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Empirical Testing of Modified Black-Scholes Option Pricing Model Formula on NSE Derivative Market in India

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ABSTRACT: The main objectives of this paper are to incorporate modification in Black-Scholes option pricing model formula by adding some new variables on the basis of given assumption related to risk-free interest rate, and also shows the calculation process of new risk-free interest rate on the basis of modified variable. This paper also identifies the various situations in empirical testing of modified and original Black-Scholes formula with respect to the market value on the basis of assumed and calculated risk-free interest rate.

Keywords: Black-Scholes Option Pricing Model; Empirical testing; Suggested modification
JEL Classifications: C30; G13; G17

1. Introduction

Financial derivative is a core area of financial mathematics, under financial derivative Black-Scholes Option Pricing Model shows suitable use of financial mathematics to derive the formula of valuation of Call and Put option but the derivation of Black-Scholes formula is more problematical in nature due to the use of Partial Differential Equation (PDE), Partial Differentiation, Definite Integration, Logarithmic Properties, Exponential, Maxima & Minima and other mathematical function. On the basis of formula complication, researchers and other scholars who are interested to understand the valuation formula of BSM they derive different types of meanings and conclusions from that formula. Black-Scholes Formal is a combine work of two persons Fischer Black and Myron Scholes, maximum contribution to B-S model is by Fischer Black but due to less knowledge of Partial Differential Equation (PDE) he failed to derive final result. On the other hand Myron Scholes work on PDE on the movement of stock price helped him to contribute to the field. Finally in 1973, both of them worked together and shared their ideas, and developed Black-Scholes Option Pricing Formula for Call and Put option valuation.

Structure of Black-Scholes Option Pricing Formula for Call or Put option given as:

$$v_{cop} = PN\left(\frac{\log\frac{P}{P_E} + (R_f + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}\right) - e^{-R_f t}P_E N\left(\frac{\log\frac{P}{P_E} + \left(R_f - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right) \quad (1)$$

and

$$V_{\text{pop}} = e^{-R_f t} P_E N\left(-\left(\frac{\log\frac{P}{P_E}+\left(R_f-\frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right)\right) - PN\left(-\left(\frac{\log\frac{P}{P_E}+\left(R_f+\frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right)\right) \quad (2)$$

Where:

P = Current price of underlying stock;
P_E = The exercise or execute or strike price;
R_f = Risk free interest rate or Risk free rate of returns;
N = The value of the cumulative normal distribution at $\left(\frac{\log\frac{P}{P_E}+\left(R_f+\frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right)$ ***and*** $\left(\frac{\log\frac{P}{P_E}+\left(R_f-\frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right)$;
t = The time remaining to expiration on an annual basis;
$e^{-R_f t} = 1 + (-R_f t) + \frac{(-R_f t)^2}{2!} + \frac{(-R_f t)^3}{3!} + \frac{(-R_f t)^4}{4!} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \approx 2.7183$; and
σ = The standard deviation of the continuously compounded annual rate of return or long-returns of the share and commonly referred to as volatility;
Above BSM formula is based on various assumption details are given blow:

- Trading in stock take place continuously and market are always open;
- Interest rate risk is well-known and constant over the life of option;
- The stock pays no dividend on any type of underlying assets or security;
- There is zero transaction cost on buying and selling of assets in option contract;
- There is no provision of any type of tax;
- The stock price follows a Geometric Brownian motion process with μ and σ as a constant.
- The assets are completely divisible in nature;
- There is no penalties on short selling of shares and investor also get full use of short-sell procedure; and
- Stock price option follows a explicit type of stochastic process called diffusion process;

It is a logical approach to pricing options. The Black-Scholes Model captures most of the property but some restrictions with practical function have to be required. The hypothesis of being able to set-up a risk-less hedge by rebalancing endlessly and immediately is not reasonable in real trading because contract or transaction cost barricade continuous process and scatter investment returns. Price changes on circumstance can be quite considerable and hypothesis of risk-free rate is idealistic.

2. Literature Review

Black and Scholes (1972) them self test the result of his formula by using the data of over-the-counter market (OTC), they found that the result of his formula give lower value then actual market value. Geske, Roll and Shastri (1983) found that these differences create due to imperfect protection of dividend in the OTC market. Merton (1973) extended the Black formula and showed that the basic form of model was the same if the payment structure was increase or lifted, the interest rate is stochastic and the option is exercisable prior to maturity. Black and Scholes (1973) use Itos's lemma mathematical tools which are used to calculate particular type of stochastic process, it also provides helps in the derivation of B-S formula. Thorp (1973) has shows that the Black-Scholes formula still holds if there are limitations on the use of the proceeds of short sales. Merton (1975), Cox and Ross (1975) have shows that if the return on ordinary stock do not follow a stochastic process with a continuous path, basically a convinced form of Jump process, the hedge mechanism that was used by Black-Scholes will not be suitable. Ingersoll (1975) alluded to the tax question but did not develop the effect of taxes on listed call option. Latane and Rendleman (1976), Mac-Beth and Merville (1979) solve the B-S formula in the form of implied variance rates by taking a sample of period 1975-1976 and found a result that strike price is bias but this bias was exactly opposite which is reported by Black. Galai (1977) use the data of Chicago Board Option Exchange and found a result that the excess daily returns on the hedged portfolio are significantly different from zero, and addition of one percent transaction cost eliminates the positive excess return. Bhattacharaya (1980) tested BSM under ideal condition, he initiated the hedge by buying or selling the call at the model price and thus eliminated the efficiency of the option market and use the actual variance from the investigation period and thus isolated from the measurement of formula inputs, and reported the over valuation of model in case of at-the-money option under a maturity of less than three week. On the other hand near-the-money option is undervalued by the model, and this valuation error decrease as the time of maturity increases. Liu (2007) found an uncertainty theory that is a branch of mathematics based on normality, monotonicity, self-duality and countable subadditivity axioms. Khan et al., (2012) suggested modification in Black-Scholes option pricing model on the basis of risk-free interest rate assumption without given its practical implementation. One of the meaningful outcomes of Black-Scholes equation is the fact that the expected return on the underlying assets, and the expected return on the derivative itself, does not appear in the equation.

3. Objectives

- Incorporate modification in Black-Scholes option pricing model formula by adding some new variables on the basis of given assumption related to risk-free interest rate;
- Calculation process of new risk-free interest rate on the basis of modified variable; and
- Situation identification in empirical testing of modified and original Black-Scholes formula with respect to the market value on the basis of assumed and calculated risk-free interest rate.

4. Methodology

The research methodologies used in this paper are based on *'Conceptual and Experimental'* type of research. Conceptual research is that related to some abstract ideas or theory. It is generally used by philosophers and thinkers to develop new concepts or to reinterpret existing one. Experimental type of research is used in case of empirical testing.

With the help of such kind of research technique we suggested few modifications on the basis of higher order thinking related to risk-free interest rate assumption in Black-Scholes Option Pricing Model. In case of empirical testing of modified and original Black-Scholes formula with respect to the market value on the basis of assumed and calculated risk-free interest rate, we select two NSE derivative market stock CE9000 and PE8500, and compare its result on 20 experimental point of CE9000 and PE8500. Result of comparison creates different type of situations which are discussed in separate section of this paper.

5. Suggested Modification in BSM Formula

This Black–Scholes formula depends upon various assumptions which are given above. If we work on few assumptions and try to eliminate or modified these assumption, the shape of formula will change. The equations (1 and 2) are derived by considering *Risk Free Interest Rate* ($\mathrm{R_f}$) (it means interest rate is constant). If we consider '*Risk Include Interest Rate'* then valve of $\mathrm{R_f}$ will be change (it mean interest will also change according to time), so it is very difficult to make a proper mathematical equation which calculates *Risk Include Interest Rate* because stock market is always un predictable in nature and it depends upon purchase and sale of contract by thousands of people who are engaged in that process in a single time. According to that nature we are only able to include risk includes interest rate structure up to certain limit.

Basically we include three type of Risk factor which replaces the risk free interest rate variable i.e.

- Yield curve Risk (R_{YC});
- Basis Risk (R_B); and
- Reprising Risk (R_{RP});

Let R_{IIR} = Risk Include Interest rate

$$\mathrm{R_f} = \mathrm{R_{IIR}} = \frac{\sum_{i=1}^{n} \mathrm{R_{YC_i}}}{n} + \frac{\sum_{i=1}^{n} \mathrm{R_{B_i}}}{n} + \frac{\sum_{i=1}^{n} \mathrm{R_{RP_i}}}{n}$$

Or

$$\mathrm{R_f} = \mathrm{R_{IIR}} = \frac{1}{n}\sum_{i=0}^{n}(\mathrm{R_{YC_i}} + \mathrm{R_{B_i}} + \mathrm{R_{RP_i}}) \quad (3)$$

R is calculate *n* time it depend upon customer choice who purchase the contract of call option. Put new value of $\mathrm{R_f}$ in equation (1) its apply that

$$v_{cop} = PN\left(\frac{\log\frac{P}{P_E} + (R_{IIR} + \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}}\right) - e^{-R_{IIR}t}P_E N\left(\frac{\log\frac{P}{P_E} + \left(R_{IIR} - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right)$$

Or

$$V_{cop} = PN\left(\frac{\log\frac{P}{P_E} + \left(\frac{1}{n}\sum_{i=0}^{n}(R_{YC_i} + R_{B_i} + R_{RP_i}) + \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right) - e^{-\left(\frac{1}{n}\sum_{i=0}^{n}(R_{YC_i} + R_{B_i} + R_{RP_i})\right)t}P_E N\left(\frac{\log\frac{P}{P_E} + \left(\frac{1}{n}\sum_{i=0}^{n}(R_{YC_i} + R_{B_i} + R_{RP_i}) - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}\right) \quad (4)$$

6. Calculation Process of New Risk-Free Interest Rate

In Black-Scholes original formula we take current risk free rate of return is about 4.9% per-annum, which was taken on the basis of various expert group of people who are working in this sector from long time and also on the basis of various feedback given by online option premium calculator's guidelines. Basically current risk free rate of return is very difficult to calculate and there is no consensus, on how to go about direct measurement of it. However only Fisher's concept of inflationary expectations describe to calculate risk free rate of returns but it is not perfect, it is only a theoretical measurement of risk free rate of return. In our modification we try to develop some appropriate method to calculate risk free rate of return by replacing R_f by R_{IIR}; which is calculated on the basis of Average summation of Yield Curve Risk, Basis Risk and Reprising Risk. In which we take a Yield Curve Risk as a difference between short term and long term interest rate, Basis Risk as a difference between interest rate followed by different financial institute and Reprising Risk are taken on the basis of RBI guide line which increase or decrease the interest rate.

6.1 Calculation Process of Yield Curve Risk

To calculate *Yield Curve Risk* we have take three public sector banks. In which we take difference between short term and long term interest rate. These banks are State Bank of India (SBI), Bank of Baroda (BOB) and Punjab National Bank (PNB), Calculations are given blow:

Yield Curve Risk SBI = $R_{YC_{1_{SBI}}} = S_{TIR_{SBI}} - L_{TIR_{SBI}} = 15.91 - 11 = 4.91\%$

Yield Curve Risk BOB = $R_{YC_{2_{BOB}}} = S_{TIR_{BOB}} - L_{TIR_{BOB}} = 14 - 11.75 = 2.25\%$

Yield Curve Risk PNB = $R_{YC_{3_{PNB}}} = S_{TIR_{PNB}} - L_{TIR_{PNB}} = 12.84 - 10.75 = 2.0905\%$

Average Summation of SBI, BOB and PND = $\sum_{i=1}^{3}\frac{1}{3}(R_{YC_{1_{SBI}}} + R_{YC_{2_{BOB}}} + R_{YC_{3_{PNB}}})$

$$= \frac{\mathbf{4.91 + 2.25 + 2.0905}}{\mathbf{3}} = \mathbf{3.0833\%}$$

$$\textbf{Yield Curve Risk}(\tfrac{1}{n}\textstyle\sum_{i=1}^{n}\mathbf{R_{YC_i}} = \mathbf{3.0833\%} \quad (5)$$

Where:

S_{TIR} = Short term interest rate; and

L_{TIR} = Long term interest rate;

6.2 Calculation Process of Basis Risk

To calculate *Basis Risk* we have to take a separate difference within short term and long term interest rate between above three financial institutes, Calculation are given blow:

Difference within short term interest rate of SBI, BOB and PNB are:

$$\begin{cases} S_{TIR_{SBI}} \\ S_{TIR_{BOB}} \\ S_{TIR_{PNB}} \end{cases} \rightarrow \begin{cases} S_{TIR_{SBI}} - S_{TIR_{BOB}} \\ S_{TIR_{BOB}} - S_{TIR_{PNB}} \\ S_{TIR_{SBI}} - S_{TIR_{PNB}} \end{cases} = \begin{cases} 15.91 - 14.00 \\ 14.00 - 12.84 \\ 15.91 - 12.84 \end{cases} = \begin{cases} 1.91 \\ 1.16 \\ 3.07 \end{cases} = \frac{1.91+1.16+3.07}{3} = 2.046\% \quad (6)$$

Difference within long term interest rate of SBI, BOB and PNB are:

$$\begin{cases} L_{TIR_{SBI}} \\ L_{TIR_{BOB}} \\ L_{TIR_{PNB}} \end{cases} \rightarrow \begin{cases} L_{TIR_{BOB}} - L_{TIR_{SBI}} \\ L_{TIR_{BOB}} - L_{TIR_{PNB}} \\ L_{TIR_{SBI}} - L_{TIR_{PNB}} \end{cases} = \begin{cases} 11.75 - 11.00 \\ 11.75 - 10.75 \\ 11.00 - 10.75 \end{cases} = \begin{cases} 0.75 \\ 1.00 \\ 0.25 \end{cases} = \frac{0.75+1.00+0.25}{3} = 0.66\% \quad (7)$$

Average between short term and long term interest rate $= \frac{2.046+0.66}{2} = 1.356\%$

$$\textbf{Basis Risk}(\tfrac{1}{n}\textstyle\sum_{i=1}^{n}\mathbf{R_{B_i}}) = \mathbf{1.356\%} \quad (8)$$

6.3 Calculation Process of Reprising Risk

To calculate ***Reprising Risk*** we have to take a difference within Repo-rate increase or decrease in a given period of time and also take difference within reverse repo-rate increase or decrease in a given period of time. (See table 1 & 2)

Table 1. Repo-rate change by RBI over the last 25 months

Date	Repo-rate (%)	Difference B/w Change
19 March 2010	5.00	
20 April 2010	5.25	0.25
2 August 2010	5.50	0.25
27 August 2010	5.75	0.25
2 September 2010	6.00	0.25
2 November 2010	6.25	0.25
25 January 2011	6.50	0.25
17 march 2011	6.75	0.25
3 May 2011	7.25	0.50
16 June 2011	7.50	0.25
26 July 2011	8.00	0.50
16 September 2011	8.25	0.25
25 October 2011	8.50	0.25
17 April 2012	8.00	0.50

Source: http://www.allbankingsolutions.com/Banking-Tutor/Chronology-Repo-Rate-India.shtml

From the above table 1, we found that within 25 months RBI have changed Repo-rate 14 times. In which 11 times they increase repo-rate by 0.25%, two times increase repo-rate by 0.5% and one time decrease repo-rate by 0.5%. On the basis of above analysis we found that Indian interest rate market is inconsistent in nature because increase or decrease in repo-rate directly affect the interest rates. This type of interest rate fluctuations are came under reprising risk. To calculate reprising risk in repo-rate we have to take an average within repo-rate difference during the period of 25 month (See table 1)

Reprising Risk in Repo-rate= $\left|\frac{1}{13}\sum_{i=1}^{13}\{(0.25 \times 10) + (0.5 \times 3)\}\right| = 0.307\%$ (9)

Similarly from table 2, we found that within 25 months RBI have also changed Reverse Repo-rate 14 times. In which eight times they increase reverse repo-rate by 0.25%, four times increase reverse repo-rate by 0.5% and one time decrease reverse repo-rate by 0.5%.Similar calculation technique will be applied to calculate Reprising Risk in Reverse repo-rate.

Reprising Risk in Reverse Repo-rate= $\left|\frac{1}{13}\sum_{i=1}^{n}\{(0.25 \times 8) + (0.5 \times 5)\}\right| = 0.346\%$ (10)

Final Reprising Risk $= \frac{\{(Reprising\ Risk\ in\ Repo-rate) + (Reprising\ Risk\ in\ Reveerse\ Repo-rate)\}}{2}$

$$= \frac{0.307 + 0.346}{2} = 0.326\%$$

Reprising Risk $\left(\frac{1}{n}\sum_{i=1}^{n} \mathbf{R}_{\mathbf{RP_i}}\right) = \mathbf{0.326}\%$ (11)

Table 2. Reverse Repo-rate change by RBI over the last 25 months

Date	Reverse Repo-rate (%)	Difference B/w Change
19 March 2010	3.50	0.25
20 April 2010	3.75	0.25
2 August 2010	4.00	0.25
27 August 2010	4.25	0.50
2 September 2010	4.75	0.50
2 November 2010	5.25	0.25
25 January 2011	5.50	0.25
17 march 2011	5.75	0.50
3 May 2011	6.25	0.25
16 June 2011	6.50	0.50
26 July 2011	7.00	0.25
16 September 2011	7.25	0.25
25 October 2011	7.50	0.50
17 April 2012	7.00	

Source:http://www.allbankingsolutions.com/Banking-Tutor/Chronology-Reverse-Repo-Rate-India.shtml

6.4 Final Calculation of New Risk-Free Interest Rate

Put the value of equation 5, 8 and 11 in equation (3), in which we calculate Risk free interest rate. It's applied that:

$$\mathbf{R_f = R_{IIR} = \frac{\sum_{i=1}^{n} R_{YC_i}}{n} + \frac{\sum_{i=1}^{n} R_{B_i}}{n} + \frac{\sum_{i=1}^{n} R_{RP_i}}{n}}$$

$$\mathbf{R_f = R_{IIR} = 3.0833 + 1.356 + 0.326 = 4.765\%} \quad (12)$$

7. Situation Identification and Empirical Testing of Assumed and Calculated value of Risk Free Interest Rate

Now we test our calculated risk free interest rate in Black-Scholes Option Pricing Model in livestock of option with respect to assumed risk free interest rate.

7.1 We take live derivative Stock on 22nd May 2012

Where:

Option Type: CE; (Call European)
Strike Price : 9000;
Symbol : BANKNIFTY;
Expiry Date : 31st May 2012;
Market Lot : 25;
MV = Market Value;
OBSM = Original Black-Scholes Model; and
MBSM = Modified Black-Scholes model;

Calculated Standard Deviation or Volatility of Stock CE 9000 on 22rd May 2012

$$\sigma = \sqrt{\frac{\sum X^2}{N}},$$

$$where\ X = x - \bar{x}\ and\ N = Number\ of\ Observation\ point$$

$$\bar{x} = \frac{4844.41}{20} = 242.2205$$

$$\sum (x - \bar{x})^2 = 10461.403$$

$$\sigma = \sqrt{\frac{10461.403}{20}} = 22.870$$

Table 3. Calculation Table of BSM at Different R_f

S. No	Time in 24 hour	Market Price of CE 9000 (MV) Rs.	BSM price $R_f = 4.9\%$ (OBSM) Rs.	BSM price on Calculated $R_{IIR} = 4.765\%$ (MBSM) Rs.	Situation Result for CE 9000
1	09 : 19 am	235.00	255.38	243.98	$OBSM \neq MV \& MBSM \neq MV$
2	09 : 45 am	255.35	253.79	238.91	$OBSM \cong MV \& MBSM \neq MV$
3	09 : 54 am	240.21	230.71	242.37	$OBSM \neq MV \& MBSM \cong MV$
4	10: 15 am	259.42	256.34	256.83	$OBSM \cong MV \& MBSM \cong MV$
5	10 : 29 am	259.42	241.69	247.46	$OBSM \neq MV \& MBSM \neq MV$
6	10 : 45 am	243.34	244.9	252.71	$OBSM \cong MV \& MBSM \neq MV$
7	11 : 04 am	225.42	225.82	224.31	$OBSM \cong MV \& MBSM \cong MV$
8	11 : 15 am	225.42	233.75	223.97	$OBSM \neq MV \& MBSM \cong MV$
9	11 : 34 am	209.93	212.5	210.82	$OBSM \cong MV \& MBSM \cong MV$
10	11 : 45 am	219.87	201.57	206.37	$OBSM \neq MV \& MBSM \neq MV$
11	12 : 04 pm	203.35	192.78	205.12	$OBSM \neq MV \& MBSM \cong MV$
12	12 : 15 pm	222.32	224.81	221.24	$OBSM \cong MV \& MBSM \cong MV$
13	12 : 39 pm	265.47	282.7	276.81	$OBSM \neq MV \& MBSM \neq MV$
14	13 : 00 pm	244.79	242.61	231.59	$OBSM \cong MV \& MBSM \neq MV$
15	13 : 14 pm	285.36	297.32	284.51	$OBSM \neq MV \& MBSM \cong MV$
16	13 : 30 pm	295.00	297.7	293.98	$OBSM \cong MV \& MBSM \cong MV$
17	13 : 44 pm	222.17	210.57	224.37	$OBSM \neq MV \& MBSM \cong MV$
18	14 : 19 pm	241.99	243.98	253.31	$OBSM \cong MV \& MBSM \neq MV$
19	14 : 54 pm	243.61	240.87	244.97	$OBSM \cong MV \& MBSM \cong MV$
20	15 : 29 pm	246.97	223.71	230.61	$OBSM \neq MV \& MBSM \neq MV$

Source: Manual calculation will be done on the basis of BSM formula given in Equation 1 and 4

From the above Table 3, we easily found that out of 20 experimental point of CE 9000, six experimental points shows common results that Original Black-Scholes Model (OBSM) and Modified Black-Scholes Model (MBSM) values are approximately equal to market value, except that four results are in favour of OBSM and five results are in favour of MBSM with respect to market value, and remaining five results are not in favour of both.

If we convert the above results of CE 9000 is in the form of percentage on the basis of given situations then result are more understandable.

Situation 1

$[OBSM \cong MV \& MBSM \cong MV] \rightarrow$ This situation covers 30% out of 20 experimental points.

Situation 2

$[OBSM \cong MV \& MBSM \neq MV] \rightarrow$ This situation covers 20% out of 20 experimental points.

Situation 3

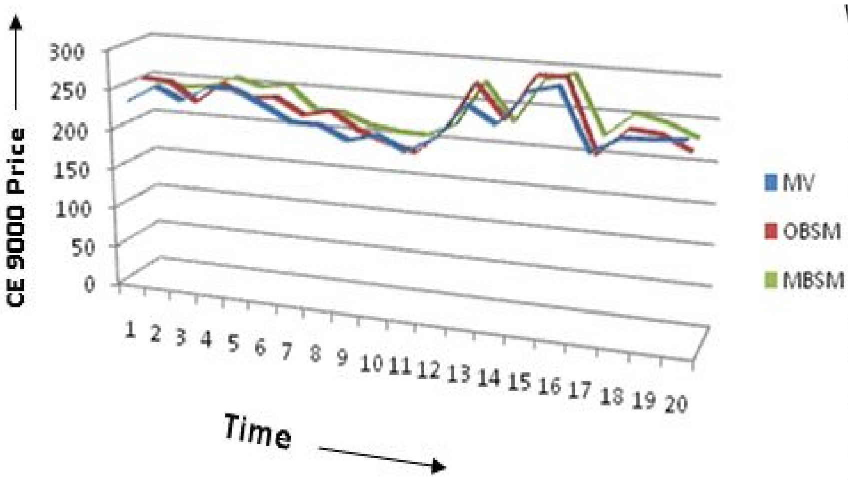
$[OBSM \neq MV \& MBSM \cong MV] \rightarrow$This situation covers 25% out of 20 experimental points.

Situation 4

$[OBSM \neq MV \& MBSM \neq MV] \rightarrow$This situation covers 25% out of 20 experimental points.

Above situations shows that out of 20 experimental trial 30% of the result are common in both OBSM and MBSM it means $[(OBSM \cong MV \& MBSM \cong MV) + (OBSM \cong MV \& MBSM \neq MV)]50\%$ results are in favour of OBSM and similarly $[(OBSM \cong MV \& MBSM \cong MV) + (OBSM \neq MV \& MBSM \cong MV)]55\%$ results are in favour of MBSM.

Figure 1. Graphical Representation of Table 3, Related to CE 9000 Price



Where

01 =	09 : 19 am	11=	12 : 04pm
02 =	09 : 45 am	12=	12 : 15pm
03 =	09 : 54 am	13=	12 : 39pm
04 =	10: 15 am	14=	13 : 00pm
05 =	10 : 29 am	15=	13 : 14pm
06 =	10 : 45 am	16=	13 : 30pm
07 =	11 : 04 am	17=	13 : 44pm
08 =	11 : 15 am	18=	14 : 19pm
09 =	11 : 34 am	19=	14 : 54pm
10 =	11 : 45 am	20=	15 : 29pm

On the basis of above graphical representation of CE 9000, we easily conclude that OBSM, MBSM and MV shows approximately similar path, which shows that all of these values are positively correlated with each other. Level of correlation between CE9000 values are shown in table 4.

Table 4. Correlation Between MV, OBSM and MBSM

		MVCE9000	OBSMCE9000	MBSMCE9000
MVCE9000	Pearson Correlation	1	.921**	.935**
	Sig. (2-tailed)		.000	.000
	N	20	20	20
OBSMCE9000	Pearson Correlation	.921**	1	.952**
	Sig. (2-tailed)	.000		.000
	N	20	20	20
MBSMCE9000	Pearson Correlation	.935**	.952**	1
	Sig. (2-tailed)	.000	.000	
	N	20	20	20

**. Correlation is significant at the 0.01 level (2-tailed).

On the basis of correlation values which are given in above table, we found that correlation between MVCE9000, OBSMCE9000 and MBSMCE9000 are shows highly positive correlation with each other. In which correlation between MVCE9000 and OBSMCE9000 is little bit lower than correlation between MVCE9000 and MBSMCE9000.This shows that MBSM give little bit better result than OBSM in the calculation of value of Call Option (CE9000).

7.2 Take another live derivative stock on 22nd May 2012

Where:

Option Type: PE; (Put European)
Strike Price : 8500;
Symbol : BANKNIFTY;
Expiry Date : 31st May 2012;
Market Lot : 25;
MV = Market Value;
OBSM = Original Black-Scholes Model; and
MBSM = Modified Black-Scholes model;

Calculated Standard Deviation or Volatility of Stock PE 8500 on 22nd May 2012

$$\sigma = \sqrt{\frac{\sum X^2}{N}},$$
$$where\ X = x - \bar{x}\ and\ N = Number\ of\ Observation\ point$$
$$\bar{x} = \frac{455.86}{20} = 22.793$$
$$\sum (x - \bar{x})^2 = 342.9029$$
$$\sigma = \sqrt{\frac{342.9029}{20}} = 4.14066$$

Table 5. Calculation Table of BSM at Different R_f

S. No	Time in 24 hour	Market Price of PE 8500 (MV) Rs.	BSM price at $R_f = 4.9\%$ (OBSM) Rs.	BSM price on Calculated $R_{IIR} = 4.765\%$ (MBSM) Rs.	Situation Result for PE 8500
1	09 : 19 am	25	36.8	38.92	OBSM ≠ MV & $MBSM \neq MV$
2	09 : 45 am	20	22.32	21.96	OBSM ≅ MV & $MBSM \cong MV$
3	09 : 54 am	20	21.9	13.72	OBSM ≅ MV & $MBSM \neq MV$
4	10: 15 am	20	28.3	21.9	OBSM ≠ MV & $MBSM \cong MV$
5	10 : 29 am	22.31	13.73	16.21	OBSM ≠ MV & $MBSM \neq MV$
6	10 : 45 am	27.82	25.9	26.12	OBSM ≅ MV & $MBSM \cong MV$
7	11 : 04 am	26.96	27.81	37.52	OBSM ≅ MV & $MBSM \neq MV$
8	11 : 15 am	25.81	33.83	23.83	OBSM ≠ MV & $MBSM \cong MV$
9	11 : 34 am	28.71	40.3	42.67	OBSM ≠ MV & $MBSM \neq MV$
10	11 : 45 am	27.32	28.57	28.98	OBSM ≅ MV & $MBSM \cong MV$
11	12 : 04 pm	24.12	37.62	35.97	OBSM ≠ MV & $MBSM \neq MV$
12	12 : 15 pm	17.27	16.5	18.2	OBSM ≅ MV & $MBSM \cong MV$
13	12 : 39 pm	18.91	13.73	21.73	OBSM ≠ MV & $MBSM \cong MV$
14	13 : 00 pm	17.42	16.12	18.72	OBSM ≅ MV & $MBSM \cong MV$
15	13 : 14 pm	15.05	16.32	16.99	OBSM ≅ MV & $MBSM \cong MV$
16	13 : 30 pm	20.21	19.13	29.56	OBSM ≅ MV & $MBSM \neq MV$
17	13 : 44 pm	27.12	21.37	29.12	OBSM ≠ MV & $MBSM \cong MV$
18	14 : 19 pm	29.1	26.5	25.5	OBSM ≅ MV & $MBSM \cong MV$
19	14 : 54 pm	20.52	11.52	14.62	OBSM ≠ MV & $MBSM \neq MV$
20	15 : 29 pm	22.31	36.67	31.78	OBSM ≠ MV & $MBSM \neq MV$

Source: Manual calculation will be done on the basis of BSM formula given in Equation 1 & 4.

From the above Table 5, we easily found that out of 20 experimental point of PE 98500, seven experimental points shows common results that Original Black-Scholes Model (OBSM) and Modified Black-Scholes Model (MBSM) values are approximately equal to market value, except that three results are in favour of OBSM and four results are in favour of MBSM with respect to market value, and remaining six results are not in favour of both.

If we convert the above results of PE 8500 is in the form of percentage on the basis of given situations then result are more understandable.

Situation 1

[OBSM ≅ MV & MBSM ≅ MV] → This situation covers 35% out of 20 experimental points.

Situation 2

[OBSM ≅ MV & MBSM ≠ MV] → This situation covers 15% out of 20 experimental points.

Situation 3

[OBSM ≠ MV & MBSM ≅ MV] →This situation covers 20% out of 20 experimental points.

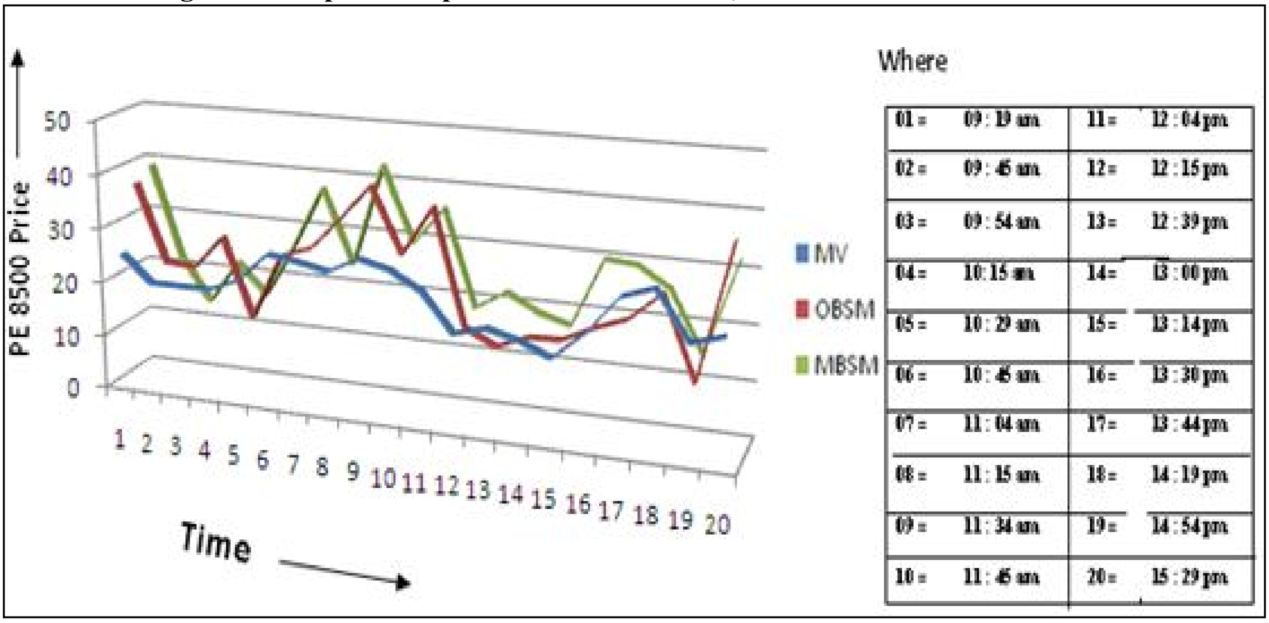
Situation 4

[OBSM ≠ MV & MBSM ≠ MV] → This situation covers 30% out of 20 experimental points.

Above situations shows that out of 20 experimental trial 35% of the result are common in both OBSM and MBSM it means $[(\text{OBSM} \cong \text{MV}\ \&\ \text{MBSM} \cong \text{MV}) + (\text{OBSM} \cong \text{MV}\ \&\ \text{MBSM} \neq \text{MV})]$

50% results are in favour of OBSM and similarly $[(\text{OBSM} \cong \text{MV} \;\&\; \text{MBSM} \cong \text{MV}) + (\text{OBSM} \neq \text{MV} \;\&\; \text{MBSM} \cong \text{MV})]$ 55% results are in favour of MBSM.

Figure 2. Graphical Representation of Table 5, Related to PE 8500 Price



On the basis of above graphical representation of PE 8500, we easily concluded that OBSM, MBSM and MV shows inconsistent pattern of price movement according to time , which conclude that all of these values are positively correlated with each other but level of movement of PE 8500 price is not much positive then CE 9000 price. Level of correlation between PE8500 values are shown in table 6.

Table 6. Correlation Between MVPE8500, OBSMPE8500 and MBSMPE8500

		MVPE8500	OBSMPE8500	MBSMPE8500
MVPE8500	Pearson Correlation	1	.619**	.658**
	Sig. (2-tailed)		.004	.002
	N	20	20	20
OBSMPE8500	Pearson Correlation	.619**	1	.799**
	Sig. (2-tailed)	.004		.000
	N	20	20	20
MBSMPE8500	Pearson Correlation	.658**	.799**	1
	Sig. (2-tailed)	.002	.000	
	N	20	20	20

**. Correlation is significant at the 0.01 level (2-tailed).

On the basis of correlation values which are given in above table, we found that correlation between MVPE8500, OBSMPE8500 and MBSMPE8500 are positively correlated with each other. In which correlation between MVPE8500 and OBSMPE8500 is little bit lower than correlation between MVPE8500 and MBSMPE8500. This shows that MBSM give little bit better result than OBSM in the calculation of value of Put Option (PE8500). When we compare correlation values of PE8500 with CE9000, we found that correlation values of PE8500 are much lower than correlation values of CE9000.

8. Conclusion and Discussion

On the basis of situation analysis in small empirical test of Black-Scholes Model (BSM), on two derivative market stock CE9000 and PE8500. The values of call and put option of these stocks are calculated on the basis of assumed and calculated risk free interest rate. Calculated risk free interest rate results are changed due to a small correction in the value of risk free interest rate. This corrected or calculated risk free interest rate gives better result with respect to existing value of risk free interest rate in the calculation of the value of call and put option in CE9000 and PE8500. From the above empirical test of two derivative market stocks'CE9000 and PE8500', proved that existing Black-Scholes Model have powerful outcome which can't be neglected. In case of situation analysis this can also be proved because *Situation First* indicate that our MBSM is somehow similar to OBSM in both empirical test of CE9000 and PE8500. It covers 30% and 35% experimental point out of 20 experimental points respectively. *Situation Second* is entirely in favour of OBSM, which covers 20% and 15% experimental points out of 20 experimental points in CE9000 and PE8500 respectively. *Situation Third* is entirely in favour of MBSM, which covers 25% and 20% experimental points out of 20 experimental points in CE9000 and PE8500 respectively but it cover five percent more than OBSM. This shows MBSM formula gives positive outcome with respect to OBSM formula. The *Fourth and Final Situation* shows unpredictable type of result which occur due to the dynamic property of market behaviour which neither came under the range of MBSM nor in the range of OBSM its cover 25% and 30% experimental points out of 20 experimental points in CE9000 and PE8500 respectively.

In case of correlation analysis we found that CE9000 shows high level of positive correlation between MVCE9000, OBSMCE9000 and MBSMCE9000. On the other hand PE8500 also shows positive correlation between MVPE8500, OBSMPE8500 and MBSMPE8500 but the correlation values of PE8500 are not that much high as CE9000. We may consider that this might be happen due to the high level of standard deviation or volatility difference between two derivative market stocks. This situation of correlation is also more understandable when we test our result on at least 50 stock of derivative market in which we are easily able to trace the actual pattern between volatility and correlation level. This research work is only a type of working paper; more results are calculated in future but as per our opinion this research paper is useful for Indian investors who are interested to calculate value of call and put option in more appropriate manner, which reduces his or her risk factor while trading in option derivative market in India.

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