International Journal of Economics and Financial Issues

ISSN: 2146-4138

available at http: www.econjournals.com

International Journal of Economics and Financial Issues, 2026, 16(4), 108-117.

Transition Matrix Instability and Credit Risk

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Received: 25 March 2026 **Accepted:** 18 June 2026 **DOI:** https://doi.org/10.32479/ijefi.24251

ABSTRACT

Credit risk stress testing has emerged as an essential risk management tool employed by both by financial institutions and regulatory authorities. Its implementation involves substantial complexity stemming from the requirement to forecast complete portfolio creditworthiness changes under specified macroeconomic scenarios across multi-year horizons. This complexity arises from the integration of numerous model parameters governing changes over time. With the standard practice involving the specification of baseline parameters calibrated to average economic conditions that are subsequently transformed to stressed states through macroeconomic models. A critical – but often overlooked – consequence of this parameterisation and calibration approach is that it implicitly defines a unique equilibrium portfolio that exists independently of the financial institution's actual portfolio composition which emerges purely from the interaction of model parameters rather than from existing exposure characteristics. The mathematical structure of these models creates an inherent tendency for current portfolios to converge toward this parameter-implied portfolio over the projection horizon. When stress test parameters are inconsistent with actual portfolio characteristics the convergence process generates spurious effects in projected portfolio default rates (a common situation arising when banks utilise external data sources or industry benchmarks due to insufficient internal historical data). The projected portfolio defaults can originate from the parameterisation itself rather than from the economic stress being modelled. This effect can potentially produce misleading risk assessments that compromise both internal capital allocation decisions and regulatory capital requirements.

Keywords: IFRS9, Provisions, Capital, Expected Credit Loss, Stress Testing
JEL Classifications: C5, C15, G23, G24

1. INTRODUCTION

Since the global financial crisis (GFC) of 2008, stress testing has transformed from a peripheral risk assessment tool into a fundamental component of banking risk management. This fundamental shift reflects the widespread recognition among both risk managers and banking supervisors that pre-crisis risk measurement frameworks failed to capture systemic vulnerabilities and tail risks.

More recent regulatory frameworks now mandate that financial institutions conduct more regular and thorough stress testing exercises, requiring institutions to develop internally relevant scenarios while simultaneously responding and participating in standardised assessments prescribed by regulatory bodies (BCBS, 2018). In the United States (US) the Federal Reserve conducts annual stress tests for the largest banking institutions (US Federal Reserve, 2025), while in Europe the European Banking Authority (2025) performs analogous assessments amongst their member countries. These traditional regulatory stress tests typically employ 3-year projection horizons which focuses primarily on assessing institutional resilience to near-term economic shocks.

A critical methodology shift in stress testing emerged in 2021 with the introduction of climate risk scenarios, fundamentally altering both the temporal scope and complexity of risk assessment frameworks. Unlike conventional stress tests with their 3-year horizons, climate scenarios extend projection periods up to 30 years, reflecting the long-term nature of physical and transition risks associated with climate change. The new approach towards stress testing includes the Bank of England's 2021 climate stress test, the European Central Bank's 2022 climate assessment (ECB, 2022), the Reserve Bank of New Zealand's 2023 exercise (Reserve Bank of New Zealand, 2024), and the Federal Reserve's 2023 (US Federal Reserve, 2024) climate scenario analysis. Given that credit risk constitutes the dominant risk category for most banking institutions, regulatory stress tests tend to focus on credit risk modelling methodologies, while acknowledging the presence of other risk categories such as market, operational and liquidity risks.

In the absence of universally or holistic mandated modelling standards by regulatory bodies the banking sector has converged toward a common methodological framework for implementing credit risk stress tests. This framework centres on three core parameters that govern credit risk dynamics: the probability of default (PD) – representing the likelihood of borrower default – loss given default (LGD) - capturing the expected loss percentage conditional on default occurrence – and exposure at default (EAD) which quantifies the anticipated exposure in the event of default. These parameters are typically estimated under average economic conditions, yielding "Through-the-Cycle" (TTC) risk parameters. The framework also incorporates rating migration dynamics through transition matrices that capture movements between credit quality categories. Stress scenarios are operationalised through macroeconomic variables, with specialised models translating economic scenarios into abstract state factors. These factors subsequently transform TTC parameters into "Point-in-Time" (PIT) or stressed risk parameters, enabling quantification of scenario impacts on default rates, portfolio losses, capital adequacy, loan loss provisions and net interest income.

The extended horizons characteristic from 3 to 30 years of climate stress tests necessitate abandoning static portfolio assumptions in favour of dynamic modelling approaches. Realistic long-term projections require explicit modelling of portfolio evolution mechanisms including write-offs, contractual run-offs, prepayments, and new loan originations. These dynamic parameters, in conjunction with TTC risk parameters, implicitly define an equilibrium TTC portfolio that exists independently of a bank's current portfolio composition. Current portfolios also exhibit a natural tendency to converge toward this TTC equilibrium state over time. This convergence property introduces a critical methodological challenge for financial institutions implementing credit stress scenarios and when stress test parameters are calibrated inconsistently with current portfolio characteristics, models may generate spurious projections of average portfolio PDs that originate from the parameterisation itself rather than from the stress scenario. These hidden distortions can significantly compromise stress test validity and interpretation. Such inconsistencies frequently arise when financial institutions rely on external data sources due to insufficient internal data: A common occurrence for large corporate portfolios. Financial institutions may lack sufficient default observations to estimate reliable internal rating transition matrices and instead substitute transition matrices published by external rating agencies.

Questions financial institutions must consider is how transition matrix stability, measured through the stability of the diagonal based elements, impacts the magnitude of spurious effects in the projection of TTC and PIT PDs. Another layer of quantification could involve an investigation of how instability sensitivity varies across different rating grades.

The remainder of this article is structured as follows: Section 2 reviews the existing literature around constructing credit risk stress approaches commonly used by financial institutions. Section 3 outlines the methodology employed in the analysis using simulations to explore a wide range of parameter inputs. It describes the underlying mathematical methodology stipulating the characteristics of the TTC portfolio as well as the conditions for the existence of a unique TTC portfolio. Section 4 presents the results of the analysis and provides insights into potential explanations for the observed outcomes. Section 5 concludes.

2. LITERATURE SURVEY

A key measure of credit risk within the stress testing framework is the PD. The PD represents a percentage value which can be described as an evaluation of the credit risk for a counterparty to the financial institution. Establishing the PD of a counterparty is also the practice of predicting or forecasting the ability of a counterparty to pay back the debt or default. Financial institutions use a standardised classification systems to assess and communicate the creditworthiness of counterparties. The classifications represent the likelihood that a borrower can default on their debt obligations, serving as a critical tool for lenders, investors, and regulators to evaluate credit risk systematically. The relationship between the PD and credit rating classifications form the quantitative foundation of credit risk assessment in modern banking, while credit ratings provide ordinal risk rankings, PDs translate these qualitative assessments into cardinal measures of default likelihood, enabling precise risk quantification essential for risk management practises such as capital provisioning and capital allocation. Credit rating grades and PDs are intrinsically linked through a monotonic relationship where each successive rating grade corresponds to a higher expected probability of default. This mapping transforms the ordinal scale of ratings into a continuous probability measure ranging from near zero for the highest ratings to certainty of default for the lowest.

Credit rating transition matrices represent the probabilistic framework that captures how counterparties migrate between different credit classifications over time. These matrices transform the static concept of credit ratings into a dynamic system that models credit quality evolution, serving as the mathematical foundation for portfolio credit risk modelling, stress testing, and forward-looking risk assessment, which ultimately impacts the accuracy of financial institution capital levels (Jarrow et al., 1997; Israel et al., 2001; Boreiko et al., 2019).

The transition matrix in credit risk is a fundamental tool that captures the dynamic nature of borrower creditworthiness over time, representing probabilities of borrowers migrating between different credit rating categories over a specific period. A credit transition matrix is a square matrix with several credit rating categories or classifications, including a default state. Credit ratings typically follow an alphabetical grading system although the specific scales vary between external rating agencies (such as Moody's, Standard & Poors (S&P) and Fitch) and internal bank rating systems. Each element of the transition matrix represents the probability that a borrower currently in a rating grade could transition (up or down) to a different rating grade by the end of the observation period. The transition matrix encodes several critical aspects of credit risk, such as the migration patterns of borrowers between credit ratings represented by the off-diagonal elements of the matrix (stability or instability). The matrix also represents credit risk where the rightmost column captures the PD from each rating grade, thus providing a direct measure of credit risk across the portfolio (Gavalas and Syriopoulos, 2014).

Morgan (1997) first employed an unconditional approach towards transition matrices by introducing a transition matrix driven by the current state of the economy or business cycle. As a well-known condition of transition matrices is that the distributions of PDs vary and is time dependent on the issuer of the instrument. Similar approaches were later introduced by Nickell et al. (2000) for dynamic transition matrices based on the ordered probit model approach. Risk factors such as stage of the business cycle (prevailing economic conditions) and the business sector of the counterparty are driving factors in probability changes for transition matrices. Nyström and Skoglund (2006) then employed a more general approach towards the multinomial logit model extension by including competing non-default states as opposed to just default risk.

Gunnvald (2014) then showed that the time-homogeneous assumption used as industry standard does not hold up in certain estimates for transition matrices when estimating PDs over the entire business cycle. This assumption is then attributed towards the simplification of estimation. There is however no market consensus over how to account for the time-inhomogeneity assumption widely used. which was later extended by Skoglund and Chen (2016) to devise an extended approach to rather make use of a multinomial logit model for estimating transition matrices by more effectively implemented a simple Markov chains iteration process. The approach using Markov chains was introduced historically to estimate credit rating migration and calculate the transition matrices themselves to become the industry standard. These developments established the Markov chain framework as the standard approach for modelling credit rating migration, while highlighting the persistent challenge of ac-counting for time-varying transition dynamics.

To ensure holistic scenario inclusion in credit stress testing turbulent market conditions such as the GFC is commonly included. The International Accounting Standard Board (IASB) and Financial Accounting Standard Board (FASB) collaborated to revamp accounting standards during this period, aiming for a more effective and streamlined Expected Credit Loss (ECL) framework. This effort resulted in the release of the IFRS 9 in 2014 (IFRS, 2014). IFRS 9 addresses how financial institutions should recognise their financial assets and liabilities in its financial statements, emphasising an ECL framework for identifying impairments. ECL quantification typically involves assessing three key components: PD, loss LGD, and EAD and it mandates that ECL models consider both current and forecasted macroeconomic conditions to assess ECL. This approach facilitates the calculation of forward-looking impairment estimates which can be obtained by credit stress scenarios.

IFRS 9 further stipules a provisions amount calculated based on the stage of a loan, driven by the significant increase in credit risk (SICR) indicator. The stage of the underlying loan prescribes whether a 12-month or lifetime ECL amount should be used in the provisions amount. To calculate the provisions, amount the TTC PD is converted to a more market condition relevant PIT PD. Both the TTC -and PIT PDs are needed within the ECL provision calculation process and therefore also critical within credit stress testing. It is in financial institutions best interest to fully understand the risk parameters (such as PD) used in the credit stress testing framework, as well as how the risk parameters are used and evolve in over time in the principal component of transition matrices.

While there is no universally or holistic accepted model or framework for implementing credit risk stress tests, financial institutions adhere to a common modelling philosophy. Migration between rating categories is typically incorporated and modelled using transition matrices. The key component risk parameters are generally calibrated for an average economic state and are referred to as through-the-cycle risk parameters. Stress scenarios are usually derived from macroeconomic projections, which are processed through a macroeconomic model to generate an abstract factor representing the overall economic condition. This factor is then applied to adjust TTC risk parameters, converting them into PIT or stressed risk parameters, which are subsequently used to assess the impact of the stress scenario on aspects of a financial institutions' provision capital.

Engelmann (2025) demonstrated that the transition matrices used by financial institutions may exhibit spurious behaviour which potentially can distort the stress test and produce misleading results. This has implications for aspects of provision capital calculations as well. This means the stress test might signal a recession or boom for a financial institution that is not observed in the macroeconomic scenario but is instead caused by the mismatch between the transition matrix and the portfolio.

Engelmann (2025) also showed when the parameters included in the stress test framework are inconsistent with the current portfolio of the financial institution this could produce spurious projections of PDs and therefore inaccurate or inconsistent results for capital provision calculations. Such discrepancies are embedded within the stress test parameterisation and is particularly likely to occur when a financial institution relies on external data (rating agencies) due to insufficient or incomplete internal data. Transition matrix stability therefore plays a critical role in credit stress testing approach as it directly impacts the accuracy and reliability of projected portfolio PDs.

The distribution of ratings change probabilities plays a crucial role in credit risk models. As is mentioned these distributions may vary across time and ignoring or misalignment of such dependencies may lead to inaccurate assessments of credit risk. The need to better understand the stochastic nature of rating transitions has become a critical part in credit stress testing frameworks given their key component of credit risk modelling techniques.

Credit portfolio modelling originally emerged in the 1990s as a transformative field, driven by the need to measure credit portfolio risk using metrics such as Value-at-Risk (VaR) and C (ES). Among the pioneering innovations, CreditMetrics (Gupton et al., 1997) provided the foundational framework approach in this regard. Its influence extends to the Basel II capital functions and the stress testing models widely applied today (Gordy, 2003). At its core, this framework introduces a one-period, one-factor model for credit risk, an elegant and enduring construct that revolutionised the assessment of portfolio dynamics and laid the groundwork for advances in regulatory and institutional practices. The model is presented by:

$$r = \sqrt{\rho} Z + \sqrt{1-\rho\epsilon} \tag{1}$$

Where r represents the log-return of a borrower's assets, Z is a random systemic single factor shared among all borrowers, ϵ is a borrower-specific random factor, and ρ is the correlation between the asset log-returns of any two borrowers. Both Z and ϵ are assumed to be independent and follow standard normal distributions. Borrower-specific random factors ϵ are assumed to be independent across different borrowers and a borrower is considered to default if r falls below a specified threshold, θ. By construction, r is modelled as a standard normally distributed variable which facilitates the expression of the threshold θ in terms of the unconditional borrower default probability p:

$$p = P(r < \theta) = \Phi(\theta) \Rightarrow \theta = \Phi^{-1}(p) \tag{2}$$

From (2), Φ denotes the cumulative distribution function of the standard normal distribution. The systemic factor Z (1) serves as an abstract representation of the economy's overall state. A positive value of Z signifies an economic boom, decreasing the probability that a borrower's asset returns fall below the threshold θ, given Z. Negative Z values indicate a recession, increasing the likelihood of default. The conditional probability of borrower default, represented as $p(z)$, is calculated as:

$$p(z) = P(r < \theta) = \mathrm{P}\left(\epsilon < \frac{\theta - \sqrt{\rho} z}{\sqrt{1-\rho}} \right) = \Phi\left(\frac{\Phi^{-1}(p) - \sqrt{\rho} z}{\sqrt{1-\rho}} \right) \tag{3}$$

Where (3) represents a transformation between a PIT PD, conditional on the state of the economy Z, and an unconditional TTC PD. This relationship has been leveraged in the development of PIT PD to TTC PD conversion frameworks for both regulatory compliance and internal risk management purposes (Aguais et al., 2007; Carlehed and Petrov, 2012). In stress testing, the TTC parameter typically serves as an input, the state of the economy z is obtained from macroeconomic models, and (3) is employed to derive the risk parameters necessary to assess the scenario's impact on a lender's portfolio.

In Gupton et al. (1997), an extension of (3) is introduced in the form of transition matrices. The foundation of this extension is the TTC PD transition matrix T, which is defined as:

$$T = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,n-1} & p_{1,n} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,n-1} & p_{2,1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ p_{n-1,1} & p_{n-1,2} & \cdots & p_{n-1,n-1} & p_{n-1,n} \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \tag{4}$$

Here, n represents the number of rating grades, and $p_{i,j}$ is the probability that a borrower in rating grade i transitions into rating grade j within 1 year. The n^{th} grade corresponds to the default grade. The default grade is widely considered or assumed to be absorbing in the aspect that once an obligor has entered the default grade, they cannot migrate to another bucket. Convention is to have the highest grade of the transition matrix on the left and then in a descending order towards the default grade on the right (Gunnvald, 2014).

If the default grade is absorbing over enough time elapsed, all obligors will eventually migrate to the default grade. This time estimate for all obligors to migrate into the default bucket is observed by Jafry and Shuermann (2003) or to its steady-state is a very long period in economic terms. This process also relies on assumptions such as time-inhomogeneity in terms of the transition matrix used, which are also considered simplistic over longer periods. Economic condition changes consistently which drives changes in the used transition matrix. This process again alters the use of a consistent transition matrix in the credit risk stress testing approach even before a steady-state can be reached (Gunnvald, 2014). Each row of the transition matri T sums to 1 (5):

$$\sum_{j=1}^{n} p_{i,j} = 1 \tag{5}$$

If the default grade is absorbing, meaning that once a borrower transitions to this grade, they cannot recover under the migration rules defined by T. To compute a transition matrix conditional on the state of the economy z, for each rating grade i, $n-1$ thresholds $\theta_{i,j}$ are introduced. A borrower in rating i migrates to rating j if the log-return of their assets r satisfies $\theta_{i,j} \leq r < \theta_{i,j-1}$. To derive the transition matrix $T(z) = (p_{i,j}(z))$, a generalisation of (3) is required:

$$p_{i,j}(z) = \begin{cases} \Phi\left(\dfrac{\Phi^{-1}(p_{i,n}) - \sqrt{\rho} z}{\sqrt{1-\rho}} \right) & \text{if } j = n \\ \Phi\left(\dfrac{\Phi^{-1}\left(\sum_{\ell=j}^{n} p_{i,\ell} \right) - \sqrt{\rho} z}{\sqrt{1-\rho}} \right) - \sum_{\ell=j+1}^{n} p_{i,j}(z) & \text{if } j = 2, \ldots, n-1 \\ 1 - \sum_{\ell=2}^{n} p_{i,j}(z) & \text{if } j = 1 \end{cases} \tag{6}$$

Applying (6) in stress-testing involves specifying a stressed value z_{stress} for the state of the economy to transform an *average* transition matrix into a *stressed* transition matrix. Non-negativity is guaranteed by the recursive structure of the computation because computing from j = n means each step subtracts only the already-computed positive probabilities from a cumulative normal value (Gupton et al., 1997). This stressed transition matrix is then used to assess the impact of a stress scenario on a lender's portfolio. Such an approach has been employed in studies such as Bangia et al. (2002), Ozdemir (2009), de Bandt et al. (2013), Miu and Ozdemir (2009) and Witzany (2022).

The stressed state of the economy (z_{stress}) is typically derived from a macroeconomic model. At the core of this model is a credit index *C*, which captures the economic cycle. This index can be constructed from the time series of a financial institution's portfolio default rate or externally published default or bankruptcy rates. The macroeconomic model establishes a relationship between the time series of the credit index C_t and macroeconomic variables $X_{k,t}$, such as unemployment rates, gross domestic product (GDP) growth, or other relevant indicators.

Macroeconomic models are extensively utilised in risk management beyond stress testing, for example, in projecting PD's for ECL calculations under IFRS9 (Pesaran et al., 2006; Schechtman and Gaglianone, 2012; Skoglund and Chen, 2016). To derive a stressed state of the economy, z_{stress}, from a macroeconomic scenario, the process begins with a macroeconomic model, such as:

$$\Phi^{-1}\left(C_t\right)=\beta_0+\beta_1 X_{1,t-\ell}+\ldots+\beta_h X_{h,t-\ell} \tag{7}$$

Here, ℓ represents the time lag used during model estimation. The historically observed credit index C_t can be interpreted as a realisation of a PIT PD *p*(*z*). Using (3):

$$\Phi^{-1}\left(C_t\right)=\frac{\Phi^{-1}(p)-\sqrt{\rho}z_t}{\sqrt{1-\rho}} \tag{8}$$

Parameters p and ρ can be estimated from the time series C_t. This estimation can be performed using the method of moments, as outlined in Carlehed and Petrov (2012), or a more advanced approach, such as the one described in Duellmann et al. (2010). Once p and ρ are determined, a connection between macroeconomic variables and the state of the economy z_t can be established by combining (7) and (8).

$$z_t=\frac{\sqrt{1-\rho}\left(\beta_0+\beta_1 X_{1,t-\ell}+\ldots+\beta_h X_{h,t-\ell}\right)-p}{\sqrt{\rho}} \tag{9}$$

A detailed example of this approach, applied to residential mortgages using data from a Comprehensive Capital Analysis and Review (CCAR) stress test conducted by the US Federal Reserve, can be found in Engelmann (2021) who provides a macroeconomic stress scenario transformed via (9) into a stressed state of the economy (z_{stress}). The stressed state is then used to adjust a TTC transition matrix into a stressed transition matrix using (6). The economic stress is fully encapsulated in *z*, while the transition matrix serves to allocate loan balances appropriately across the different rating grades.

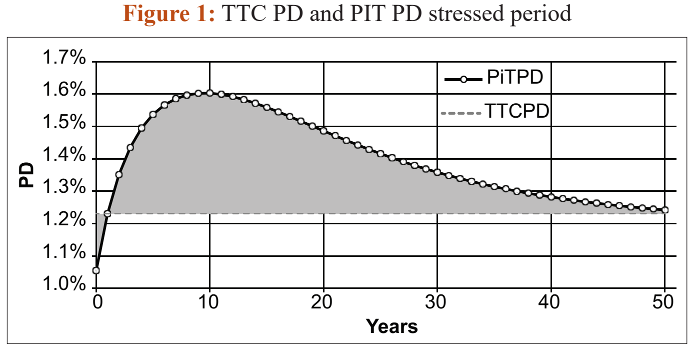
Engelmann (2025) demonstrated that the transition matrix may exhibit spurious behaviour, potentially distort the stress test and produce misleading results. Engelmann (2025) showed when the parameters included in the stress test methodology are inconsistent with the current portfolio, spurious projections of the average portfolio PD might occur that are hidden in the stress test parameterisation and distort the stress test outcome. A typical situation where this scenario could be observed is when a financial institution chooses to use external data (from rating agencies) due to the lack of internal data (Figure 1).

The examples presented by Engelmann (2025) also showed when a stressed state of portfolio PDs is used in stress testing the outcome can produce results which severely underestimates the estimated PDs when using a transition matrix that is not representative of a financial institutions portfolio which can undermine the validity and accuracy of a stress test's outcome and conclusions. A validation step can be included in the stress testing process to first compute the TTC portfolio weight distribution and compared with the financial institution's current portfolio weight distribution across the grades.

Also using the financial institution's current portfolio weight distribution without a stressed condition applied, calculating the portfolios average PD over the stressed period could signal a spurious event occurring in the stress test and lead to understating the effect of applying a stressed scenario on the credit stress approach. This shows the impact of using inconsistent parameters in the credit stress approach and could be visible in the early years of the stress period in Figure 1. The shaded area in Figure 1 shows the importance of the difference, as these estimated PDs indicates the amount of capital under IFRS9 or provisions that needs to be calculated accurately under the ECL framework. This estimation drives the importance of accurate stress testing to be conducted by using consistent risk parameters in this process.

This article introduces a quantifiable scalar metric (S) which through financial institutions can calculate and calibrate against their current vulnerability towards spurious effects, identified by Engelmann (2025). Financial institutions will be able to assess their vulnerability before implementing a stress test and use it as an early

Figure 1: TTC PD and PIT PD stressed period



warning indicator to assess their applicable vulnerability to spurious projections. Our framework provides financial institutions with a controlled and parametrised approach to explore the sensitivity of a stress test outcome towards transition matrix stability.

3. METHODOLOGY AND DATA

Our approach develops a dynamic transition matrix that is conditional on macro state variables aligning with modelling objectives. To estimate the approach, the historical average transition matrix based on available data is calculated (Belkin et al., 1998). Smoothing techniques are applied to ensure monotonicity in the transition matrix, grade specific monotonicity, non-zero migration probabilities and that all grade/rows sum to 1 (Bolder, 2022). While specific implementations and methodologies may vary across financial institutions, the Engelmann (2025) approach serves as a solid starting point for evaluating credit stress testing models.

3.1. Methodology

To explore the dynamics driven by a stress test model's parameterisation, the following model structure is assumed:

3.2. Portfolio Structure

A portfolio comprises n rating grades, with its initial distribution represented as $W = (w_1, w_2, \ldots, w_n)$. These values are expressed as percentages, satisfying the condition $\sum_{i=1}^{n} w_i = 1$.

3.3. Inputs

The model uses a TTC transition matrix T (4) and an origination vector $\mathcal{O} = (o_1, o_2, \ldots, o_n$, as inputs. The origination vector also satisfies the condition $\sum_{i=1}^{n} \mathcal{O}_i = 1$.

3.4. Macroeconomic Influence

A time series for the economic state, z_t for $t = 1, \ldots m$ is derived from a macroeconomic model (9).

3.5. Portfolio Evolution

At each period t, the transition matrix $T(z_t)$ is computed, and the portfolio W_{t-1} is updated using $T(z_t)$ to produce W_t.

3.6. Default and Origination Adjustment

The defaulted portion of the portfolio, $W_{t,n}$ is immediately written off. To maintain a constant total loan balance over time, this defaulted amount is replaced by new originations. The origination vector $\mathcal{O}$ determines the percentage of new loans allocated to each rating grade.

3.7. Matrix Stability

The stability factor S is defined as the average of the diagonal elements of the TTC transition matrix T (4):

$$S = \frac{1}{n-1} \sum_{i=1}^{n-1} p_{i,i} \tag{10}$$

Where n is the number of rating grades (including default grade) and $p_{i,i}$ represents the diagonal elements (self-transition probabilities) for non-default grades i = 1, 2., $n-1$.

3.8. Diagonal Elements

For the diagonal elements of the scaled transition matrix T the stability factor S is applied through the convex combination approach:

$$p_{i,i}(\lambda) = (1-\lambda) \cdot 1 + \cdot \lambda\, p_{i,i} \tag{11}$$

for $i = 1, 2., n-1$ (non-default grades) and $\lambda \in [0,1]$ serves as the scaling parameter, which may be rewritten as:

$$p_{i,i}(\lambda) = 1 - \lambda(1 - p_{i,i}) \tag{12}$$

3.9. Off-Diagonal Elements

For the off-diagonal elements ($i \neq j$ excluding transitions to default), the scaling is applied as:

$$p_{i,j}(\lambda) = (1-\lambda) \cdot 0 + \lambda \cdot p_{i,j} \tag{13}$$

for $i,j = 1, 2., n-1$ where $i \neq j$. This may also be rewritten as:

$$p_{i,j}(\lambda) = \lambda\, p_{i,j} \tag{14}$$

This approach ensures a consistent approach to stress testing while accommodating varying economic conditions and portfolio dynamics. The origination of new loans to defaulted borrowers is generally not part of a financial institution's standard business model, aside from the relatively uncommon practice of investing in distressed loans. These conditions imply that it must be possible for any performing rating grade i to migrate to any other performing rating grade j. While such migrations do not need to occur within a single period, over multiple periods, every transition between performing grades must be achievable. Also to note in this simulation process, we observe that the transition matrix can both depend on economic variables through time and on past transition states. Once the current state of the transition matrix has been observed, determining the next state is trivial (Skoglund and Chen, 2016).

3.10. Data and Simulation Design

To provide a simulated illustrative example, transition matrix T (4) is used from Trueck and Rachev (2009):

$$T = \begin{pmatrix} 0.9276 & 0.0662 & 0.0050 & 0.0009 & 0.0003 & 0.0000 & 0.0000 & 0.0000 \\ 0.0064 & 0.9152 & 0.0700 & 0.0062 & 0.0008 & 0.0011 & 0.0002 & 0.0001 \\ 0.0007 & 0.0221 & 0.9137 & 0.0546 & 0.0058 & 0.0024 & 0.0003 & 0.0005 \\ 0.0005 & 0.0029 & 0.0550 & 0.8753 & 0.0506 & 0.0108 & 0.0021 & 0.0029 \\ 0.0002 & 0.0011 & 0.0052 & 0.0712 & 0.8229 & 0.0741 & 0.0111 & 0.0141 \\ 0.0000 & 0.0010 & 0.0035 & 0.0047 & 0.0588 & 0.8323 & 0.0385 & 0.0612 \\ 0.0012 & 0.0000 & 0.0029 & 0.0053 & 0.0157 & 0.1121 & 0.6238 & 0.2389 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 1.0000 \end{pmatrix}$$

Replicating the approach by Engelmann (2025) for a stylised portfolio with seven grades and starting weights or originating vector as $\mathcal{O}$ = (0.00, 0.20, 0.30, 0.30, 0.20, 0.00, 0.00, 0.00). Following the same approach, the TTC portfolio is produced with weights as (0.0183, 0.1423, 0.3379, 0.2633, 0.1321, 0.0911, 0.0150, 0) (Figure 1).

A key assumption that is made is that the simulation approach is not a standard absorbing Markov chain due to the continuous effect of replacing the defaults with new originations. Our approach (transition matrix with the default row replaced by the origination vector) instead forms an ergodic Markov chain under mild conditions (irreducibility and aperiodicity), which guarantees a unique stationary distribution (Meyer and Stewart, 2023). The conditions for the modified system are that every performing grade has the probability of eventually reaching every other performing grade through T (4) and the origination vector $\mathcal{O}$ has at least one non-zero probability in a grade that has the probability of migrating to all other grades.

The constant-rebalancing assumption (Engelmann, 2025) serves as a simplifying approach that isolates the effect of transition matrix instability from balance-sheet dynamics. In practice, financial institutions adjust origination volumes under stress, which interact with the effects mentioned in this article.

4. RESULTS AND DISCUSSION

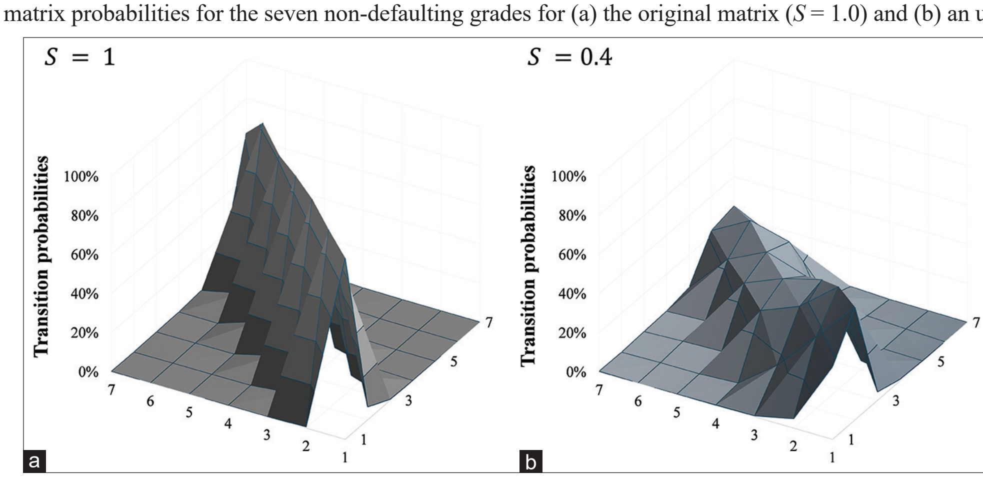
A fundamental consideration in this process lies in the distinction between TTC and PIT risk parameters. Conventionally, key component risk parameters are calibrated for average economic conditions as TTC measures, which are subsequently adjusted through macroeconomic modelling to reflect stressed scenarios. This transformation process, while theoretically sound, introduces potential discrepancies when applied to actual or stylistic loan portfolios. The divergence between the idealised TTC portfolio – defined independently of a financial institution's current portfolio - and the actual portfolio composition can lead to misaligned projections, particularly in average PD estimations. These structural inconsistencies embedded within stress test parameterisation frameworks have significant implications for the accuracy and reliability of stress test outcomes, warranting careful consideration in the interpretation and application of results.

The ramifications of these discrepancies extend significantly to the ECL framework estimation, creating a cascade of potential measurement errors. When TTC parameters fail to accurately represent the current portfolio composition, the resulting PIT transformations may systematically over- or underestimate credit losses under stressed conditions. This misalignment can manifest in several ways: Portfolios with higher-quality assets than the TTC benchmark may show artificially inflated stress losses, while riskier portfolios might appear deceptively resilient. The temporal dynamics of portfolio convergence toward the TTC state introduce additional complexity, as the speed and pattern of this convergence directly influence the accuracy of forward-looking ECL calculations. These estimation errors not only affect regulatory capital requirements and strategic decision-making but may also compromise the fundamental purpose of stress testing as a risk management tool, potentially leading to misallocation of resources and inadequate provisioning for future credit losses.

Central to understanding these dynamics is the stability factor (S (10)), represented by the diagonal elements of the TTC transition matrix. These diagonal entries capture the probability that a borrower remains in the same credit rating category over time and serve as a critical determinant of portfolio evolution patterns. The stability factor directly influences how quickly the current portfolio converges to the theoretical TTC portfolio, with higher stability values indicating slower convergence and more persistent portfolio characteristics. This persistence has profound implications for stress testing accuracy: When stability factors are high, the current portfolio's distinctive features remain relevant for longer periods, amplifying the impact of any misalignment between actual and TTC portfolio compositions. Conversely, low stability factors suggest rapid portfolio turnover, potentially reducing but not eliminating the temporal mismatch effects. The proper calibration and incorporation of these stability parameters are therefore essential for developing more accurate stress testing frameworks that better account for the intertemporal dynamics of credit portfolio evolution.

Given the dynamics of S, in the context of transition matrix transformations, several mathematical approaches exist for implementing scaling operations that preserve essential structural properties. Consider the transition matrix T (4) with elements $p_{i,j}$ representing the probability of transitioning from state i to state j. Figure 2 shows the impact on transition probability of varying S from 1 and 0.4. A fundamental requirement for any scaling operation is that when the scaling parameter equals unity, the original matrix is recovered exactly. The most elementary

Figure 2: Transition matrix probabilities for the seven non-defaulting grades for (a) the original matrix (S = 1.0) and (b) an unstable matrix (S = 0.4)



form of matrix scaling employs direct multiplication; however, this approach fails to preserve the stochastic nature of transition matrices, specifically the constraint that row sums must equal unity. To address this limitation, a convex combination approach can be employed:

$$T(\lambda) = (1-\lambda)\mathrm{I} + \lambda T \quad (15)$$

where I denotes the identity matrix and again $\lambda \in [0,1]$ serves as the scaling parameter.

The stability factor S (10) serves as a diagnostic measure of a given transition matrix's stability. In the simulation framework, the scaling parameter (λ) from (15) is varied to construct matrices with different stability characteristics (where S of the resulting matrix varies monotonically with λ). This formulation possesses several desirable properties: when λ equals unity, the transformation yields the original matrix T, ensuring identity preservation; when λ equals zero, the result is the identity matrix I, representing the limiting case of maximum stability; and crucially, the row-stochastic property is maintained throughout the scaling process, with $\sum_j p_{i,j}(\lambda) = 1$ for all rows i, thereby preserving the fundamental probabilistic interpretation of the transition matrix.

Considering (15), Figure 3 shows varying intervals of S implemented to produce the TTC PD for a given portfolio (W) over 50 years for each interval. Relatively smaller scaling factors produces a more pronounced spurious event in the estimation of the TTC PD over the 50-year period, indicating divergence from the original transition matrix T or the scenario for $\lambda = 1$ has a pronounced impact on estimation of the TTC of the underlying portfolio. This further affects the portfolios' ECL estimation under IFRS 9 over the stressed period and ultimately the accuracy of capital requirements.

Considering a stylistic portfolio constructed with n = 8 and using (4), the effect of varying the S (10) [0,5;0,7;0,9] on the effect can be observed on the PIT PD on grades where n = [2,5,7]. This is shown in Figures 4-6 with the effect on the PIT PD on each of the named grades in the stylistic portfolio the transition matrices with identical default probabilities can create very different PIT PD projections driven by the instability factor (S). Especially, zero versus non-zero transition probabilities between rating grades can have a strong impact on projected PIT PDs. The z-range is adjusted per grade to capture the full range of materially different PIT PD responses.

Figure 3: TTC PD versus S. Arrow indicates decreasing transition matrix stability

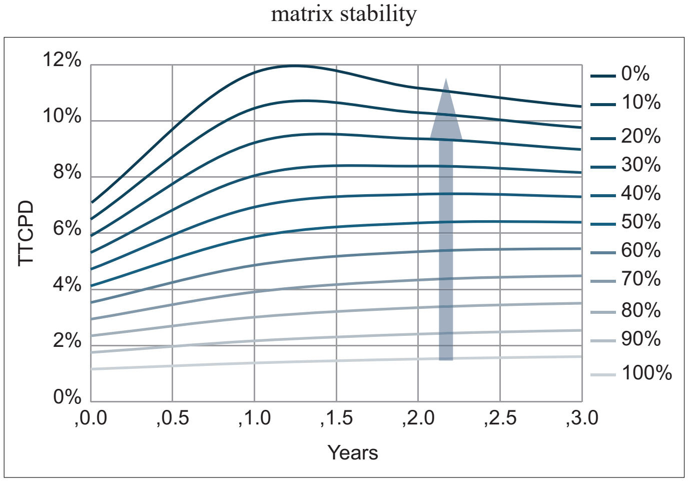


Figure 4: PIT PD for n = 2 versus S. Arrow indicates *decreasing* transition matrix stability

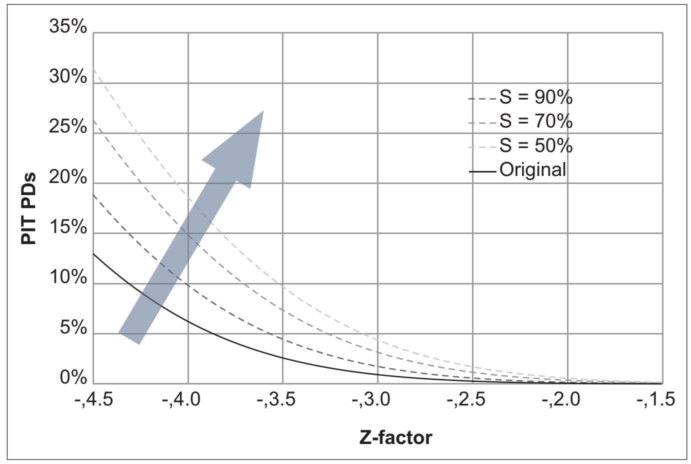


Figure 5: PIT PD for n = 5 versus S. Arrow indicates decreasing transition matrix stability

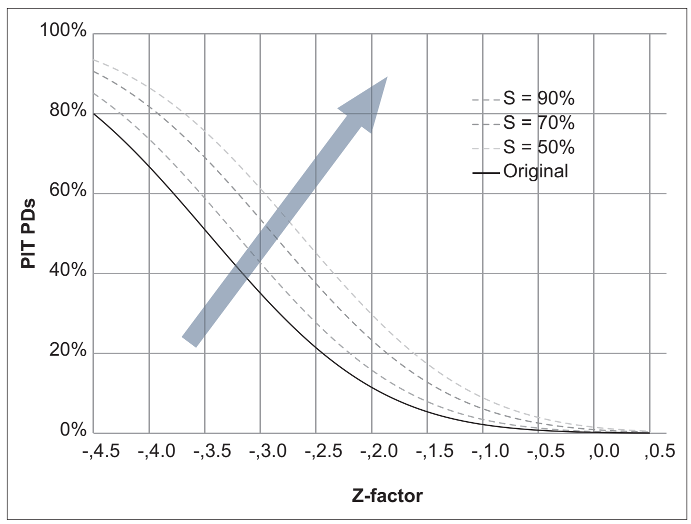
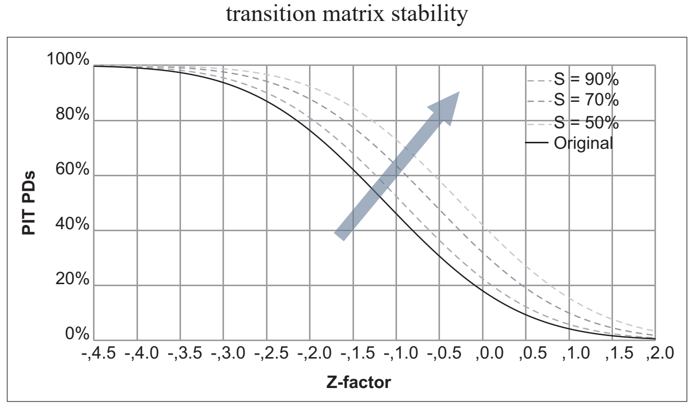


Figure 6: PIT PD for n = 7 versus S. Arrow indicates decreasing transition matrix stability



Internal matrices encode financial institution-specific rating philosophies which make them comparable across time but not across financial institutions. External matrices produced by rating agencies ensure that cross-financial institution comparability is viable but guarantees misalignment with individual portfolios. Hybrid approaches introduce a third and less transparent class of misalignment. By implementing and assessing S before a stress test is undertaken provides a diagnostic tool that is useful regardless of which approach is implemented by a financial institution. It provides the institution with early warning indicators to which degree of potential spurious effects before the stress test is run.

5. CONCLUSION

In conclusion, the extreme portfolio examples presented demonstrate that migration matrices inconsistent with a financial institution's actual portfolio can generate highly volatile and unrealistic outcomes in terms of credit stress tests. This article underscores the critical importance of validating stress test parameters, particularly when financial institutions rely on external transition matrices or have limited internal data history. It is proposed that financial institutions start the validation process by calculating their portfolio's long-term stable distribution and comparing it against their current portfolio composition. Significant deviations across rating grades serve as early warning indicators of potential instabilities in the credit risk stress testing framework, highlighting the need for more robust parameter calibration before implementing stress scenarios.

Financial institutions should also conduct portfolio projections using the unstressed transition matrices and carefully examine both the average portfolio probability of default and the evolution of rating distributions over time. This supplementary analysis enables the detection of spurious boom or recession cycles embedded within the stress test model parameters. When such artificial patterns are identified, the parameterisation must be refined and corrected before proceeding with any credit risk stress testing exercise, ensuring that the stress scenarios reflect realistic economic conditions rather than modelling artifacts.

The implications of these findings extend beyond academic interest to practical risk management applications. Under the IFRS 9 framework, parameter misalignment can systematically bias ECL calculations, affecting both regulatory capital requirements and strategic decision-making processes. Portfolios with credit quality characteristics that differ from TTC benchmarks may experience artificially inflated or deflated stress losses, potentially leading to misallocation of resources and inadequate provisioning for future credit losses.

Future research could extend this analysis to incorporate operational and market risk considerations, examine the interaction between parameter misalignment and different stress scenario designs, and develop more sophisticated calibration techniques for multi-factor stress testing models. The constant-rebalancing assumption could also be relaxed which could be an important direction for future research.

REFERENCES

Aguais, S.D., Forest, L.R., King, M., Lennon, M.C., Lordkipanidze, B. (2007), Designing and implementing a Basel II compliant PIT–TTC ratings framework. In: Ong, M., editor. The Basel Handbook: A Guide for Financial Practitioners. 2nd ed. London: Risk Books. p. 267-298.

Bangia, A., Diebold, F.X., Kronimus, A., Schagen, C., Schuermann, T. (2002), Ratings migration and the business cycle, with application to credit portfolio stress testing. Journal of Banking and Finance, 26(2-3), 445-474.

Bank of England. (2021), Key Elements of the 2021 Biennial Exploratory Scenario: Financial Risks from Climate Change. Available from: https://www.bankofengland.co.uk/stress-testing/2021/key-elements-2021-biennial-exploratory-scenario-financial-risks-climate-change

BCBS. (2018), Stress Testing Principles. Available from: https://www.bis.org/bcbs/publ/d450.pdf [Last accessed on 2025 Jan 08].

Belkin, B., Suchower, S., Forest, L. (1998), A one-parameter representation of credit risk and transition matrices. CreditMetrics Monitor, 1(3), 46-56.

Bolder, D.J. (2022), Modelling Economic Capital. Boston, MA: Springer. p. 75-78.

Boreiko, D.V., Kaniovski, S.Y., Kaniovski, Y.M., Pflug, G.C. (2019), Identification of hidden Markov chains governing dependent credit-rating migrations. Communications in Statistics-Theory and Methods, 48(1), 75-87.

Carlehed, M., Petrov, A. (2012), A methodology for point-in-time-through-the-cycle probability of default decomposition in risk classification systems. The Journal of Risk Model Validation, 6(3), 3-25.

De Bandt, O., Dumontaux, N., Martin, V., Médée, D. (2013), Stress-Testing Banks' Corporate Credit Portfolio. Technical Report. Banque de France.

Duellmann, K., Küll, J., Kunisch, M. (2010), Estimating asset correlations from stock prices or default rates - which method is superior? Journal of Economic Dynamics and Control, 34(11), 2341-2357.

ECB. (2022), 2022 Climate Risk Stress Test. Available from: https://www.bankingsupervision.europa.eu/ecb/pub/pdf/ssm.climatestresstestreport.20220708~2e3cc0999f.en.pdf [Last accessed on 2025 Jan 08].

Engelmann, B. (2021), A Simple and Consistent Credit Risk Model for Basel II/III, IFRS 9 and Stress Testing when Loan Data History is Short. Technical Report.

Engelmann, B. (2025), The hidden risks of credit risk stress test parameterizations: Spurious default probability projections unveiled. Annals of Operations Research, 2025, 1-20.

European Banking Authority. (2025), EU-Wide Stress Testing. Available from: https://www.eba.europa.eu/risk-and-data-analysis/risk-analysis/eu-wide-stress-testing

Gavalas, D., Syriopoulos, T. (2014), Bank credit risk management and migration analysis; conditioning transition matrices on the stage of the business cycle. International Advances in Economic Research, 20, 151-166.

Gordy, M.B. (2003), A risk-factor model foundation for ratings-based bank capital rules, Journal of Financial Intermediation, 12(3), 199-232.

Gunnvald, R. (2014), Estimating Probability of Default using Rating Migrations in Discrete and Continuous Time. Available from: https://urn.kb.se/resolve?urn=urn: nbn:se: kth:diva-150428

Gupton, G.M., Finger, C.C., Bhatia, M. (1997), CreditMetrics - Technical Document. Technical Report. New York: Morgan Guaranty Trust Co.

IFRS Financial Instruments. (2014), Available from: https://www.ifrs.org/content/dam/ifrs/project/fi-impairment/ifrs-standard/published-documents/project-summary-july-2014.pdf

Israel, R.B., Rosenthal, J.S., Wei, J.Z. (2001), Finding generators for

Markov chains via empirical transition matrices, with applications to credit ratings. Mathematical Finance, 11(2), 245-265.

Jafry, Y., Schuermann, T. (2003), Metrics for Comparing Credit Migration Matrices. Wharton Financial Institutions Center [Working Paper No. 03-09]. Available from: https://ssrn.com/abstract=394020

Jarrow, R.A., Lando, D., Turnbull, S.M. (1997), A Markov model for the term structure of credit risk spreads. The Review of Financial Studies, 10(2), 481-523.

Meyer, C.D., Stewart, I. (2023), Matrix Analysis and Applied Linear Algebra. 2nd ed. United States: SIAM.

Miu, P., Ozdemir, B. (2009), Stress testing probability of default and migration rate with respect to Basel II requirements. The Journal of Risk Model Validation, 3(4), 3-38.

Morgan, J.P. (1997), Creditmetrics-Technical Document. Vol. 1. New York: JP Morgan. p. 102-127.

Nickell, P., Perraudin, W., Varotto, S. (2000), Stability of rating transitions. Journal of Banking and Finance, 24(1-2), 203-227.

Nyström, K., Skoglund, J. (2006), A credit risk model for large dimensional portfolios with application to economic capital. Journal of Banking and Finance, 30(8), 2163-2197.

Pesaran, M.H., Schuermann, T., Treutler, B.J., Weiner, S.M. (2006), Macroeconomic dynamics and credit risk: A global perspective. Journal of Money, Credit and Banking, 28, 1211-1261.

Reserve Bank of New Zealand. (2024), 2023 Climate Stress Test Results. Available from: https://www.rbnz.govt.nz/-/media/project/sites/rbnz/files/publications/bulletins/2024/rbb-2024-87-05.pdf

Schechtman, R., Gaglianone, W.P. (2012), Macro stress testing of credit risk focused on the tails. Journal of Financial Stability, 8(3), 174-192.

Skoglund, J., Chen, W. (2016), The application of credit risk models to macroeconomic scenario analysis and stress testing. Journal of Credit Risk, 12(2), 1-45.

Trueck, S., Rachev, S.T. (2009), Rating Based Modeling of Credit Risk: Theory and Application of Migration Matrices. Massachusetts: Academic Press.

US Federal Reserve. (2024), Climate Scenario Analysis Exercise Results. Available from: https://www.federalreserve.gov/publications/files/csa-exercise-summary-20240509.pdf

US Federal Reserve. (2025), Stress Tests. Available from: https://www.federalreserve.gov/supervisionreg/stress-tests-capital-planning.htm

Witzany, J. (2022), Stressing of migration matrixes for International Financial Reporting Standard 9 and Internal Capital Adequacy Assessment Process calculations. Journal of Credit Risk, 18(3), 49-76.