1 * x1 - 2 * rho * x1 * x2 + x2 * x2))
9130 rho2 = (rho * x2 - x1) * sgn2 / (Sqr(x1 * x1 - 2 * rho * x1 * x2 + x2 * x2))
9140 delta = (1 - sgn1 * sgn2) / 4
9150 If x1 > 0 And rho1 >= 0 Then q1 = 0.5 - altbivar(-x1, 0, -rho1): GoTo 9300
9160 If x1 > 0 And rho1 < 0 Then q1 = cumnorm(x1) - 0.5 + altbivar(-x1, 0, rho1): GoTo 9300
9170 If x1 < 0 And rho1 >= 0 Then q1 = cumnorm(x1) - altbivar(x1, 0, -rho1): GoTo 9300
9180 q1 = altbivar(x1, 0, rho1)
9300 If x2 > 0 And rho2 >= 0 Then q2 = 0.5 - altbivar(-x2, 0, -rho2): GoTo 9400
9310 If x2 > 0 And rho2 < 0 Then q2 = cumnorm(x2) - 0.5 + altbivar(-x2, 0, rho2): GoTo 9400
9320 If x2 < 0 And rho2 >= 0 Then q2 = cumnorm(x2) - altbivar(x2, 0, -rho2): GoTo 9400
9330 q2 = altbivar(x2, 0, rho2)
9400 BIVAR = q1 + q2 - delta
9500 End Function
```

```
Function altbivar(x1 As Double, x2 As Double, rho As Double)
Dim a(4) As Double, b(4) As Double, aa As Double, bb As Double
Dim cum As Double, i As Integer, j As Integer
Dim f As Double
If rho > 0.999999 Then rho = 0.999999
If rho < -0.999999 Then rho = -0.999999
9600 a(1) = 0.325303: a(2) = 0.4211071: a(3) = 0.1334425: a(4) = 0.006374323
9610 b(1) = 0.1337764: b(2) = 0.6243247: b(3) = 1.3425378: b(4) = 2.2626645
9620 aa = x1 / (Sqr(2 * (1 - rho * rho))): bb = x2 / (Sqr(2 * (1 - rho * rho)))
9630 cum = 0
9640 For i = 1 To 4
9650 For j = 1 To 4
9660 f = Exp(aa * (2 * b(i) - aa) + bb * (2 * b(j) - bb) + 2 * rho * (b(i) - aa) * (b(j) - bb))
9670 cum = cum + a(i) * a(j) * f
9680 Next j
9690 Next i
9700 altbivar = cum * Sqr(1 - rho * rho) / 3.1415927
End Function
```

```
Public Function cumnorm(x As Double) As Double
Dim q As Double, primex As Double, y As Double
Dim alpha As Double, a1 As Double, a2 As Double
Dim a3 As Double, a4 As Double, a5 As Double, dum As Double
'   This calculates the cumulative normal function.
alpha = 0.2316419
a1 = 0.31938153
a2 = -0.356563782
a3 = 1.781477937
a4 = -1.821255978
a5 = 1.330274429
q = 1 / (1 + alpha * Abs(x))
primex = 0.3989423 * Exp(-x * x * 0.5)
dum = (a3 + q * (a4 + a5 * q))
primex = primex * (q * (a1 + q * (a2 + q * dum)))
If x < 0 Then
    y = primex
Else
    y = 1 - primex
End If
cumnorm = y
End Function
```

```
 Sub AssetCorrelation()
 Dim p1, p2, epsilon As Variant
 epsilon = 0.001

 Dim L As Double
 Dim R As Double
 Dim M As Double

  L = -1
 R = 1

'Start loop
 Do While (R - L) > epsilon
  'Calcule le milieu du domaine de définition
  M = (R + L) / 2

  'Find f(xM)
  p1 = BIVAR(0.03, 0.12, L) - 0.2994
  p2 = BIVAR(0.03, 0.12, M) - 0.2994

  If p1 * p2 > 0 Then

    'jette la moitié de gauche
    L = M
  Else
    'jette la moitié de droite
    R = M
  End If
Loop
MsgBox M
 End Sub
```