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Market Risk Assessment: A Comparison between CAPM and VaR Methodologies with High Volatility Episodes in the Mexican Stock Market

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ABSTRACT

This study aims to estimate the β-CAPM for firms listed on the Mexican Stock Exchange Price Index (IPC) by incorporating Value at Risk (VaR) measures during periods of high market volatility. We apply five risk estimation methodologies: (1) Historical Simulation VaR (VaR-SH), (2) Delta-Normal VaR (VaR-δN), (3) Monte Carlo Simulation VaR (VaR-MC), (4) Threshold-GARCH-based VaR (TGARCH-VaR), and (5) Expected Shortfall (ES). The results demonstrate consistency across the different methodologies during high-volatility periods. Nonetheless, we identify two important limitations: β-CAPM is constrained to equity portfolios, and VaR-based methods may underestimate tail risk during extreme market events. This study enhances the understanding of market risk exposure in the Mexican stock market by offering a comparative analysis of multiple VaR methodologies. It provides a valuable reference for assessing the relative riskiness of individual securities against the market portfolio, especially under conditions of financial stress.

Keywords: Value at Risk, Historical Simulation, Monte Carlo Simulation, Capital Asset Pricing Model, TGARCH, Expected Shortfall
JEL Classifications: G11, C52, G17, C58

1. INTRODUCTION

Global economic and financial integration encompasses systemic risks that are, at best, predicted using historical data and projections of potential future scenarios. Financial and monetary authorities have made numerous efforts to reach a consensus on the various instruments financial institutions can implement to mitigate systemic risks and prevent contagion across different financial and economic systems.

In this context, the economic and financial integration processes among various world regions have intensified to the extent that monetary and financial authorities regulate the identification and measurement of market risks through the Basel Accords. This is reflected in a series of prudential macro-financial guidelines aimed at minimizing exposure to risks and potential contagion between specific markets, particularly during periods of financial and economic stress and high volatility[1].

The financial and economic crises experienced in 2008-2009 and 2020-2021 contributed to the theoretical discussion on assessing

1 In the late 1970s and early 1980s, some companies began designing systems for risk measurement. One of the most well-known systems was RiskMetrics, developed by JP Morgan.

the market risk measures applied under the Basel III Agreements on Banking Regulation[2]. In general, these analyses examine the sensitivity of risk measurement to various market variables, including interest rates, volatility, and exchange rates, as well as other economic and financial indicators[3].

Several methodological proposals exist for estimating market risks, including Value at Risk (VaR) and Expected Shortfall (ES). However, for market operators and financial intermediaries, the β-CAPM metric remains the most widely used indicator for quantifying the risk of a specific portfolio being valued in relation to the risk of the estimated market portfolio[4].

Historically, the β-CAPM has been the predominant methodology among financial institutions and intermediaries for measuring the market risk of stock portfolios. Its prominence originates in the seminal works of Fama (1965; 1973). In contrast, the VaR methodology has only recently gained traction, emerging from the various Basel Agreements on capital requirements and risk measures that have been gradually implemented since 1996[5].

Essentially, the CAPM was designed to identify potential discrepancies in the risk premiums of different financial assets, which are partially explained by the differences in the inherent risk of each asset's return. Specifically, the CAPM estimates a consistent risk measure through the calculation of Beta so that the observed differences in the expected returns of different assets are explained by the fact that the Betas of each asset are specific in each case.

However, each financial asset has characteristics that the estimation of Betas, as a standardized risk measure, might not reflect at a given time. McGrattan and Jagannathan (1995) point out that even though there is some debate on the accuracy of risk estimations that depend heavily on the selected time windows, such as Betas' estimations, CAPM's estimates are still a reference for financial economists and policy makers. This study does not aim to contribute to this debate.

The purpose of this study is to assess the market risk of stock portfolios in the Mexican financial market during periods of high volatility using six complementary methodologies: (i) The CAPM Beta (β-CAPM), (ii) Historical Simulation-VaR (VaR-HS), (iii) Delta-Normal VaR (VaR-δN), (iv) Monte Carlo Simulation-VaR (VaR-MC), (v) VaR-TGARCH, and (vi) Expected Shortfall (ES)[6] to better capture tail risks and provide a more accurate estimation of potential losses in extreme market conditions. These methodologies were chosen for easy implementation because of their parsimony.

2. LITERATURE REVIEW

Black (1972; 1993) notes that discrepancies in the CAPM arise from its design limitations, particularly the use of Beta with a simulated portfolio rather than a real one. To address this, this study uses actual market portfolio composition. Black (1972; 1993) also points out that the regression intercept might be misestimated without risk-free assets, biasing the Beta risk return slope. Fama and MacBeth (1973) further investigate whether the square of Beta and the volatility of a financial asset's return help explain the residual variation in average returns not accounted for by Beta.

Banz (1981) finds one specific characteristic that Betas does not explain: the relative size of companies listed in stock markets. This helps explain part of the residual variation in the average returns. Additionally, Hwang and Pedersen (2004) note that differences in economic conditions between advanced and emerging economies also contribute to residual variations in average returns, influencing expectations and the risk-return relationship in financial markets.

Estrada and Serra (2005), and Estrada (2002) note that while there is no consensus on the differences between emerging and advanced stock markets regarding risk-return estimation and cross-sectional analysis, traditional risk variables used for advanced markets show a weak relationship with observed returns. This suggests that local and global economic-financial conditions influence stock prices in emerging economies[7].

Sarwar (2019) noted that high economic and financial integration reduces the benefits of diversification in advanced and emerging economies. This is because of the high correlation between stock market returns in these economies, driven by trade relations, foreign direct investment flows, and portfolio investments. Additionally, volatility dispersion through indices such as the Volatility Index (VIX)[8], which is widely used in emerging economies, indicates the volatility of U.S. financial markets[9].

Bisias et al. (2012) suggest that several risk exposure measures on stock market instruments apply prudential measures for

2 The Basel I, II, and III accords represent efforts in banking regulation and international norms conducted by the Basel Committee of the Bank for International Settlements since 1974. Accordingly, financial institutions are required to report market risks frequently. Refer to the Comprehensive Circulars for Banks, Pension Funds (Afores), Specialized Retirement Funds (Siefores), Credit Institutions, Financial Institutions, Circular 4/2012 (31 Points from the Bank of Mexico).

3 The estimation based on the β-CAPM structurally does not include these variables.

4 For a review of the theoretical and empirical scope of the □-CAPM methodology, see Fama and French (2004).

5 By consensus, its origin is assumed to lie in the Technical RiskMetrics document that the investment bank J.P. Morgan released in the mid-1990s.

6 The Basel Committee on Banking Supervision (BCBS) recommended the use of Expected Shortfall (ES) alongside Value at Risk (VaR) to estimate market risks for financial institutions as part of its Fundamental Review of the Trading Book (FRTB). This recommendation was finalized and published in January 2016. The FRTB aimed to address weaknesses in the existing market risk framework and enhance the banking sector's resilience by introducing a more risk-sensitive approach to capital requirements for market risk. The Basel Committee's revised market risk framework under the FRTB was set to be implemented by January 1, 2023.

7 Estrada and Serra (2005) implicitly suggest that the stock markets are not fully integrated.

8 Since 1993, the volatility index VIX is calculated by the Chicago Board of Options Exchange (CBOE) with options from the Index Standard and Poor's 100, and later with the Index Standard and Poor's 500.

9 Review Bekaert et al. (2009; 2014) and Phylaktis and Xia (2006).

financial institutions with more extensive market participation than commercial banks. These include: i) institutional leverage as in Merton and Bodie (1993) and Geanakoplos (2010), ii) Credit Default Swaps (CDS) from Jobst and Gray (2013), and iii) the Co-Value at Risk (Co-VaR) models of the International Monetary Fund (2009), which quantify risk based on the market prices of issued stocks[10].

This study compares these proposed methodologies to calculate market portfolio composition and evaluate price realizations during periods of high volatility. This approach prevents the underestimation of risks that might otherwise occur.

The Value at Risk (VaR) methodology calculates the maximum expected loss of a stock portfolio and, while relatively recent, traces its origins to the capital requirements of the New York Stock Exchange in 1922[11]. The CAPM by Sharpe (1964), Lintner (1965), and Mossin (1969) initiated the Asset Pricing Theory, enabling the quantification of the risk-return balance in stock market investments. Unlike Black (1972), Black et al. (1972), Fama and MacBeth (1973), and Black (1993), who emphasize risk measurement through β-CAPM, this study aims to estimate and compare the market risk of β-CAPM for portfolios of Mexican stock market companies with the VaR methodology during periods of high volatility and economic-financial stress.

Grajales and Pérez (2010) note that volatility is a crucial factor to consider when identifying the efficiency of risk estimates using different methodologies, including VaR. These methods produce varying risk estimates associated with asset returns, which are not necessarily normally distributed. However, in this study, the VaR-δN, VaR-SM, VaR-TGARCH, and β-CAPM methods assume that stock portfolio returns follow a normal distribution, while VaR-SH uses an empirical density function.

In this context, statistical evidence shows that portfolio returns do not behave as a normal distribution function because extreme events occur more frequently and predictably in stock portfolio return series than in the normal distribution function estimates.

However, parametric methods typically assume that stock return distributions are normally distributed. Applying other distribution functions, such as the Inverse Gaussian distribution, complicates the implementation of the risk quantification methodology due to the complexity of the specific function[12].

In this study, although each quantitative method for risk estimation assumes a symmetric and normal distribution function for stock portfolio returns, with only one case using an empirical density function, no other proposed model calibration is used to correct the bias generated otherwise. It acknowledges that the distributions are neither normal nor symmetric and exhibit fat tails. However, the study aims to make the different VaR measures and β-CAPM comparable, without weakening the validity of estimates due to the self-imposed limitations inherent in the normality assumption.

Engle and Manganelli (2004) classified VaR measurement methodologies into three main groups: i) parametric methods that assume a distribution function for stock returns, ii) non-parametric methods that do not assume any known distribution function and use an empirical distribution function, and iii) hybrid methods that combine parametric and non-parametric approaches. Each of these methods has its strengths and weaknesses.

Abad et al. (2014) pointed out that VaR estimations using extreme value theory and filtered historical simulation methodology also improve risk estimates. This study proposes a VaR method for each group identified in Engle and Manganelli's classification: VaR-δN and VaR-TGARCH for parametric methods, VaR-SH for non-parametric methods, and VaR-SM for hybrid methods[13].

Rossignolo (2017; 2019) and Álvarez and Rossignolo (2015) note that while the Bassel Accords have incorporated the volatile and concentrated nature of financial markets in general, their implementation through the Standardized Approach (SA) and the Internal Models Approach (IM) have resulted in some discrepancy regarding the proper incentives to apply the most appropriate technical and economic policy methodologies. This suggests a significant gap in the implementation of risk measurement methodologies, their proper calibration, and their expression in the minimum capital requirements imposed by current regulations[14].

In the theoretical and regulatory framework of financial institutions such as Investment Funds, Brokerage Houses, Pension Funds Administrators (AFORES), Insurers, Commercial and Investment Banks, and other Financial Entities operating in the Mexican Financial System, the most used quantitative methodologies for calculating market risk are VaR-SH, VaR-SM, VaR-δN, and β-CAPM. These methodologies are preferred for their simplicity, ease of implementation and interpretation, and because more sophisticated models require higher technological and human capital requirements[15].

In contrast to Rossignolo (2017, 2019), Álvarez and Rossignolo (2015), Benoit et al. (2017), and Angelidis et al. (2004), whose

10 Refer to Bisias et al. (2012) for a literature review about systemic risks.

11 For a historical revision for using VaR as a quantitative methodology to estimate market risks, read Holton, G. (2002, 2003), and Morgan, J.P. (1996).

12 Review Trejo et al. (2006) for a detailed treatment of the Inverse Normal Gaussian function and its implementation to measure asset portfolio risks.

13 For a broader discussion about the implications of VaR on the return's distribution function, review Angelidis et al. (2004).

14 Rossignolo (2017; 2019) suggests conducting additional tests that could strengthen the case for adjustments to the Basel Committee's regulations for risk measurement, considering abnormal distributions and high volatility in Latin American financial markets. He proposes adopting specific and flexible calibration coefficients for each financial market to adjust excessively high minimum capital requirements resulting from underestimating risk during economic and financial stress periods.

15 In the case of VaR-SM, for instance, the technological requirements are higher because it requires a minimum of 10,000 simulations *per* market variable, which increases resource usage and the execution time of this method.

main objective was to identify alternative methodologies consistent with internal models to enhance efficiency in risk measurement during periods of high volatility and financial stress, this study measures and compares the risk of β-CAPM and VaR in their different forms: VaR-SH, VaR-δN, VaR-TGARCH and VaR-SM. These methodologies were re-expressed to facilitate a comparison with the results of the β-CAPM risk estimation[16].

This study contributes to the literature not by identifying the limitations of VaR models through adjustments and corrections, as suggested by Rossignolo (2017; 2019), Álvarez and Rossignolo (2015), Grajales and Pérez (2010), and Salinas Ávila (2009). While acknowledging that the distribution of stock portfolio returns is neither symmetric nor normal, and has fat tails, this study does not propose alternative methodologies to address these structural limitations.

The primary purpose of this study is to integrate the estimation and interpretation of risk measurement using β-CAPM with a quantitative comparative analysis of risk measurement for stock portfolios through the VaR methodologies proposed in this study during periods of high and low volatility. This approach contributes to identifying the consistency of risk measurement using different methods[17].

Álvarez and Rossignolo (2015) argue that a proper calibration of VaR methodologies to estimate risk should consider that the stock portfolio returns series distribution exhibits "fat tails " and asymmetry. Thus, both tails of the distribution should be employed to enhance the performance of applied Extreme Value Theory models (EVT)[18].

Following Artzner et al. (1997; 1999), we estimated the Expected Shortfall (ES) using different VaR methodologies, considering that Yamai and Toshiba (2002) pointed out that VaR and ES estimates under stress conditions could lead to underestimated risks[19].

In this study, we utilize the VIX volatility index, combined with S&P 500 options, to identify periods of high volatility and the windows where the β-CAPM and VaR may overestimate or underestimate risks. The total data sample covers five specific periods where episodes of high volatility and extreme stock market conditions are: (i) September 15, 2008, to May 19, 2009, (ii) May 6, 2010, to July 8, 2010, (iii) August 4, 2011, to December 9, 2011, (iv) February 27, 2020, to July 14, 2020, and (v) April 26, 2022 to June 22, 2022[20].

16 Ospina Duque and Tangarife Trujillo (2008) compare VaR-SH, VaR-δN, and VaR-SM estimates for a portfolio of Colombian financial market stocks, but they do not integrate β-CAPM into their analysis.

17 While the CAPM Beta measures the risk of a stock portfolio relative to the market portfolio, VaR indicates the maximum expected loss that a given investment portfolio can experience over a time horizon and at a confidence level. It might seem that these metrics are not directly comparable; however, both are used to measure market risk for such portfolios. The risk of committing a type beta error is high in terms of false positives.

18 The extreme value theory has been used to model the tails of stock portfolio return distributions to incorporate the probability associated with periods of stress or high volatility, which are, by definition, extreme events.

19 Lazar and Zhang (2019) pointed out that ES risk estimates outperform those of the VaR tests, suggesting that the latter should be substituted in the Basel normativity.

The remainder of this paper is structured as follows. Section 3 describes the methodology of the risk estimation models used in this study, namely, β-CAPM, VaR-SH, VaR-δN, VaR-SM, VaR-TGARCH, and ES. Section 4 compares the results obtained for the estimated risk using these four models based on the sample's observed periods of high and low volatility. Finally, Section 5 concludes the study and identifies avenues for future research.

3. METHOD

The Value at Risk (VaR) is a statistical measure of the potential losses associated with a specific set of stocks an investor might invest in. This measurement had two essential characteristics. The primary benefit is that, through its measurement, it provides a brief representation of the maximum amount of loss associated with an investment in a portfolio of stocks, given a specific probability. The second characteristic is that VaR estimation considers the correlations between risk factors. These two features enable the VaR to allocate capital across different assets.

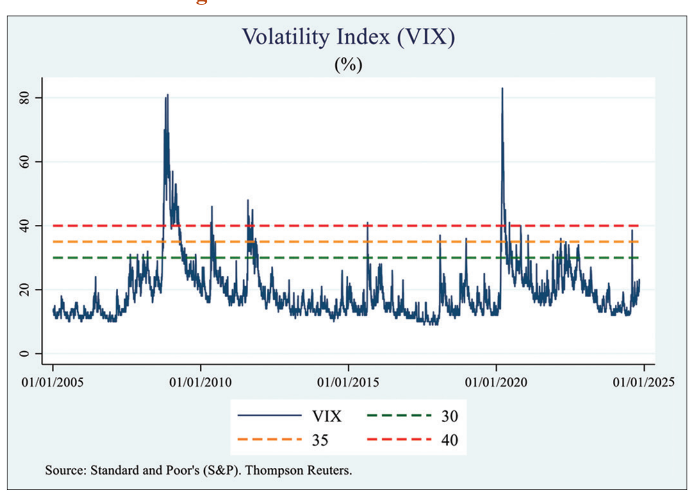
The calculation of Historical Simulation VaR is as follows:

Let be R, the set of all historical[21] market risk factors impacting a specific portfolio of financial instruments, where R is the matrix of dimension $(n+1)\times m$, $(n+1)$ represents the number of periods, and m is the number of risk factors. In other words:

20 Based on the VIX (Volatility Index) associated with the S&P 500 stock index, five specific periods of financial stress are worth noting where a pronounced deterioration was observed in global economic activity indicators, accompanied by increased volatility in international financial markets. These episodes of high volatility reflected concerns regarding, *inter alia*, the fiscal sustainability of the United States and the peripheral countries within the eurozone and the declining global growth prospects, along with the unprecedented economic challenges caused by the pandemic in industrialized and emerging market economies. Overall, it reflected a significant deterioration in global economic activity indicators, signaling recession episodes in several countries. These economic and financial downturns, accompanied by heightened risk aversion, raised in stock and debt markets worldwide.

21 In practice, it is the database of market risk factors.

Figure 1: Periods of financial stress



$$R = \{\bar{r}_0, \bar{r}_1, \bar{r}_2, \ldots, \bar{r}_n\} \quad (1)$$

Each component in equation (1) represents the value that all risk factors take on a specific date, with *m* as the number of factors for each component. Thus, the dimension of each component $\bar{r}_i$ is *m*, i.e., $|\bar{r}_i| = m$. Therefore, $\bar{r}_i^T = \{r_i^1, r_i^2, \ldots, r_i^{m-1}, r_i^m\}$ represents the risk market factors vector, where *m* is the number of risk market factors affecting the theoretical portfolio at the time *i*, and *T* denotes the transposed vector. In doing so, the value of the risk factors for the current scenario is:.

Table 1. Expected Short Fall Summary statistics

	ES-Delta Normal	ES-SH	ES-GARCH	ES-Monte Carlo
May 31, 2017	**0.02193**	**0.02642**	**0.02010**	**0.02166**
Period 1				
September 15, 2008	0.03098	0.04169	0.03268	0.03045
May 19, 2009	0.05322	0.06109	0.04905	0.05411
Period 2				
May 6, 2010	0.05090	0.06109	0.03866	0.04780
July 8, 2010	0.05108	0.06109	0.03624	0.05065
Period 3				
August 4, 2011	0.02278	0.03025	0.03356	0.02041
December 9, 2011	0.02789	0.04026	0.03273	0.02886
Period 4				
February 27, 2020	0.02580	0.03353	0.03055	0.02487
July 14, 2020	0.03649	0.05710	0.02940	0.03554
Period 5				
April 26, 2022	0.02901	0.03270	0.02910	0.02436
June 22, 2022	0.02678	0.02575	0.03092	0.02561

Source: own calculations based on data from Thompson Reuters

Figure 2: Expected shortfall for the proposed methodologies

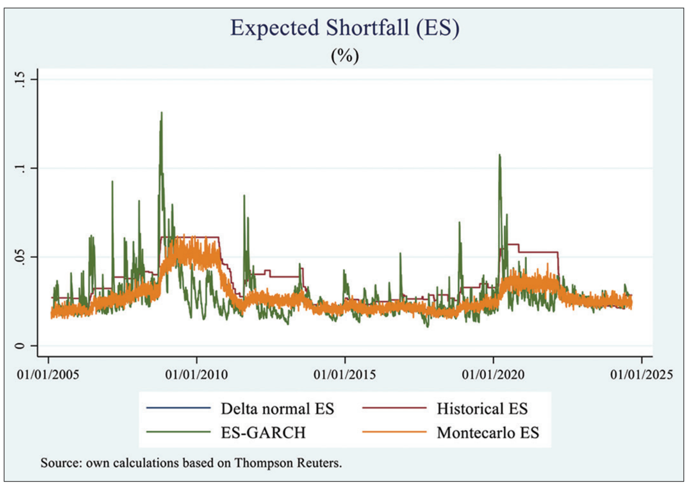
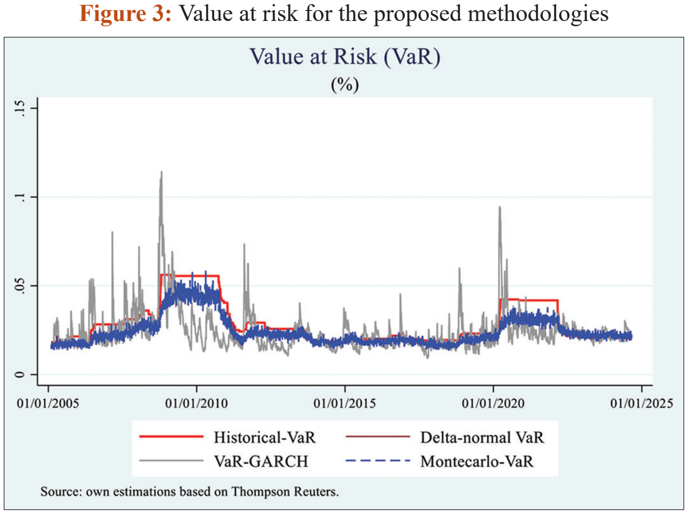


Figure 3: Value at risk for the proposed methodologies



In this context, for the data in *R*, we design a set of scenarios $\bar{s}_1, \bar{s}_2, \bar{s}_3, \ldots, \bar{s}_n$, where each $\bar{s}_j \forall j = 1, 2, \ldots, n$, is used to identify the risk factor scenario, given the observations in $\bar{r}_j \wedge \bar{r}_{j+1}$. It is important to note that $\bar{s}_j$ is the forecast of the risk factors for a certain period, known as the "holding period", given the current level of these risk factors and their levels at the dates of $j \wedge j + 1$.

3.1. Calculation of Scenarios

The scenarios were calculated as follows: Let the *j-th* scenario $\bar{s}_j$ be defined as:

$$\bar{s}_j = g\left(\bar{r}_0, \mathrm{f}\left(\bar{r}_j, \bar{r}_{j+1}\right)\right) \forall j = 1, 2, \ldots, n \quad (2)$$

Where in equation (2): $\mathrm{f}\left(\bar{r}_j, \bar{r}_{j+1}\right) = \left(\frac{r_j^{\,1}}{r_{j+1}^{\,1}}, \frac{r_j^{\,2}}{r_{j+1}^{\,2}}, \ldots, \frac{r_j^{\,m}}{r_{j+1}^{\,m}}\right)$ is the rate of change between the factors given two dates, and $g(\cdot)$ is the value of the risk factors in the scenario *j* given the rate of change between data at time $j \wedge j + 1$ and the current value of those factors, that is,

$$\bar{s}_j^{\,T} = g\left(\bar{r}_0, \mathrm{f}\left(\bar{r}_j, \bar{r}_{j+1}\right)\right) = \left(r_0^1 \cdot \frac{r_j^1}{r_{j+1}^1},\ r_0^2 \cdot \frac{r_j^2}{r_{j+1}^2}, \cdots, r_0^m \cdot \frac{r_j^m}{r_{j+1}^m}\right) \quad (3)$$

This general methodology considers a database of risk factors; however, focusing on the case of a single risk factor, equation (3) can be rewritten as follows:

Table 2. Value at Risk Summary statistics

	VaR-Delta Normal	VaR-SH	VaR-TGARCH	VaR-Monte Carlo
May 31, 2017	**0.01911**	**0.02011**	**0.01756**	**0.01939**
Period 1				
September 15, 2008	0.02701	0.03379	0.02857	0.02610
May 19, 2009	0.04651	0.05555	0.04284	0.04943
Period 2				
May 6, 2010	0.04441	0.05555	0.03371	0.04117
July 8, 2010	0.04453	0.05555	0.03183	0.04481
Period 3				
August 4, 2011	0.01983	0.02440	0.02924	0.01911
December 9, 2011	0.02431	0.02922	0.02852	0.02716
Period 4				
February 27, 2020	0.02257	0.02310	0.02647	0.02228
July 14, 2020	0.03194	0.04230	0.02560	0.03316
Period 5				
April 26, 2022	0.02522	0.02709	0.02552	0.02163
June 22, 2022	0.02331	0.02195	0.02701	0.02239

Source: own calculations based on data from Thompson Reuters

Figure 4: Comparison of expected shortfall and VaR measures

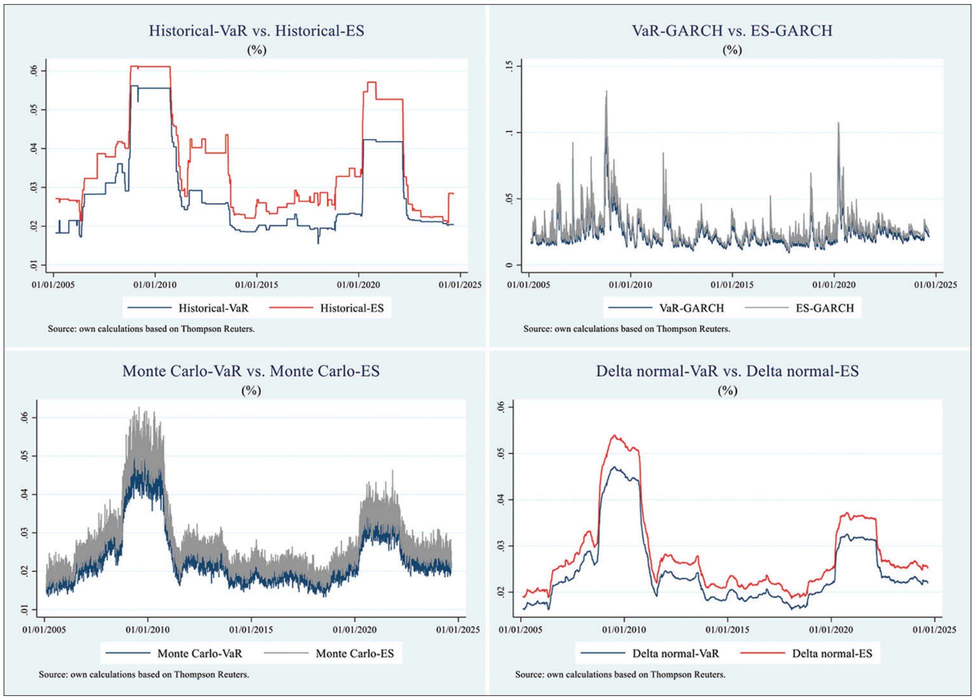


Table 3. BETAS Results Base Period

	Base Period: May 31, 2017			
	VaR-Delta Normal	VaR-SH	VaR-TGARCH	VaR-Monte Carlo
GMEXICO	1.07528	1.06816	1.07062	1.08808
ALSEA	0.88531	0.85661	0.91259	0.95635
KIMBER	0.73414	0.79371	0.73422	0.77480
PINFRA	0.82767	0.89941	0.83035	0.82304
GFNORTE	1.27436	1.31604	1.27774	1.33371
ALPEK	0.60886	0.78214	0.60320	0.58127
LAB	0.52064	0.58247	0.56046	0.61350
CEMEX	1.85570	1.60320	1.84793	1.80051
BOLSA	0.53839	0.59646	0.55088	0.60703
BIMBO	0.99300	1.06660	0.98540	1.00579
FEMSA	0.80143	1.06518	0.81339	0.83776
OMA	0.79167	0.86903	0.81308	0.92425
GRUMA	0.49311	0.63012	0.50765	0.55991
ASUR	0.80198	0.86457	0.84299	0.88208
GENTERA	0.83870	0.77174	0.84086	0.85695
TLEVISA	0.92538	0.84691	0.88714	0.97369
GAP	0.80039	0.93129	0.84843	0.79572
GCARSO	1.15259	1.04332	1.16122	1.21433
LIVEPOL	0.94769	1.04237	0.91239	0.90182
MEXCHEM	0.95871	0.83695	0.95561	1.03700
AC	0.60232	0.79152	0.63085	0.58979
PEÑOLES	0.72197	0.84045	0.76713	0.94336
ALFA	1.09006	0.99515	1.05626	0.94126
GFINBUR	1.14784	0.96157	1.11883	1.16512
WALMEX	0.82048	1.12299	0.81984	0.72673
ELEKTRA	0.44352	0.35424	0.52046	0.50505
GFREGIO	0.64665	0.50983	0.66193	0.56199
LALA	0.76682	0.81248	0.75511	0.78685
NEMAK	0.54922	0.30328	0.53588	0.42647
OHLMEX	1.05297	0.65939	1.04466	1.05852
VOLAR	0.59828	0.86082	0.62082	0.70602
IENOVA	0.48447	0.46502	0.48570	0.50471
SANMEX	0.90386	0.72407	0.90591	1.02576
KOF	0.75320	0.82818	0.76050	0.76079
AMX	1.18830	0.88080	1.16966	1.07288

Source: own calculations based on data from Thompson Reuters

$$\bar{s}_j = g\left(r_0, \mathrm{f}\left(r_j, r_{j+1}\right)\right) = r_0\, \mathrm{f}\left(r_j, r_{j+1}\right) = r_0\left[1+\frac{r_{j+1}}{r_j - 1}\right] = r_0\left[1+ {}_j\rho_{j+1}\right] \quad (4)$$

where: ρ_{j+1} represents the percentage change in the observed risk factor from date j to date $j+1$[22], as identified in equation (5).

$${}_j\rho_{j+1} = \frac{r_{j+1}-r_j}{r_j} = \frac{r_{j+1}}{r_j} - 1 \quad (5)$$

In this way, equations (3), (4), and (5) show how the multiplicative scenarios for a historical simulation are calculated for VaR estimation.

3.2. Profits and Losses Forecast

After obtaining the scenarios, we calculate the portfolio's theoretical value in each scenario, which constitutes the forecast for the *"holding period"*. Let $V_j = V\left(\bar{s}_j\right)$ be the hypothetical value of the instrument in scenario j, $\bar{V} = \left(V\left(\bar{s}_1\right), V\left(\bar{s}_2\right), \ldots, V\left(\bar{s}_n\right)\right)$ represents the series of theoretical values of instrument i in each scenario.

Additionally, it is worth noting that the function $V(\cdot)$ [23], and the risk factors involved may vary depending on the portfolio under analysis since it represents the aggregated valuation of the portfolio components. Therefore, given that $V_0 = V\left(\bar{r}_0\right)$ identifies the baseline value of the portfolio or *"mark to market"*, we calculate the profit or loss in each scenario as: $PnL_j = V_j - V_o \forall j = 1, \ldots, n$. The difference represents the change in the portfolio's value relative to its baseline value under the scenario j[24].

22 Period corresponds to the "holding period," typically one day.
23 Function $V(\cdot)$implies underlying assessment models.
24 This change is known as "*Profit and Loss*" or *PnL*.

Table 4. BETAS Results Period 1

Period 1	September 15, 2008				May 19, 2009			
	VaR-Delta Normal	VaR-SH	VaR-TGARCH	VaR-Monte Carlo	VaR-Delta Normal	VaR-SH	VaR-TGARCH	VaR-Monte Carlo
GMEXICO	1.51682	2.24780	1.51186	1.53921	1.51417	1.72236	1.51703	1.53851
ALSEA	0.85657	0.95041	0.84103	0.91640	0.98215	0.61979	0.95125	0.85470
KIMBER	0.73046	0.65648	0.72566	0.70155	0.63376	0.70544	0.64520	0.68305
PINFRA	0.82979	1.44052	0.85646	0.84544	0.80163	0.89825	0.79992	0.78845
GFNORTE	1.25176	1.17509	1.23589	1.32244	1.60227	1.18166	1.59673	1.52816
ALPEK	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LAB	0.06157	0.04793	0.05791	0.01487	0.36490	0.29097	0.36204	0.31730
CEMEX	1.43128	0.99964	1.38348	1.31282	1.76525	0.99586	1.74496	1.79128
BOLSA	0.13463	0.11866	0.09416	0.10216	0.59322	0.27051	0.58529	0.58917
BIMBO	0.80404	0.94729	0.84136	0.77203	0.73511	1.00398	0.74376	0.75497
FEMSA	1.16322	1.03598	1.17224	1.18057	1.02061	1.19056	1.03024	1.06745
OMA	0.65701	0.55102	0.61912	0.63025	0.69691	0.57568	0.66223	0.64410
GRUMA	0.53487	0.55271	0.51647	0.53092	0.73593	0.48452	0.71729	0.66825
ASUR	0.62447	0.58869	0.63995	0.63075	0.60131	0.76782	0.59745	0.49399
GENTERA	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
TLEVISA	1.08191	1.04957	1.07435	1.06341	0.95362	0.86943	0.94766	0.90805
GAP	0.92311	0.85699	0.89645	0.92621	0.73245	0.77469	0.71537	0.76655
GCARSO	0.94646	1.12921	0.96010	0.82240	1.01871	1.16119	1.02944	0.99122
LIVEPOL	-0.05288	-0.14629	-0.00127	0.06369	0.00244	0.04979	-0.01184	-0.03380
MEXCHEM	0.52045	0.92317	0.63916	0.63463	1.02841	1.74214	1.05453	1.03240
AC	0.20174	0.23360	0.21036	0.22651	0.15450	0.19912	0.14877	0.15637
PEÑOLES	1.07584	1.61363	1.10654	1.03797	1.16557	1.74587	1.20677	1.25510
ALFA	0.88159	0.87559	0.87136	0.88381	1.04481	0.77357	1.01151	0.99536
GFINBUR	0.12224	0.13071	0.18000	0.07125	0.37364	0.54170	0.40931	0.49195
WALMEX	1.22012	1.02158	1.20706	1.23156	0.78225	0.83080	0.78781	0.81455
ELEKTRA	0.37261	0.42358	0.46871	0.45420	0.55063	1.02932	0.60943	0.61391
GFREGIO	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LALA	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
NEMAK	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
OHLMEX	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
VOLAR	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
IENOVA	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
SANMEX	0.57580	0.47803	0.56823	0.65798	0.85332	0.73164	0.85043	0.90203
KOF	0.56119	0.47993	0.60065	0.66959	0.60715	0.93994	0.62350	0.63690
AMX	1.37902	1.46434	1.37621	1.32597	1.16354	1.41917	1.16401	1.15089

Thus, the series of profits or losses *PnL* for the portfolio obtained after revaluing it in each scenario *j* is:

$$PnL = \left(PnL_1, PnL_2, \ldots, PnL_n\right) \tag{6}$$

Finally, in equation (6) we arrange the elements of *PnL* in ascending order and obtain:

$$PnL = \left(PnL_{j:1}, PnL_{j:2}, \ldots, PnL_{j:n}\right) \tag{7}$$

Where (:) in equation (7) indicates that the series is ordered, so it holds that:

$$PnL_{j:1} \leq PnL_{j:2} \leq, \ldots, \leq PnL_{j:n} \tag{8}$$

In equation (8) *j* has been appended to represent that any *PnL* from the original series can be in the first position, any other *PnL* in the second position, and so on. By contrast, the index numbers now represent an ascending order, that is, a statistical order.

3.3. Calculation of Historical Simulation (VaR) and Total Valuation

The estimation of the Historical Simulation VaR with a confidence level α for the portfolio (VaR^{α}_SH) summarizes the identification of the element *k* or $PnL_{j:k}$ in the *PnL* that corresponds to the desired confidence level α[25]:

$$\begin{aligned} k &= n\left(1-\alpha\right) \\ VaR^{\alpha}\_SH &= PnL_{j:k} \end{aligned} \tag{9}$$

This way, equation (9) allows us to calculate the VaR using the Historical Simulation method. VaR analysis helps to identify the maximum expected loss for a time horizon and a specific confidence level:

25 For instance, for n = 500 scenarios and a confidence level of α = 99%, we have k = 500 (1−0.99) = 5. The *VaR* is given by the fifth element of *PnL* which corresponds with the worst fifth loss of the PnL series.

Table 5. BETAS Results Period 2

Period 2	May 6, 2010				July 8, 2010			
	VaR-Delta Normal	VaR-SH	VaR-TGARCH	VaR-Monte Carlo	VaR-Delta Normal	VaR-SH	VaR-TGARCH	VaR-Monte Carlo
GMEXICO	1.46994	1.72236	1.47853	1.49974	1.48046	1.72236	1.49621	1.44369
ALSEA	1.00849	0.61979	1.00572	0.95759	1.01559	0.61979	1.01185	1.00691
KIMBER	0.63933	0.70544	0.65038	0.60014	0.62272	0.70544	0.64764	0.65420
PINFRA	0.77963	0.89825	0.76518	0.71273	0.77957	0.89825	0.76343	0.75615
GFNORTE	1.67756	1.18166	1.67919	1.66333	1.68680	1.18166	1.68212	1.68588
ALPEK	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LAB	0.48716	0.29097	0.52971	0.42817	0.50000	0.29097	0.55058	0.58064
CEMEX	1.84983	0.99586	1.81823	1.89780	1.85344	0.99586	1.81924	1.84692
BOLSA	0.71623	0.27051	0.72226	0.75662	0.70519	0.27051	0.71802	0.67710
BIMBO	0.70547	1.00398	0.72047	0.77562	0.71660	1.00398	0.72962	0.67339
FEMSA	0.98876	1.19056	0.99255	0.96067	0.97825	1.19056	0.97811	0.98321
OMA	0.72072	0.57568	0.70904	0.75665	0.71995	0.57568	0.71412	0.75228
GRUMA	0.86561	0.48452	0.87323	0.85661	0.87221	0.48452	0.87572	1.10542
ASUR	0.66201	0.76782	0.66642	0.65272	0.68076	0.76782	0.68739	0.62984
GENTERA	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
TLEVISA	0.93326	0.86943	0.92403	0.90191	0.92026	0.86943	0.91157	0.93724
GAP	0.71190	0.77469	0.70938	0.63416	0.68936	0.77469	0.70128	0.68944
GCARSO	1.07686	1.16119	1.06975	1.09423	1.08052	1.16119	1.07130	1.10380
LIVEPOL	-0.01308	0.04979	-0.00538	-0.01782	-0.00690	0.04979	-0.00021	-0.02799
MEXCHEM	1.17034	1.74214	1.18452	1.11260	1.18514	1.74214	1.18809	1.09864
AC	0.19505	0.19912	0.20147	0.17514	0.19056	0.19912	0.19768	0.20684
PEÑOLES	1.20374	1.74587	1.19751	1.27474	1.21768	1.74587	1.21413	1.17580
ALFA	1.05536	0.77357	1.06513	1.01060	1.05183	0.77357	1.06245	0.99979
GFINBUR	0.60614	0.54170	0.61448	0.55140	0.63329	0.54170	0.63937	0.64653
WALMEX	0.63930	0.83080	0.64983	0.71237	0.63946	0.83080	0.65129	0.70299
ELEKTRA	0.64279	1.02932	0.66423	0.57435	0.66947	1.02932	0.68070	0.63235
GFREGIO	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LALA	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
NEMAK	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
OHLMEX	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
VOLAR	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
IENOVA	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
SANMEX	0.96701	0.73164	0.94617	0.89579	0.99838	0.73164	0.99109	0.88123
KOF	0.64068	0.93994	0.65572	0.58863	0.63560	0.93994	0.64940	0.58395
AMX	1.06462	1.41917	1.06050	1.11430	1.05441	1.41917	1.05160	1.06078

Source: own calculations based on data from Thompson Reuters

$$VaR_{0.99} = \inf\left\{PnL_{\{K\}} : P\left(L > PnL_{\{K\}}\right) \leq 1 - 0.99\right\} \tag{10}$$

In equation (10), $VaR_{0.99}$ represents the 99 percent Value at Risk, $PnL_{\{K\}}$ represents the *PnL* in the scenario *k* of the ordered series of *PnL* and P(L>$PnL_{\{K\}}$)≤1−0.99 indicates the probability that the portfolio's loss or gain is greater than *VaR* is 0.01.

The calculation of *VaR* using the Historical Simulation method in this study has the following characteristics: (i) Multiplicative scenarios, in which it is assumed that the one-day return on stocks will be a return that has already occurred in the past, and (ii) 500 scenarios will be used for periods of high volatility.

3.4. Calculation of Value at Risk Monte Carlo Simulation (VaR-SM)

As mentioned earlier, *VaR−SM* belongs to the hybrid model group, in which the movements of market variables are modeled by applying multiple scenarios. These scenarios can utilize historical data to calculate the parameters of a distribution function assumed to follow the variables, or they can be estimated independently of historical data.

For the calculation of scenarios with this method, it was assumed that stock prices follow a Geometric Brownian Motion with a zero trend, that is,

$$dP(t) = \sigma P(t)\, dB(t) \tag{11}$$

For scenario generation, the variance-covariance matrix of the returns for each stock and the Cholesky decomposition must be calculated. Using these inputs, 10,000 simulations were conducted for each variable. To achieve the scenarios for each variable, we calculated *VaR* at a confidence level of 99% and a 1-day horizon following the methodology of *VaR−SH*.

While the estimation is a widely standardized measure of market risk, Hoppe (1998; 1999) pointed out that one of the estimators' structural weaknesses is that mathematical models do not apply to a financial reality in which investors interact with gradual learning processes about market conditions.

Beder (1995) noted that different *VaR* calculation specifications could generate different market risk measures, suggesting that

Table 6. BETAS Results Period 3

Period 3	August 4, 2011				December 9, 2011			
	VaR-Delta Normal	VaR-SH	VaR-TGARCH	VaR-Monte Carlo	VaR-Delta Normal	VaR-SH	VaR-TGARCH	VaR-Monte Carlo
GMEXICO	1.50562	2.30123	1.53938	1.63316	1.42275	1.94733	1.42328	1.42496
ALSEA	0.89140	1.03529	0.91278	0.97557	0.79737	1.05074	0.81827	0.81878
KIMBER	0.76199	0.55445	0.75915	0.69596	0.72119	0.61892	0.73684	0.77900
PINFRA	0.58141	0.45067	0.65160	0.54192	0.54482	0.55942	0.59432	0.60195
GFNORTE	1.40984	1.61110	1.40445	1.37209	1.31290	1.47495	1.29527	1.33487
ALPEK	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
LAB	0.90227	2.09058	1.01094	0.93990	0.98188	1.50085	1.02272	1.03382
CEMEX	1.82774	1.51645	1.72669	1.69182	2.16399	1.42860	2.08558	2.11197
BOLSA	0.74083	1.03737	0.77722	0.71957	0.83240	1.00284	0.85856	0.93002
BIMBO	0.83255	0.85124	0.84663	0.83717	0.81956	0.86664	0.82762	0.82058
FEMSA	0.83289	0.70276	0.85922	0.78295	0.88825	0.80566	0.90905	0.89224
OMA	0.77362	0.53520	0.75953	0.77041	0.57598	0.45336	0.57710	0.58170
GRUMA	1.05953	1.02091	1.05162	1.05879	0.95877	0.83938	0.95873	0.94066
ASUR	0.99185	0.83856	0.98152	0.86014	0.83793	0.85988	0.83395	0.85499
GENTERA	0.27004	0.25167	0.23370	0.18576	0.79095	0.39002	0.75375	0.67846
TLEVISA	0.99466	0.87405	0.97190	0.99017	0.97169	0.89692	0.96271	1.00985
GAP	0.86323	0.64374	0.86447	0.85051	0.52454	0.62318	0.53358	0.61158
GCARSO	1.09106	1.05165	1.10185	1.18430	0.98254	1.10428	1.02099	1.00821
LIVEPOL	0.10056	0.01654	0.16700	0.13783	0.26783	0.14753	0.31240	0.26650
MEXCHEM	1.16216	1.62404	1.22848	1.19083	1.10159	1.75706	1.13518	1.17414
AC	0.33368	0.27135	0.39598	0.37020	0.50448	0.48160	0.52816	0.64094
PEÑOLES	1.15293	1.30525	1.20550	1.20607	1.03170	1.33795	1.08166	1.17242
ALFA	1.01312	1.31813	1.09668	1.17072	1.04718	1.38645	1.08743	1.03885
GFINBUR	1.02126	0.99424	1.02535	1.01242	0.86139	1.00967	0.87976	0.90725
WALMEX	0.77467	0.79632	0.78195	0.85945	0.80496	0.91374	0.81969	0.79576
ELEKTRA	0.90835	0.60918	0.90038	1.01532	1.08739	0.99564	1.13730	1.09863
GFREGIO	0.04399	0.02785	0.03716	0.02944	0.30750	0.11163	0.29491	0.28736
LALA	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
NEMAK	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
OHLMEX	0.16859	0.18726	0.16301	0.19656	0.65371	0.37507	0.63759	0.58183
VOLAR	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
IENOVA	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
SANMEX	1.35638	0.91863	1.28354	1.15579	1.15719	0.75624	1.09862	1.04572
KOF	0.67434	0.63730	0.71542	0.70327	0.66304	0.63285	0.68895	0.69043
AMX	0.92841	0.83928	0.90233	0.96598	0.86214	0.86643	0.85070	0.78423

Source: own calculations based on data from Thompson Reuters

risk estimates could be imprecise. Marshall and Siegel (1997) identify an additional risk to the market risk that the *VaR* methodology aims to estimate, related to its implementation risk. In other words, differences in how the model is implemented could lead to different market risk measurements for a specific stock portfolio.

3.5. Calculation of Expected Shortfall

Expected Shortfall (ES) represents the expected value of losses exceeding the VaR at the confidence α level. ES was calculated using the following formula[26]:

$$ES_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 VaR_u du \qquad (12)$$

where VaR_u is the VaR at confidence level u, with u ranging from α to 1. For discrete distributions or historical simulation methods in which the VaR is estimated at discrete confidence levels, the ES calculation is as follows:

$$ES_\alpha = \frac{1}{N}\sum_{i=1}^{N} L_i \cdot I\{L_i \geq VaR_\alpha\} \qquad (13)$$

where L_i is the loss, N is the number of observations, and $I\{L_i \geq VaR_\alpha\}$ is an indicator function, where:

$$I = \begin{cases} 1 \text{ if } L_i \geq VaR_\alpha \\ 0 \text{ if } L_i < VaR_\alpha \end{cases} \qquad (14)$$

3.6. Calculation of CAPM Beta

The Capital Asset Pricing Model (CAPM) Beta was calculated using the following formula:

$$\beta_{im} = \frac{\text{cov}(R_i, R_M)}{\text{var}(R_M)} \qquad (15)$$

where:

β_{iM} is the Beta of the asset i and M is the market portfolio, $cov(R_i,R_M)$ is the covariance between the returns of assets and the market return M, and var(R_M) is the variance of the market return

26 The Expected Shortfall measures the expected loss given that losses exceed the *VaR* threshold. It focuses on the tail of the loss distribution.

M. This calculation helps measure the sensitivity of asset returns to overall market returns, providing insights into the asset's risk and potential performance in the market.

The Capital Asset Pricing Model (CAPM) Beta estimation involves plotting feasible portfolios, followed by the calculation of efficient portfolios by solving the next problem:

$$\min \frac{1}{2}\operatorname{var}(Rp) \quad s.a. \quad \sum_i w_i = 1, \quad \sum_i w_i R_i = R \tag{16}$$

Equation (17) provides a detailed description of the problem to be solved by using equation (16):

$$\min \frac{1}{2}\sum_{i,j=1} w_i w_j \sigma_{ij} \quad s.a. \quad \sum_i w_i = 1, \quad \sum_i w_i R_i = R \tag{17}$$

where w_i is the proportion of money invested in stock *i*, R_i is the expected return of stock *i*, *R* is the expected portfolio return, and σ_{ij} is the covariance of stock *i,j*. When $i = j$, then σ_{ij} is the variance of the return of stock *i*.

Finally, the composition of the market portfolio is obtained from the variance-covariance matrix and the risk-free instrument, as the market portfolio is the most efficient portfolio that, when combined with a risk-free asset, generates the best investment portfolios.

The problem solved is as follows:

$$\max \tan(\theta) \quad s.a. \quad \sum_{i=1}^{n} w_i = 1 \tag{18}$$

In equation (18), θ represents the angle formed between the risk of the risk-free asset and the market portfolio, and the return of the risk-free portfolio *Rf*. The previous problem is equivalent to solving the problem described by equation (19):

$$\max \frac{\sum_{i=1}^{n} w_i \cdot (R_i - R_f)}{\sqrt{\sum_{i,j=1}^{n} \sigma_{ij} w_i w_j}} \quad s.a. \quad \sum_{i=1}^{n} w_i = 1 \tag{19}$$

By solving equation (19), we obtain the optimal proportion w_i of the total financial resources that the investor allocates to each stock of the market portfolio. Once the proportions of w_i are identified and the expected returns of each stock are calculated, the expected return of the market portfolio is obtained as: $E(R_M) = \sum_i w_i E(R_i)$ and the variance of the return of the market portfolio as: $\sum_{i,j=1} w_i w_j \sigma_{ij}$. Likewise, the covariance of the market portfolio's return concerning the return of any stock can be calculated, as described in equation (20):

$$\operatorname{cov}(R_i, R_M) = \operatorname{cov}\left(R_i, \sum_{j=1}^{n} w_j R_j\right) = \sum_{j=1}^{n}\left[w_j \operatorname{cov}(R_i, R_j)\right] \quad \forall i = 1,\ldots,n \tag{20}$$

Using this information, the β-CAPM for any stock *i* can be calculated as:

$$\beta_{im} = \frac{\operatorname{cov}(R_i, R_M)}{\operatorname{var}(R_M)} \tag{15}$$

As observed in equation (20), calculating the CAPM Beta requires knowing the proportion of each stock in the market portfolio.

3.7. VaR Normal Delta (*VaR−δN*) and *VaR*-TGARCH

Among the parametric models, the *VaR−δN* estimation assumes that the density function of asset portfolio returns is known, with parameters not previously identified. Notably, this methodology assumes normality in the distribution of the daily returns with the given first and second moments $R_p \sim N(\mu\sigma^2)$. In this context, we obtained *VaR* estimations that reflect the change in asset portfolio value from 1 day to another, such as the loss or gain, as in equation (21):

$$VaR = V_0 \times R_p^* \tag{21}$$

where: V_0 is the initial value of the asset portfolio, and R_p^* represents the daily return with an σ confidence level. Given the density function $R_p \sim N(\mu\sigma^2)$, therefore $x = \frac{R_p - \mu}{\sigma} \sim N(0,1)$. Then, for a given confidence level α:

$$P\left(x \le \frac{R_p^* - \mu}{\sigma}\right) = 1 - \alpha \tag{22}$$

If Φ is the density function *N*(0,1), then:

$$\Phi^{-1}(1-\alpha) = \frac{R_p^* - \mu}{\sigma} \tag{23}$$

where: Φ^{-1} is the inverse function of the normal standard distribution. Assuming that the expected return is zero, we have:

$$R_p^* = \sigma \times \Phi^{-1}(1-\alpha) \tag{24}$$

Therefore, the *VaR* estimation for a 1-day time setting is:

$$VaR = V_0 \times \sigma \times \Phi^{-1}(1-\alpha) \tag{25}$$

Table 7. BETAS Results Period 4

Period 4	February 27, 2020				July 14, 2020			
	VaR-Delta Normal	VaR-SH	VaR-TGARCH	VaR-Monte Carlo	VaR-Delta Normal	VaR-SH	VaR-TGARCH	VaR-Monte Carlo
GMEXICO	1.25999	1.20543	1.24817	1.33401	1.04321	1.07354	1.06996	1.05362
ALSEA	0.79774	0.73599	0.76789	0.75830	1.02982	0.75760	0.98812	1.02250
KIMBER	0.79889	0.95228	0.83036	0.73929	0.66870	1.03881	0.68904	0.64457
PINFRA	0.62814	0.70682	0.63822	0.56414	0.68095	0.80101	0.68599	0.68241
GFNORTE	1.42468	1.43205	1.44361	1.43900	1.48070	1.51979	1.48744	1.49271
ALPEK	0.65046	0.84463	0.61271	0.62192	0.81542	0.68790	0.80104	0.78330
LAB	0.81093	0.55874	0.83065	0.73971	0.67523	0.69693	0.71731	0.82601
CEMEX	1.49877	1.32952	1.45964	1.53363	1.43781	1.15316	1.42518	1.47604
BOLSA	0.53638	0.58185	0.56573	0.56517	0.68009	0.92849	0.71232	0.64114
BIMBO	0.94490	1.00295	0.92597	0.94682	0.83999	1.05831	0.85518	0.80443
FEMSA	0.77455	0.90138	0.77886	0.77405	0.76108	0.91129	0.75571	0.70591
OMA	0.97848	1.21001	1.02347	1.08476	1.23563	1.40964	1.25727	1.18163
GRUMA	0.62565	0.76188	0.62574	0.67056	0.47286	0.60387	0.47943	0.51431
ASUR	0.94213	0.97914	0.95281	1.07471	1.02743	1.03692	1.02974	1.05934
GENTERA	1.02205	1.00607	1.06475	1.08022	1.13449	1.13983	1.11912	1.00447
TLEVISA	1.13953	1.01523	1.11298	1.19844	1.47829	0.94427	1.43513	1.33027
GAP	0.93395	0.97540	0.96480	0.99069	1.28519	1.29680	1.29938	1.20938
GCARSO	1.12528	1.42381	1.13195	1.33311	1.05518	1.26258	1.04972	1.14337
LIVEPOL	0.95888	1.02970	0.94279	0.99101	1.05242	1.07977	1.01952	1.05142
MEXCHEM	0.97792	0.97062	0.96733	0.98208	1.09374	0.90794	1.08542	1.17303
AC	0.54275	0.47981	0.53311	0.56625	0.46808	0.46833	0.46455	0.54736
PEÑOLES	1.16804	0.91535	1.12175	1.20252	0.71619	0.72663	0.71653	0.60539
ALFA	1.11524	1.17518	1.08131	1.08152	1.15153	0.97756	1.13864	1.15018
GFINBUR	1.08145	1.06741	1.06485	1.09284	1.22825	1.12877	1.21668	1.21968
WALMEX	0.75046	0.87688	0.78324	0.62655	0.63150	0.84209	0.64790	0.64059
ELEKTRA	0.24757	-0.12916	0.33651	0.28732	0.09692	-0.04234	0.14758	0.05428
GFREGIO	0.70754	0.74301	0.71157	0.79459	1.05741	1.03532	1.04179	1.05415
LALA	0.59250	0.71611	0.55174	0.26800	0.55849	0.70154	0.53394	0.55028
NEMAK	0.59656	0.72592	0.54244	0.36997	0.79306	0.64781	0.75110	0.79373
OHLMEX	0.23408	0.28756	0.22934	0.19604	0.31643	0.44897	0.30759	0.14188
VOLAR	0.87044	0.86776	0.91270	0.69241	1.33751	1.69582	1.37151	1.45912
IENOVA	0.80255	0.70312	0.81294	0.79330	0.74908	0.80802	0.74548	0.87622
SANMEX	0.40560	0.46450	0.36445	0.32877	0.68337	0.50200	0.66448	0.70247
KOF	0.69305	0.71184	0.69234	0.62105	0.66019	0.73909	0.65977	0.69764
AMX	1.14858	1.08381	1.15388	1.11808	0.99460	1.11082	1.00406	1.08850

Source: own calculations based on data from Thompson Reuters

Furthermore, if we assume that daily returns are independent and identically distributed, we have that the *VaR−δN* for a given time setting of *t* days is:

$$VaR - \delta N = V_0 \times \sigma \times \Phi^{-1}\left(1-\alpha\right) \times \sqrt{t}. \tag{26}$$

Finally, the calculation of *VaR*-TGARCH risk measure is:

$$VaR_\alpha^t = \mu_{t+1} + \sigma_{t+1} q_\alpha\left(Z\right) \tag{27}$$

where: *Z* is a random variable with a normal distribution function and $q_\alpha(Z)$ is the *α−quantil* of *Z*.

3.8. TGARCH modeling

If returns on m days follow a compound interest process, we use the daily prices to build the stock return series[27]. The compound interest distributions are estimations that use time-series models of the ARCH-GARCH family[28].

Here, we use the TGARCH-VaR model to estimate the marginal density distributions of innovations associated with stock market returns. This model extends Zakoian's (1994) GARCH model. The TGARCH model is among the best for describing the statistical behavior of asset returns in developing economies. Moreover, the TGARCH model can capture features that characterize many financial and economic series. The existence of skewed and leptokurtic non-constant volatilities, volatility clustering, and leverage effects are worth mentioning.

From a modeling perspective, the main feature of the TGARCH model is that it allows the volatility of the return series in period t, r_t, to depend on the "news" arriving at the market (that is, the lagged innovation u_{t-1}). We describe such volatility with the following specification of the conditional variance of innovations, σ_t^2 :

27 We define the *m−return* for an asset during the day t, r_t, as the change in logs of the price on *m* days of such asset, P_t. Therefore $r_t = \ln P_t - \ln P_{t-m}$.

28 The ARCH-GARCH family includes more than a hundred time-series models. The acronyms ARCH and GARCH stand for Auto-Regressive Conditional Heteroscedasticity and Generalized Auto-Regressive Conditional Heteroscedasticity. Engle (1982) and Bollerslev (1986) developed these models.

Table 8. BETAS Results Period 5

Period 5	April 26, 2022				June 22, 2022			
	VaR-Delta Normal	VaR-SH	VaR-TGARCH	VaR-Monte Carlo	VaR-Delta Normal	VaR-SH	VaR-TGARCH	VaR-Monte Carlo
GMEXICO	0.98483	1.34938	1.00684	0.97577	1.05472	1.33101	1.07859	1.11260
ALSEA	0.94232	1.08595	0.97773	0.91182	0.88777	0.93444	0.91621	1.00388
KIMBER	0.56738	0.36904	0.54038	0.57956	0.52947	0.30547	0.50140	0.55114
PINFRA	0.80631	0.57051	0.77323	0.73790	0.77488	0.59238	0.74810	0.73771
GFNORTE	1.41521	1.51042	1.43004	1.43476	1.43674	1.46323	1.43971	1.43036
ALPEK	0.46329	0.68242	0.51958	0.52255	0.42131	0.45590	0.45203	0.51374
LAB	0.27658	0.12386	0.28465	0.16480	0.32474	0.16600	0.31608	0.30220
CEMEX	1.52368	2.72481	1.52086	1.52530	1.63273	2.69626	1.61299	1.47207
BOLSA	0.41311	0.36281	0.39801	0.45667	0.40089	0.26099	0.38304	0.45752
BIMBO	0.69859	0.50496	0.71571	0.68605	0.70233	0.52825	0.73413	0.68119
FEMSA	1.04909	0.66599	1.01348	1.04482	1.02099	0.69207	0.99299	1.00480
OMA	1.10921	1.27163	1.11555	1.09634	1.05261	0.93504	1.05196	1.25843
GRUMA	0.35604	0.21639	0.35310	0.36597	0.41208	0.21165	0.39722	0.44134
ASUR	1.04440	0.98297	1.05731	1.06845	0.96979	0.94289	0.98859	0.97897
GENTERA	1.03481	0.61974	1.05023	1.01770	0.99161	0.54514	1.01304	1.14208
TLEVISA	1.71221	1.38996	1.69783	1.70541	1.60317	1.68882	1.60146	1.63275
GAP	1.10992	1.31855	1.13537	1.15721	1.09211	1.19801	1.11946	1.16507
GCARSO	1.16047	0.85948	1.15493	1.26548	1.16311	0.95143	1.18531	1.08305
LIVEPOL	0.69032	0.70665	0.71155	0.68977	0.63313	0.55396	0.66454	0.60687
MEXCHEM	0.85418	0.92200	0.87017	0.77912	0.82901	0.76715	0.84133	0.89008
AC	0.47091	0.23359	0.48357	0.45967	0.51340	0.29416	0.52451	0.50363
PEÑOLES	0.55930	0.93727	0.56668	0.62635	0.80626	0.86004	0.78999	0.78027
ALFA	0.85441	1.39062	0.87510	0.77087	0.87902	0.83232	0.88755	0.72513
GFINBUR	1.01856	0.70534	1.05968	1.05377	0.92572	0.80550	0.96599	1.01152
WALMEX	0.63190	0.44019	0.63433	0.69199	0.72083	0.54757	0.72205	0.79478
ELEKTRA	0.14741	0.10712	0.13580	0.09838	0.15068	0.12699	0.14077	0.11922
GFREGIO	0.85065	0.59571	0.88793	0.79573	0.77128	0.60756	0.80141	0.80310
LALA	0.25590	0.18435	0.28114	0.27410	0.25426	0.15253	0.27393	0.18307
NEMAK	0.65148	0.70411	0.64610	0.75697	0.58526	0.49270	0.56422	0.64911
OHLMEX	0.19014	0.04380	0.18625	0.16476	0.13895	0.05761	0.12901	-0.05580
VOLAR	1.20730	2.50354	1.25858	1.30579	1.15919	1.99229	1.19324	1.11751
IENOVA	0.35912	0.33095	0.34991	0.36705	0.33386	0.30956	0.33967	0.33111
SANMEX	1.13243	0.77458	1.11686	1.04887	1.03289	0.83370	1.02483	1.10462
KOF	0.73396	0.52858	0.71813	0.66445	0.66775	0.55592	0.66313	0.61759
AMX	0.85336	0.63079	0.85495	0.85194	0.83115	0.68529	0.83635	0.84939

Source: own calculations based on data from Thompson Reuters

Figure 5: Historical betas GRUMA, KIMBERLY, WALMEX, and LIVERPOOL issuers

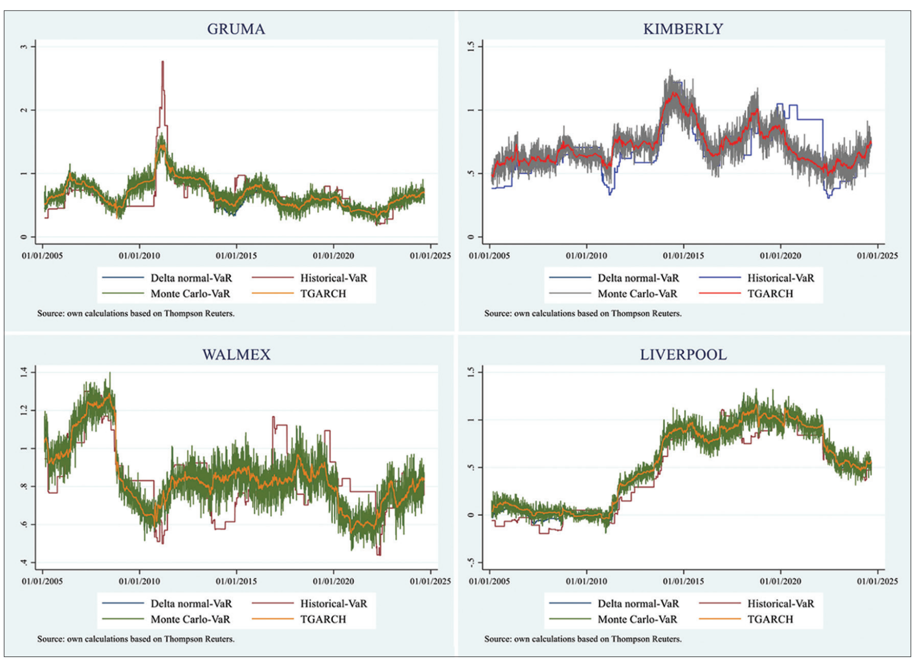


Table 9. Value at Risk Summary statistics KUPIEC P VALUE IPC PORTFOLIO

	VaR-Delta Normal	VaR-SH	VaR-TGARCH	VaR-Monte Carlo
May 31, 2017	**0.21669**	**0.99642**	**0.10711**	**0.21669**
Period 1				
September 15, 2008	0.00779	0.66638	0.04848	0.02020
May 19, 2009	0.00000	0.02020	0.00093	0.00000
Period 2				
May 6, 2010	0.00002	0.39923	0.02020	0.00000
July 8, 2010	0.00002	0.39923	0.00779	0.00000
Period 3				
August 4, 2011	0.12408	0.12408	0.39923	0.12408
December 9, 2011	0.10711	0.99642	0.10711	0.21669
Period 4				
February 27, 2020	0.00279	0.66638	0.00093	0.00093
July 14, 2020	0.00000	0.00779	0.00009	0.00000
Period 5				
April 26, 2022	0.63834	0.02798	0.66638	0.12408
June 22, 2022	0.02798	0.02798	0.99642	0.02798

Source: own calculations based on data from Thompson Reuters

Table 10. Theoretical Portfolio

Base Period: May 31st, 2017 (theoretical weights)	
GMEXICO	-0.07270
ALSEA	0.20210
KIMBER	0.34094
PINFRA	0.12818
GFNORTE	0.14519
ALPEK	0.06035
LAB	0.08900
CEMEX	0.08276
BOLSA	0.10467
BIMBO	-0.29695
FEMSA	0.13487
OMA	-0.05328
GRUMA	0.08785
ASUR	0.24277
GENTERA	-0.15510
TLEVISA	-0.16970
GAP	0.72877
GCARSO	0.21157
LIVEPOL	-0.70487
MEXCHEM	0.32224
AC	0.38122
PEÑOLES	0.38377
ALFA	-0.64082
GFINBUR	-0.60746
WALMEX	0.04122
ELEKTRA	-0.03883
GFREGIO	0.36373
LALA	-0.08302
NEMAK	0.04466
OHLMEX	-0.06838
VOLAR	0.21611
IENOVA	-0.05946
SANMEX	0.04150
KOF	-0.25895
AMX	-0.14396

Source: own calculations based on data from Thompson Reuters

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \gamma u_{t-1}^2 I\left(u_{t-1} < 0\right) + \beta\sigma_{t-1}^2 \quad (28)$$

where innovations u_t are, by assumption, normally distributed. The parameters α_0, α_1, β, and γ are assumed to be non-negative, and I defines an indicator function:

$$I = \begin{cases} 1 \; if \; u_{t-1} < 0 \\ 0 \; if \; u_{t-1} \geq 0 \end{cases} \quad (29)$$

The specification of the conditional variance given by the equation (28) allows us to analyze the effects of qualitative news on the current volatility of the return series: i) good news, $u_{t-1} > 0$, have an effect equal to α_1 on σ_t^2, and ii) bad news, $u_{t-1} < 0$, have an effect equal to $\alpha_1 + \gamma$. Thus, when $\gamma \neq 0$, bad news has measurable effects on the volatility of the series. When bad news occurs and $\gamma > 0$, these series show the "leverage effect" (that is, the volatility caused by bad news is more significant than that caused by good news). Therefore, γ could be considered a measure of sensitivity to lousy news prevailing in the market.

We use the AR(1)-TGARCH(1,1) model with a normal distribution to estimate the marginal distributions of returns. This model has a three-equation system structure. The first expression is the conditional mean of the series of returns, r_t, during period t. The second condition defines the ARCH process. The third is the specification of the conditional variance. The structure that represents the estimated TGARCH model is:

$$\begin{aligned} r_t &= \phi_0 + \phi_1 r_{t-1} + u_t \\ u_t &= \sigma_t \varepsilon_t \\ \sigma_t^2 &= \alpha_0 + \alpha_1 u_{t-1}^2 + \gamma u_{t-1}^2 I\left(u_{t-1} < 0\right) + \beta\sigma_{t-1}^2 \end{aligned} \quad (30)$$

Complementary tests were used to validate the estimation procedure. Specifically, we used ADF and KPSS tests to assess the order of integration of the log series. We used both tests because of their complementarity and to avoid spurious estimations[29]. In addition, we used ARCH-LM tests of the type proposed by Engle (1982) to examine the convenience of using models of the ARCH-GARCH family to model and analyze the series of returns. Furthermore, we use Ljung-Box tests of the type proposed by Ljung and Box (1978) to assess potential misspecification problems.

3.9. Identification of High-volatility Episodes

We identify high-volatility periods based on the volatility index (VIX) associated with the S&P 500 Stock Index. We analyzed three specific periods of financial stress:

(a) From September 15, 2008, to May 19, 2009, the Lehman Brothers bankruptcy had a negative impact on the macro-financial outlook. This marked the unprecedented onset of the economic and financial crisis of 2008-2009, which continued into the first half of 2009.

29 The augmented Dickey-Fuller (ADF) and the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) tests have complementary null hypotheses. The null hypothesis of the ADF test is that the data-generating process contains a unit root. The null hypothesis of the KPSS test is that the data-generating process is stationary. The joint use of both tests allows us to guarantee the estimation of robust results regarding the order of integration of the series.

(b) Between August 4 and December 9, 2011, global economic activity indicators deteriorated significantly, accompanied by increased volatility in the international financial markets. This volatility reflected concerns about the fiscal sustainability of the United States and peripheral countries within the Eurozone, as well as declining global growth prospects.
(c) From February 27 to July 14, 2020, this period highlighted the unprecedented economic challenges posed by the pandemic in both industrialized and emerging market economies. This reflects a significant deterioration in global economic activity indicators, signaling recession episodes in several countries. The economic and financial downturn, along with heightened risk aversion, has led to a rise in stock and debt markets worldwide.

Figure 1 shows the five identified periods of financial stress[30].

As already mentioned, the data sample covers five specific periods with episodes of high volatility and extreme stock market conditions: (i) September 15, 2008, to May 19, 2009; (ii) from May 6, 2010, to July 8, 2010; (iii) from August 4, 2011, to December 9, 2011; (iv) from February 27, 2020 to July 14, 2020; and (v) from April 26, 2022, to June 22, 2022.

This study contrasts five methods for calculating market risk for portfolios with Mexican stock market shares: VaR-SH, VaR-δN, VaR-TGARCH, and ES vs. β-CAPM. The methods are compared for a 1-day horizon, which is commonly used in market risk reports by Mexican financial institutions.

4. RESULTS

We calculated estimations for the time window from February 7, 2005, to August 30, 2024. The weights of the Mexican Stock Prices Index (IPC for its acronym in Spanish) were taken using information as of May 31, 2017; to integrate the market portfolio. The following figure shows the results using the Expected Shortfall methodology.

Figure 2 shows the Expected Shortfall measures for IPC under the selected periods of financial stress.

Table 1 shows the results of the Expected Shortfall using the proposed methodologies during critical periods of high volatility.

In Figure 2, we observe that the crises of 2008 and 2020 had a more significant impact on the calculated values of Expected Shortfall, followed by the situation that occurred at the end of 2011. Figure 2 shows the persistence of the crisis in the results. For instance, in the Historical Simulation method, there was no change in the ES value during the months following the crisis. In contrast, the ES calculated using the delta-normal method shows slight variations, with values lower than those obtained using the Historical Simulation method. On the other hand, although the GARCH results reflect market volatility, as this method is designed to model volatility, the Monte Carlo results are consistent with the delta-normal methodology, with generally lower levels than the delta-normal ones.

Table 1 shows the highest level of the Expected Shortfall Measure (ESM) observed at the end of period 1 crisis and for the remaining methodologies. These results are consistent with the VIX behavior during the periods.

The behavior observed in the Expected Shortfall results resembles that in the VaR results, as illustrated in Figure 3 and Table 2. As expected, the ES values were higher than the VaR values for all three methods (Figure 4).

Moreover, ES and VaR values differed depending on the method used. The most significant differences observed were using the Monte Carlo method.

Figure 3 shows the results obtained for the Value at Risk (VaR) measures of IPC under the selected periods of financial stress.

Table 2 shows the VaR results with the proposed methodologies during the critical periods of high volatility.

During high-volatility periods, the VaR measure of risk is higher than that in the baseline scenario (May 31, 2017). It is worth noting that the highest identified levels of risk correspond to the VaR_SH methodology, except at the end of period 5, where the highest level of risk corresponds to the VaR-T-GARCH estimations.

In all methodologies, the Expected Shortfall measure was greater than VaR. This result suggests that the left tail of the probability distribution is heavier than the right.

The results from Table 2 are the inputs to calculate the Betas for the complete set of IPC issuers of financial assets. These results are presented in Table 3.

The behavior of the VaR and ES results resembles that observed in the Betas. The Beta value for stocks is consistent with that of the four proposed methodologies. It is worth noting that some discrepancies between VaR-SH and the other methods are still present, as expected.

Tables 4-8 present the results for the five periods of high volatility. The findings indicate consistent values estimated using the VaR-Delta Normal, VaR-Monte Carlo, and VaR-TGARCH methods. There is a discrepancy in the results from the VaR-SH method, which can be attributed to the normality assumption underlying the first three methodologies. Regarding the betas estimated using the VaR-SH method, the results exhibit the same behavior as VaR: the persistence of values following a crisis. Additionally, estimations for the Betas of assets with a value of zero are self-explained because these assets are not listed on the Mexican Stock Exchange in episodes of high volatility.

Figure 5 shows the historical Betas for the four specific issuers of financial assets in the Mexican stock market in the IPC. The Annex 1 collects the remaining Figures.

30 The VIX shows the forward-looking volatility investors expect over a 30-day period. As shown in Figure 1, the red, yellow, and green lines represent 34, 37, and 42 index points, respectively.

We applied the Kupiec test to evaluate the performance of the VaR models at the dates corresponding to each economic and financial crisis examined in this study. Table 9 presents a summary of the results, which indicate that the Historical Simulation VaR outperforms the other models, while the Delta-Normal VaR exhibits the weakest performance among the four methodologies.

As previously noted, these results are consistent with the market portfolio weights of the IPC as of May 31, 2017. Moreover, we estimated the theoretical portfolio for the same date using 501 historical price observations from the Mexican stock market. The resulting portfolio betas and comparative outcomes are summarized in Table 10.

5. DISCUSSION AND CONCLUSION

Investors are always aware of their risk profile. Rational investors with complete information choose investment portfolios that are consistent with the known low, moderate, and high profiles. Two of the widely used measures to assess portfolio risks are VaR and Expected Shortfall. However, these measures do not indicate how risky the portfolio is compared with the market (IPC). For this reason, VaR measures are used to calculate the portfolio's Beta, determining the risk of the investment portfolio compared with the market portfolio (IPC).

The results obtained using the different methodologies typically show that VaR measures increase during periods of high volatility. However, risk measures based on the Monte Carlo methodology still exhibit significant volatility, and those from VaR under historical simulation measures have a lagged response when critical high-volatility scenarios are included, which modifies the VaR scenario. This also causes VaR estimates on the dates following the high-volatility periods to be overestimated with a certain persistence. This behavior is reflected in the observed results for Betas and different assets.

For all methodologies, the risk measures based on the Expected Shortfall are always greater than those based on the VaR. When comparing the Historical Simulation, T-GARCH and the Delta Normal methodologies, the risk measures do not show as much volatility as the Monte Carlo methodology.

The observed volatility in the VaR and Expected Shortfall measures is consistent with the results in the Beta analysis. This result allows us to conclude that the Betas obtained with the Delta Normal and Historical Simulation methodologies are far more consistent than those obtained with the Monte Carlo methodology. It is worth mentioning that the Betas obtained with the Historical Simulation methodology will be overestimated for a time while the critical scenario of high volatility changes again.

It is important to note that, despite the high volatility observed with the Monte Carlo method, the results remain consistent with those obtained using the Delta-Normal and T-GARCH methods. This consistency can be attributed to the assumption of normality in the estimation. Consequently, we can conclude that, for the sake of time and efficiency, the calculation of betas for different assets using VaR should be performed exclusively with the VaR-Delta Normal and VaR-SH methods, as the additional information provided by the Monte Carlo and T-GARCH methodologies has already been captured by the Delta-Normal approach.

We estimated the beta coefficients using the IPC portfolio weights as of May 31 as the market benchmark. These estimates reflect the systematic risk of each stock relative to the IPC on that specific date. This study aims to assess the consistency of the Value at Risk (VaR) metric during periods of financial distress, employing multiple estimation methodologies.

As previously noted, we also estimated a theoretical portfolio for the same reference date. However, the beta coefficients derived from this portfolio proved inconsistent, mainly due to the presence of numerous short positions. In future research, we plan to dynamically recalibrate the market portfolio, adjusting it to reflect the IPC composition relevant to each rolling window or analysis period.

Our current findings highlight the robustness of the VaR estimates during crisis episodes and reveal their influence on beta behavior in those same periods. This analysis provides valuable insights into the performance of equity investment portfolios that are anchored to a fixed benchmark date. Furthermore, it provides a useful framework for simulating the potential risk exposure of such portfolios under financial shocks within the investment horizon, ultimately supporting more informed and risk-aware decision-making.

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REFERENCES

Abad, P., Benito, S., López, C. (2014), A comprehensive review of value at risk methodologies. The Spanish Review of Financial Economics, 12(1), 15-32.

Álvarez, V.A., Rossignolo, A. (2015), Análisis Comparativo De Técnicas (IMA) Para Determinar Capitales Mínimos Regulados Por Basilea, Ante Crisis En Mercados Emergentes (Comparative Analysis of Techniques (IMA) for Determining Minimum Capital Regulated by Basel, Facing Crises in Emerging Markets). [SSRN Scholarly Paper], Rochester, NY.

Angelidis, T., Benos, A., Degiannakis, S. (2004), The use of GARCH models in VaR estimation. Statistical Methodology, 1(1-2), 105-128.

Artzner, P., Delbaen, F., Eber, J., Heath, D. (1999), Coherent measures of risk. Mathematical Finance, 9(3), 203-228.

Artzner, P., Delbaen, F., Eber, J.M., Heath, D. (1997), Thinking coherently. Risk, 10(11), 68-71.

Banz, R.W. (1981), The relationship between return and market value of common stocks. Journal of Financial Economics, 9(1), 3-18.

Beder, T.S. (1995), VAR: Seductive but Dangerous. Financial Analysts Journal, 51(5), 12-24.

Bekaert, G., Ehrmann, M., Fratzscher, M., Mehl, A. (2014), The global crisis and equity market contagion. The Journal of Finance, 69(6), 2597-2649.

Bekaert, G., Hodrick, R.J., Zhang, X. (2009), International stock return comovements. The Journal of Finance, 64(6), 2591-2626.

Benoit, S., Colletaz, G., Hurlin, C. and Pérignon, C. (2017), "A theoretical and empirical comparison of systemic risk measures", Journal of Financial and Quantitative Analysis, 52(6), 2211-2246.

Bisias, D., Flood, M., Lo, A.W., Valavanis, S. (2012), A Survey of Systemic Risk Analytics. US Department of Treasury, Office of Financial Research, Working Paper No. 0001.

Black, F. (1972), Capital market equilibrium with restricted borrowing. The Journal of Business, 45(3), 444-455.

Black, F. (1998), In: Bernstein, PL., Fabozzi, F.J. Beta and Return (Fall 1993), Streetwise: The Best of the Journal of Portfolio Management. United States: Princeton University Press, p78-84.

Black, F., Jensen, M., Scholes, M. (1972), The capital asset pricing model: Some empirical tests. In: Jensen, M., editor. Studies in the Theory of Capital Markets. New York: Praeger, p79-121.

Bollerslev, T. (1986), "Generalized autoregressive conditional heteroskedasticity", Journal of Econometrics, 31(3), 307–327.

Engle, R.F. (1982), Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. Econometrica, 50(4), 987.

Engle, R.F., Manganelli, S. (2004), CAViaR: Conditional autoregressive value at risk by regression quantiles. Journal of Business and Economic Statistics, 22(4), 367-381.

Estrada, J. (2002), "Systematic risk in emerging markets: the D-CAPM", Emerging Markets Review, Vol. 3 No. 4, pp. 365–379, doi: 10.1016/S1566-0141(02)00042-0.

Estrada, J., Serra, A.P. (2005), Risk and return in emerging markets: Family matters. Journal of Multinational Financial Management, 15(3), 257-272.

Fama, E.F. (1965), The behavior of stock-market prices. The Journal of Business, 38(1), 34-105.

Fama, E.F., French, K.R. (2004), The capital asset pricing model: Theory and evidence. Journal of Economic Perspectives, 18(3), 25-46.

Fama, E.F., MacBeth, J.D. (1973), Risk, return, and equilibrium: Empirical tests. Journal of Political Economy, 81(3), 607-636.

Geanakoplos, J. (2010), Solving the present crisis and managing the leverage cycle. SSRN Electronic Journal, doi: 10.2139/ssrn.1539488.

Grajales, C., Pérez, F. (2010), Valor en riesgo para un portafolio con opciones financieras. Revista Ingenierías Universidad de Medellín, 9(17), 105-118.

Holton, G.A. (2002), History of Value-at-Risk: 1922-1998 Method and History of Economic Thought. Germany: University Library of Munich.

Holton, G.A. (2003), Value-at-Risk: Theory and Practice. Amsterdam, Boston: Academic Press.

Hoppe, R. (1998), Var and the unreal world: A value-at-risk calculation is only as good as the statistics that back it up-and they may not be as reliable as they seem. Risk London Risk Magazine Limited, 11, 45-50.

Hoppe, R. (1999), Finance is not physics. Risk Professional, 1(7), 56-76.

Hwang, S., Pedersen, C.S. (2004), Asymmetric risk measures when modelling emerging markets equities: Evidence for regional and timing effects. Emerging Markets Review, 5(1), 109-128.

Morgan, J.P. (1996), Risk Metrics Technical Documents. 4th ed. New York: Morgan Guaranty Trust Company.

Jobst, A.A., Gray, D.F. (2013), Systemic Contingent Claims Analysis: Estimating Market-Implied Systemic Risk. IMF Working Papers. Vol. 13. United States: International Monetary Fund, p1.

Lazar, E., Zhang, N. (2019), Model risk of expected shortfall. Journal of Banking and Finance, 105, 74-93.

Lintner, J. (1965), The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. The Review of Economics and Statistics, 47(1), 13.

Ljung, G.M., Box, G.E.P. (1978), On a measure of lack of fit in time series models. Biometrika, 65(2), 297-303.

Marshall, C., Siegel, M. (1997), Value-at-risk: Implementing a risk measurement standard. SSRN Electronic , doi: 10.2139/ssrn.1212.

McGrattan, E.R., Jagannathan, R. (1995), The CAPM debate. Quarterly Review, 19(4), 1941.

Merton, R.C., Bodie, Z. (1993), Deposit insurance reform: A functional approach. Carnegie Rochester Conference Series on Public Policy, 38, 1-34.

Mossin, J. (1969), Security pricing and investment criteria in competitive markets. American Economic Review, 59(5), 749-756.

Ospina Duque, V., Tangarife Trujillo, V.E. (2008), Medición del VAR en los Portafolios de Acciones-Mercado Colombiano (Doctoral Dissertation, Universidad Tecnológica de Pereira. Facultad de Ingeniería Industrial. Ingeniería Industrial).

Phylaktis, K., Xia, L. (2006), Sources of firms' industry and country effects in emerging markets. Journal of International Money and Finance, 25(3), 459-475.

Rossignolo, A.F. (2017), Empirical approximation of the ES-VaR: Evidence from emerging and frontier stock markets during turmoil/aproximación empírica del VaR y ES-VaR: Evidencia de mercados emergentes y de frontera durante períodos de turbulencia. Estocástica Finanzas y Riesgo, 7(2), 123-175.

Rossignolo, A.F. (2019), Basel IV A gloomy future for Expected Shortfall risk models. Evidence from the Mexican stock Market. Revista Mexicana de Economía y Finanzas, 14, 559-582.

Rossignolo, A.F., Álvarez, V.A. (2015), Has the basel committee got it right? Evidence from commodity positions in turmoil. Revista Mexicana de Economía y Finanzas, 10(1), 1-38.

Salinas Ávila, J.J. (2009), Metodologías de medición del riesgo de mercado. Innovar, 19(34), 187-199.

Sarwar, G. (2019), Transmission of risk between U.S., Emerging equity markets. Emerging Markets Finance and Trade, 55(5), 1171-1183.

Sharpe, W.F. (1964), Capital asset prices: A theory of market equilibrium under conditions of risk. The Journal of Finance, 19(3), 425-442.

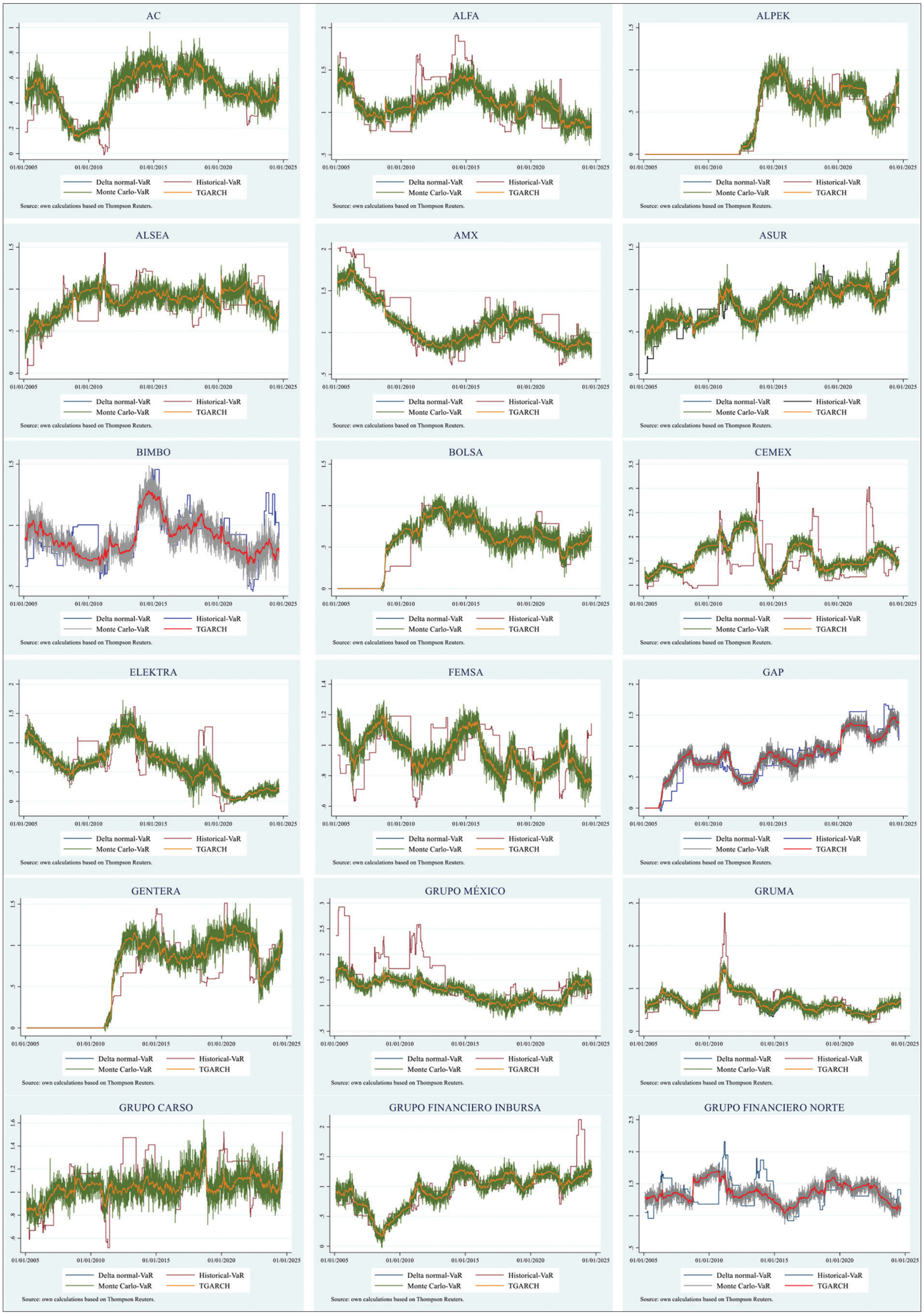
Trejo, B., Nuñez, J.A., Lorenzo, A. (2006), Distribución de los rendimientos del mercado mexicano accionario. Estudios Económicos, 21(1), 85-118.

Yamai, Y., Yoshiba, T. (2002), Comparative analyses of expected shortfall and value-at-risk (3): Their validity under market stress. Monetary and Economic Studies, 20(3), 181-237.

Zakoian, J.M. (1994), Threshold heteroskedastic models. Journal of Economic Dynamics and Control, 18(5), 931-955.

ANNEX

Annex 1: Historical betas of financial assets in mexico's stock market index



Annex 1: *(Contd...)*