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Kou Jump Diffusion Model: An Application to the Standard and Poor 500, Nasdaq 100 and Russell 2000 Index Options

Wajih Abbasi[1], Petr Hájek[2]*, Diana Ismailova[3], Saira Yessimzhanova[4], Zouhaier Ben Khelifa[5], Kholnazar Amonov[6]

[1]College of Business and Economics, Qassim University, Saudi Arabia, [2]Unicorn College and Central Bohemia University, Prague, The Czech Republic, [3]Kokshetau University Named after Abai Myrzakhmetov, Kazakhstan, [4]Narxoz University, Kazakhstan, [5]ECSTRA Center of Research on Economics and Finance, HEC Carthage, Tunisia, [6]Central Bohemia University, Prague, The Czech Republic. *Email: hajekp@gmail.com

ABSTRACT

This research focuses on the empirical comparative analysis of three models of option pricing: (a) The implied volatility daily calibrated Black–Scholes model, (b) the Cox and Ross univariate model with the volatility which is a deterministic and inverse function of the underlying asset price and (c) the Kou jump diffusion model. To conduct the empirical analysis, we use a diversified sample with options written on three US indexes during 2007: Large cap (Standard and Poor 500 [SP 500]), Hi-Tech cap (Nasdaq 100) and small cap (Russell 2000). For the estimation of models parameters, we opted for the data-fitting technique using the trust region reflective algorithm on option prices, rather than the more common maximum likelihood or generalized method of moments on the history of the underlying asset. The analysis that we conducted clearly shows the supremacy of Kou model. We also notice that it provided better results for the Nasdaq 100 and Russell 2000 index options than for the SP 500 ones. Actually, this supremacy comes from the ability of this model to be as close as possible of market participant's behavior thanks to its double exponential distribution characterized by three main properties: (a) Leptokurtic feature, (b) psychological specificity of investors and (c) memory-less feature.

Keywords: Jump-diffusion, Kou Model, Leptokurtic Feature, Trust-Region-Reflective Algorithm, US Index Options
JEL Classifications: C3, C8, G12, G13

1. INTRODUCTION

Theoretical models that are interested in evaluating options are generally based on two key elements: The process of the underlying asset and the market price of the risk factor. The Black–Scholes (BS) model (1973) is based on the assumption of a lognormal diffusion process with a constant instantaneous volatility. Being the benchmark for derivative assets valuation, this model has been, during the last 30 years, the target of several empirical studies that have revealed a number of limitations. On the one hand, the assumption of log normality of the underlying asset has been widely rejected by the autoregressive conditional heteroskedasticity literature. On the other hand, the assumption of a diffusion process was also rejected by the existence of heavy tails of the distribution of returns. Finally, the effect of debt raised (Black, 1976), and the existence of a possible correlation between the process and the volatility of the underlying asset (Heston, 1993), Nandi (2000) indicated a complex relationship between asset returns and volatility. These empirical limits pushed theorists to develop alternate models. Research undertaken thereafter considered three approaches:

The univariate models: These are models that have maintained the no-arbitrage assumption of the BS model, but gave up the assumption of geometric brownian motion. Included are the constant elasticity variance model (CEV) of Cox and Ross (1975) and Cox (1996) and more recently the trinomial or implied binomial tree models of Derman and Kani (1994) and Dupire (1994).

- The stochastic volatility models: These models are based on the assumption of a volatility of the underlying asset evolving in a stochastic manner by following a diffusion process, Heston (1993), Hull and White (1987), Wiggins (1987), and the hybrid jump-diffusion process of Duffie et al. (2000)

- The jump diffusion models: That has replaced the underlying asset classical diffusion process. Out of which the Merton model (1976) remains the most popular.

Very recent studies have attempted to combine these three approaches, such as studies by Jones (2003) and Skiadopoulos (2000) who have respectively proposed a stochastic generalization of the CEV process and the binomial tree model.

Although alternative models have started to appear few years after the original BS model, their empirical investigation was hampered by several factors. First, market data for options were and still remain difficult to collect. Then, the new models are generally much more complicated than the BS model and validating them empirically needed advanced programming works. Finally, the introduction of the concept of risk for certain jump processes or of the stochastic volatility models raised the problem of the evaluation of this variable.

It is only by the 90s that we began to witness the appearance of serious empirical research on this level with the development of computers and the progress in mathematical and econometric research such as the Fourier inversion technique used by Heston (1993), or the non-linear squares technique used by Bakshi et al. (1997), Bates (1996a; 1996b), Dumas et al. (1998).

This paper proposes to compare the empirical performance of three alternatives to the BS model that belong to three different classes. First, the *ad-hoc* BS model that is praised by practitioners for its simplicity and effectiveness. It is simply the BS classical model with a daily implied volatility calibration from option pricing. Although such procedure seems unorthodox and inconsistent with the assumptions of the BS classical model, it provides quite suitable results in the evaluation of options, where in his paper entitled "How to get the right option price with the wrong model?" Berkowitz (2001) showed that, thanks to the daily volatility calibration, the ad-hoc BS model arrived to provide close performances to those of stochastic volatility models in terms of evaluation "in sample." The second model is the CEV model developed by Cox and Ross (1975) better known by the abbreviation CEV model that belongs to the class of univariate models. By opting for a non-stationary process of the underlying asset volatility that is negatively correlated to the price of the asset, the CEV model adjusts for some empirical realities: (a) The change in volatility over time, (b) the inverse relationship between volatility and the price of the underlying asset. The third alternative is the jump-diffusion model of Kou (2002) which proposes a hybrid process for the underlying asset, consisting of a first "diffusion" component the same as in the BS model and a second "jump" component following a double exponential process. Such a model allows us to understand two major empirical phenomena: The Kou model leads to a probability distribution with heavy tails (a frequently observed phenomenon of the underlying assets distributions) which simply means a greater probability for extreme values. Then the Kou model is able to integrate the phenomenon of negative skewness (more probability for negative outcomes) through the jump signs, by proposing negative jumps for the underlying asset return, the model affects more probability for negative achievements.

The empirical approach will be structured as follows: We begin by presenting the structure of the database used in this study. Options traded on the Chicago Board Options Exchange (CBOE) during the year 2007 for the three stock indexes Standard and Poor 500 (SP 500), Nasdaq 100 and Russell 2000 for a total of 26, 968 call options (the reason is to check whether technology and or small stocks behave in a different manner than those stocks that represent best the US economy). Then we conduct a comparative analysis between the ad-hoc BS, the CEV model and the jump-diffusion Kou model. This analysis aims to verify the validity of the assumptions made by each of these models by comparing the model prices to market options prices. The comparative analysis will also detect any structural bias that would affect the performance of each of the three theoretical models.

2. CEV MODEL OF COX AND ROSS

Cox and Ross (1975) developed a pricing model of calls that verifies the negative relationship between return volatility of the underlying asset and its price. In this model, the variance of returns is a deterministic function of the underlying asset price and its elasticity with respect to price is constant. Specifically, the model assumes that the instantaneous rate of return of the underlying asset evolves according to the following process:

$$\frac{dS}{S} = \mu .dt + \delta \ .S^{\theta-1}.dz \tag{1}$$

μ is the drift rate of the underlying asset return,

$\delta.S^{\theta-1}$ is the instantaneous standard deviation of the underlying asset return with δ a strictly positive constant,

dz is a standard Wiener process which follows a normal distribution with expectation $E(dz)=0$ and variance $Var(dz)=dt$.

The major difference with the BS model is that the volatility of returns of the underlying asset $\delta.S^{\theta-1}$ is based on the price of the asset. However, if $\theta = 1$, the CEV model coincides with the BS model. Whereas, when θ deviates from 1, the process that characterizes the underlying asset becomes non-stationary. The negative correlation between asset prices and volatility, as evidenced by several empirical studies will be checked only if $\theta < 1$. h_s, the variance elasticity of the underlying asset returns is given by,

$$h_s = \left(\frac{\partial \sigma^2}{\partial S}\right) \cdot \left(\frac{S}{\sigma^2}\right) = \frac{\partial\left(\delta^2 .S^{2\theta-2}\right)}{\partial S} \cdot \frac{S}{\delta^2 .S^{2\theta-2}} = 2\left(\theta-1\right) \tag{2}$$

We notice that the elasticity is negative only if $\theta < 1$.

Considering an underlying asset that pays no dividend, a price that follows the process described in Equation (1), and a constant risk-free rate r, using the CEV model of Cox, the price of a European call under the risk-neutral probability is equal to the present value of the cash-flow expected at maturity:

$$C_v = e^{-r.\tau}.\hat{E}_t\left[Max\left(0, S^* - K\right)\right] \quad (3)$$

C_v is the theoretical price calculated using the CEV model, of a European call with exercise price K and maturing in τ years,

$\hat{E}_t$ is the expectation operator under the risk-neutral probability,

S* is the expected price of the underlying asset at the call maturity.

Once we identify the distribution of S^*, the underlying asset price at maturity following the CEV diffusion process as shown by Equation (1), we can determine the theoretical price of the call.

Cox's equation for evaluating options under the CEV assumption is defined as follows:

$$C_v = S.\sum_{n=0}^{\alpha} g\left(\lambda.S^{\phi}, n+1\right).G\left(\lambda.\left(K.e^{-r\tau}\right)^{\phi}, n+1-\frac{1}{\phi}\right) - K.e^{-r\tau}$$

$$\sum_{n=0}^{\alpha} g\left(\lambda.S^{\phi}, n+1-\frac{1}{\phi}\right).G\left(\lambda.\left(K.e^{-r\tau}\right)^{\phi}, n+1\right) \quad (4)$$

$\varnothing = 2\theta - 2$

$$\lambda = 2.\frac{r}{\delta^2}.\phi.e^{(r\phi\tau-1)}$$

$\Gamma(n) = \int_0^{\infty} e^{-v}.v^{n-1}.dv$: The gamma function,

$g(x,n) = e^{-x}.x^{n-1}.\frac{1}{\Gamma(n)}$: The gamma density function,

$G(a,n) = \int_0^{\infty} g(x,n).dx$: The gamma complementary distribution function.

However, this model continues to consider the parameters δ and θ as constants, which does not seem to be a very realistic assumption especially when it comes to evaluating options on stock indexes. Indeed, if one refers to the idea of a constant negative elasticity, we could end up in a vicious circle, since any decline in the stock index will increase volatility. This latter increases market fears and causes a further decline in the index. With such a mechanism, we may end up with a volatility that tends to infinity along with a stock index which tends to zero. Such a situation is unlikely. One solution to this problem would be to recalibrate the CEV model on a periodic basis to update its δ and θ structural parameters like the *ad-hoc* BS model, we proceed to the daily calculation of the structural parameters of the CEV model from option prices. For the remainder of this article, we will denote model with calibration by CEV.

3. KOU JUMP-DIFFUSION MODEL

The model is quite simple in its logic. The logarithm of the underlying asset price is assumed to follow a hybrid jump-diffusion process. The first component of the process is similar to that of BS geometric Brownian motion. The second component corresponds to a "Poisson" process jumps with amplitudes distributed according to the double exponential distribution. The model assumes that the underlying asset price volves according to the following process:

$$\frac{dS(t)}{S(t-)} = \mu.dt + \sigma.dW(t) + d\left(\sum_{i=1}^{N(t)}\left(V_i - 1\right)\right) \quad (5)$$

$W(t)$ is a standard Brownian motion,

$N(t)$ is a Poisson process with a frequency λ.

V_i is a sequence of positive random variables independently and identically distributed such that $Y = \mathrm{Log}(V)$ follows a distribution with an asymmetric double exponential density function:

$$f_y(y) = p.\eta_1.e^{-\eta_1.y}{}_{\{y\geq 0\}} + q.\eta_2.e^{-\eta_2.y}{}_{\{y<0\}} \quad (6)$$

$\eta_1 > 1$; $\eta_2 > 0$

Where, $q > 0$; $p + q = 1$, represent the probabilities of upward and downward jumps. The drift μ and the volatility σ are assumed to be constants and the Brownian motion and jumps are assumed to be one dimensional. In other words,

$$Log(V) = Y \triangleq \begin{cases} \xi_+ \text{ with probability}(p) \\ \xi_- \text{ with probability}(q) \end{cases} \quad (7)$$

Where ξ_+ and ξ_- are two exponential random variables with means $1/\eta_1$ and $1/\eta_2$, respectively, and the notation $\triangleq$ means equal in distribution. Y is the random variable representing the jumps that may affect the underlying asset rate of returns. ξ_+ and ξ_- are respectively the amplitudes of the upward and downward jumps.

$E(Y) = \frac{p}{\eta_1} - \frac{q}{\eta_2}$ is the average amplitude of the jump.

$Var(Y) = p.q.\left(\frac{1}{\eta_1} + \frac{1}{\eta_2}\right)^2 + \left(\frac{p}{\eta_1^2} + \frac{q}{\eta_2^2}\right)$ is part of the volatility of the underlying asset due to jump risk. This will provide:

$$Var(S) = \sigma^2 + Var(Y) \quad (8)$$

There are three interesting properties of the double exponential distribution which are fundamental to the model. First, the distribution has the leptokurtic feature. This feature that governs the jump size distribution is consistent with the empirical distribution that characterizes the underlying asset rate of return. Then, the double exponential distribution has the memory less property. In other words, the current achievements depend, in one way or another, on the past achievements. Finally, this distribution has a psychological and economic justification. Indeed, it has been demonstrated through several empirical studies that markets tend to have an overreaction and under-reaction towards various good or bad news (Fama, 1998; Barberis et al., 1998). We can then interpret the jumps as a market response to new external market information. Thus, in the absence of external information, the price of the underlying asset should move according to a Brownian motion. Good or bad news occur according to a Poisson process and the price of the underlying asset changes in response to this news, according to the distribution that governs the size of the jump. This distribution can be used to model the overreaction (through heavier tails) and the under-reaction (through a larger peak). Therefore, the diffusion model with double exponential jumps can be interpreted as an attempt to build a simple model within the traditional framework of random walk and market efficiency that takes into account investor's attitudes towards risk as well.

The European call valuation formula, according to the Kou jump-diffusion model, is given by:

$$\psi_c(0) = S(0).X\left\{r+\frac{1}{2}.\sigma^2-\lambda.\zeta,\sigma,\tilde{\lambda},\tilde{p},\widetilde{\eta_1},\widetilde{\eta_2}\ ,\log\left(\frac{K}{S}\right),T\right\}$$
$$-K.e^{-rT}.X\left\{r-\frac{1}{2}.\sigma^2-\lambda.\zeta,\sigma,\lambda,p,\eta_1,\eta_2\ ,\log\left(\frac{K}{S}\right),T\right\} \quad (9)$$

X: The probability function of the Kou jump-diffusion model.

$$\zeta=\frac{p.\eta_1}{\eta_1-1}+\frac{q.\eta_2}{\eta_2+1}-1\ ,\tilde{p}=\frac{p}{1+\zeta}.\frac{\eta_1}{\eta_1-1}\ ,$$
$$\widetilde{\eta_1}=\eta_1-1\ ,\widetilde{\eta_2}=\eta_2+1\ ,\tilde{\lambda}=\lambda.(\zeta+1)$$

The price of the corresponding put $\Psi_p(0)$, can be inferred through the call-put parity:

$$\Psi_p(0)-\Psi_c(0)=K.e^{-r.T}-S(0) \quad (10)$$

The Kou model also presents analytical solutions for the evaluation of American options, look back options, and other exotic options.

4. OVERVIEW OF THE DATABASE

The final sample used concerns call options on three U.S. indexes, the Nasdaq 100, SP 500 and Russell 2000. The final sample that combines all categories of options includes 26,968 call options traded on the CBOE during 2007and distributed as follows: 7151 options for the Nasdaq 100 index; 12,499 options for the SP 500 and finally 7138 options for the Russell 2000 Index. We then compiled abstract tables which show the main properties for each of the three groups identified.

The final sample is obtained by applying five filters. First, all the options with an average price <50 cents were removed. Then the options with a spread which is the difference between the ask price and bid price divided by the mid-price of this option, where that spread represents more than 50% of the average call price are removed. These first two filters are meant to eliminate calls with a large spread in relation to bid-ask quotations reported by the database. We also removed options with a moneyness which deviates from the range (−10%, 10%). Indeed, the options that are deep out-of-the-money (OTM) or deep-in-the-money (ITM) are illiquid and have a low time value which substantially affects the predictive power of the estimated parameters value.

Next, we eliminated options with <6 days or over 100 days to expiration. The former have almost zero time premiums while the latter are illiquid. Finally, all options that do not meet the no-arbitrage assumption are eliminated. The majority of observations eliminated correspond to deep ITM calls.

Table 1 describes the properties of the final sample of SP 500 calls to be used for our empirical study. The sample is dominated by at-the-money (ATM) options with 5599 observations (44.8% of the final sample) followed by ITM options with 3970 observations (31.7% of the sample) and finally OTM options with 2930 comments (23.5% of the sample). Referring to the criterion of time to expiration, we realize that the sample is dominated by options of short and medium term maturities with respectively 4792 (38.3% of the sample) and 4496 observations (36% of the sample). The long-term options represent only 25.7% of the final sample with 3211 observations. The average price of SP 500 calls varies from $ 89.13 (long term deep ITM options) to $ 0.67 (short term deep OTM calls). The spread ranges from 2.2% of the call mid-price (long term ITM calls) to 48.2% (short term deep OTM calls).

Similarly, the properties of the final sample for Nasdaq 100 and Russell 2000 are respectively summarized in Tables 2 and 3.

Table 1: Properties of the final sample of the SP500 calls

Moneyness (%)	Time-to-expiration (days)			Sub-total
	6-30	31-60	61-100	
OTM				
−10, −6	$0.67	$1.1	$2.31	
	0.482	0.408	0.305	
	33	363	599	995
−6, −3	$1.3	$3.72	$7.84	
	0.317	0.194	0.136	
	571	843	521	1935
ATM				
−3, 0	$5.75	$12.74	$20.11	
	0.149	0.103	0.083	
	1221	1089	581	2891
0, 3	$22.94	$29.6	$37.31	
	0.073	0.064	0.053	
	1147	1012	549	2708
ITM				
3, 6	$82.34	$84.58	$89.13	
	0.024	0.024	0.022	
	693	730	502	1925
6, 10	$49.68	$53.88	$59.95	
	0.039	0.037	0.033	
	831	755	459	2045
Sub-total	4496	4792	3211	12,499

Prices reported in the table respectively represent the calls mid-price, the effective spread (defined as the difference between the bid and ask price of the option divided by its average price) and finally the total number of observations for each sample subcategory moneyness/time-to-expiration. The sample period is spread over the whole of 2007 for a total of 12,499 observations. The moneyness equals $(S-K.e^{-r.t})/K.e^{-r.t}$. *S* means the spot level of the SP 500. *K* stands for the strike price, (*r*) for the risk-free interest rate which corresponds to the maturity of the call and (*t*) the call time-to-expiration. OTM, ATM and ITM calls denote the out-of-the-money, at-the-money and in-the-money options. Source: Authors

Table 2: Properties of the final sample of the Nasdaq 100 calls

Moneyness (%)	Time-to-expiration (days)			Sub-total
	6-30	31-60	61-100	
OTM				
−10, −6	\$1.72	\$5.92	\$14.03	
	26.5%	18.2%	12.8%	
	213	712	755	1680
−6, −3	\$5.18	\$16.71	\$29.09	
	16.9%	11.5%	12.4%	
	339	486	416	1241
ATM				
−3, 0	\$16.36	\$36.06	\$49.44	
	10.6%	11.3%	11.9%	
	356	455	401	1, 212
0, 3	\$39.9	\$58.16	\$72.51	
	9.8%	9.5%	8.8%	
	336	439	377	1152
ITM				
3, 6	\$71.77	\$86.46	\$100.04	
	7.3%	6.9%	6.6%	
	276	382	258	916
6, 10	\$111.57	\$121.26	\$133.81	
	4.9%	5%	5%	
	274	422	254	950
Sub-total	1794	2896	2461	7151

Prices reported in the table respectively represent the calls mid-price, the effective spread (defined as the difference between the bid and ask price of the option divided by its average price) and finally the total number of observations for each sample subcategory moneyness/time-to-expiration. The sample period is spread over the whole of 2007 for a total of 7, 151 observations. The moneyness equals $(S-K.e^{-r.t})/K.e^{-r.t}$. S means the spot level of the Nasdaq 100. K stands for the strike price, (r) for the risk-free interest rate which corresponds to the maturity of the call and (t) the call time to expiration. OTM, ATM and ITM calls denote the out-of-the-money, at-the-money and in-the-money. Source: Authors

Table 3: Properties of the final sample of Russell 2000 calls

Moneyness (%)	Time-to-expiration (days)			Sub-total
	6-30	31-60	61-100	
OTM				
−10, −6	\$1.17	\$2.69	\$5.72	
	29.7%	17.2%	9.6%	
	122	541	568	1231
−6, −3	\$2.35	\$6.25	\$10.89	
	19.6%	8.8%	6.9%	
	284	470	484	1238
ATM				
−3, 0	\$5.98	\$12.36	\$17.7	
	9.8%	5.9%	4.7%	
	350	457	463	1270
0, 3	\$14.7	\$21.4	\$26.68	
	5.2%	3.9%	3.4%	
	330	441	466	1237
ITM				
3, 6	\$27.24	\$32.59	\$37.51	
	3.1%	2.6%	2.4%	
	290	411	423	1124
6, 10	\$43.13	\$47.72	\$51.03	
	1.9%	1.8%	1.8%	
	257	497	464	1218
Sub-total	1633	2817	2868	7318

Prices reported in the table respectively represent the calls mid-price, the effective spread (defined as the difference between the bid and ask price of the option divided by its average price) and finally the total number of observations for each sample subcategory moneyness/time-to-expiration. The sample period is spread over the whole of 2007 for a total of 7, 318 observations. The moneyness equals $(S-K.e^{-r.t})/K.e^{-r.t}$. S means the spot level of the Russell 2000. K stands for the strike price, (r) for the risk-free interest rate which corresponds to the maturity of the call and (t) the call time to expiration. OTM, ATM and ITM calls denote the out-of-the-money, at-the-money and in-the-money options. Source: Authors

5. PARAMETERS ESTIMATION AND PERFORMANCE MODELS

In order to have a clearer view of the limits of the BS model, we represented the evolution of the implied volatility as a function of moneyness and time-to-maturity for 2 days arbitrarily chosen in our sample. We then obtained two surfaces of the implied volatility that highlight the dual structural bias plaguing the BS model (Figure 1a and b). The surface traces the evolution of volatility across different levels of moneyness and time-to-expiration. Each point on the surface corresponds to an implied volatility obtained through reversing of the BS formula. Indeed, referring to these surfaces, we realize that the implied volatility generated from the BS model is not unique in space or constant in time, which is inconsistent with the hypothesis of log normality of the price of the underlying asset on which is based the BS model.

Figure 1 the shows two surfaces of the implied volatility for 2 separate days in the sample. The surface traces the evolution of volatility across different levels of moneyness and time-to-expiration. Each point on the surface corresponds to an implied volatility obtained from the SP500 call mid-price, through reversing of the BS formula.

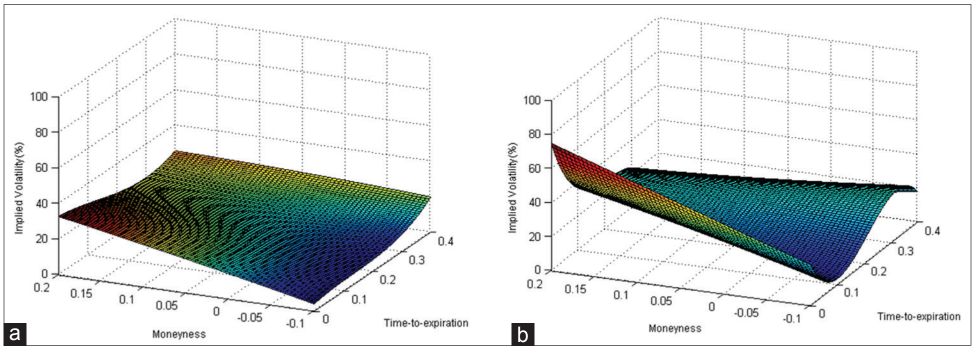
The most dramatic change in the volatility is recorded for short term options with a volatility smile where OTM and ITM options show significantly different volatilities than ATM options. Any theoretical model, which presents itself as a serious alternative to the BS model, should provide a significant improvement mainly to short term options. As the time-to-expiration increases, the change in implied volatility becomes more moderate with a decreasing pace, commonly called the sneer where the most ITM options show the highest volatility.

As both phenomena smile and sneer are synonymous with a probability distribution with negative skewness and excess kurtosis, any acceptable alternative model to BS should propose a distribution that integrates these two aspects. Thus, one can moderate the effect of the time-to-expiration and the moneyness as two generating sources of estimation bias.

5.1. Alternate Models Parameter Estimation: Trust-Region-Reflective (TRR) Algorithm

A solution to the parameters estimation problem would be to use the maximum likelihood or generalized method of moments to identify these estimates from the history of the underlying asset. Maekawa et al. (2008) have used this technique to estimate the parameters of the Kou model in the Japanese market with more than 1, 000 observations out of Nikkei 225 from June 1, 1992 to December 31, 2002. Then they used the market prices of European call options for Nikkei 225 from September 10, 1999 to December 12, 2002 to evaluate the empirical performance of the Kou model. Such a solution can be binding as it requires the collection of a large volume of historical data that eventually leads to low predictive power estimates. In order to address this gap, practitioners and researchers have chosen to derive the estimates

Figure 1: (a) Standard and Poor (SP 500) calls implied volatility surface, June 8, 2007, (b) SP 500 calls implied volatility surface, December 10, 2007



Source: Authors

of the structural parameters from observed option prices. This solution has introduced the concept of the implied volatility for the BS model. However, the application of such a technique is more complicated with models that involve several structural parameters at the same time and using much more developed mathematical tools than for the case of the BS model.

For this study, we chose to derive the estimates of the structural parameters of the Kou model from instant cross sectional price of options for each day of the sample solving nonlinear curve-fitting (data-fitting) problems in least-squares sense means to find a set of parameters β that solve the least squares minimization problem.

$$\min_{\beta} f(\beta) = \min_{\beta} \sum_{i} \left[\left(F(\beta, xdata_i) - ydata_i \right) \right]^2 \tag{11}$$

The optimization algorithm used for the least squares minimization is the TRR. The basic idea behind the TRR approach is as follows: Suppose you are at a point β in *n*-space and you want to move to a point with a lower function value *f*. First, we have to approximate *f* (β) with a simpler function *q* which is defined by the first two terms of the Taylor approximation to *F* at β. This approximation should reasonably reflect the behavior of *f* in a neighborhood *N* around the point β_i. A trial step s_i is computed by minimizing (or approximately minimizing) over *N*. This *N* is called the trust region and the improved point β_{i+1} should also be in this region. *N* is usually spherical or ellipsoidal in shape. The trial step $s_i = \beta_{i+1} - \beta_i$ is found by approximately solving the equation.

$$\min_{\beta} \left(q_i(s), s \in N \right) \tag{12}$$

Where,

$$q_i(s) = s^T g + \frac{1}{2} s^T H_s \quad \text{such that } \|D_i s\| \le \Delta_i \tag{13}$$

Where, *g* is the gradient of *f* at the current point β, *H* is the Hessian matrix, *D* is a diagonal scaling matrix, Δ_i is a positive scalar corresponding to the trust region size. The trust region is adjusted from iteration to iteration. If the computations show that the approximate function q_i at the current point β_i fit the original problem well, the trust region can be enlarged. Otherwise, it must be shrunk, Byrd et al. (2000). The approximation approach followed is to restrict the trust-region sub problem to a two-dimensional subspace *V*. In our algorithm, we choose *V* as the linear space spanned by v_1 and v_2, where v_1 is in the direction of the gradient *g*, and v_2 is either an approximate Newton direction, i.e., a solution to,

$$H.v_2 = -g \tag{14}$$

In summary, the TRR algorithm involves the following steps:

1. Formulate the two-dimensional trust-region subproblem
2. Solve (12) to determine the trial step s_i.
3. If $f(\beta_i + s_i) < f(\beta_i)$, then $\beta_i + 1$ becomes the current point; otherwise $\beta_i + 1 = \beta i$
4. Update Δ_i
5. If the gradient is below a chosen tolerance, the algorithm ends; otherwise, repeat and increment *i*.

Such a technique can significantly reduce the number of observations required to estimate and leads to a significant improvement in the performance of the evaluation models, Bates (1996a; 1996b), Dumas et al. (1998), Bakshi et al. (1997). The estimation procedure is as follows:

Step 1: For a well-defined sample, we collect m options, such as m is greater than or equal to (*n* + 1) where *n* is the number of parameters to estimate. In the case of the Kou model, *n* = 4. $C_{i_{market}}$ is the market price of the i^{th} call. $C_{i_{Kou}}$ is the theoretical price of the ith call calculated using the Kou model. The difference between these two prices will depend on the vector $\phi = \{\sigma, \lambda, \eta_1, \eta_2\}$. For each option (*i*), we define:

$$\varepsilon_i(\phi) = C_{i_{market}}(t, \tau_i, K_i) - C_{i_{Kou}}(t, \tau_i, K_i) \tag{15}$$

Step 2: We find the vector of parameters that minimizes the sum of squared errors between the observed prices and the theoretical prices of options.

$$SSE \equiv \min_{\phi} \sum_{i=1}^{N} \left| \varepsilon_i(\phi) \right|^2 \tag{16}$$

These two steps are repeated for each option and for each day in our sample. The objective function SSE is defined as the sum of squared errors, in dollars, of call options prices. The use of nonlinear least squares should provide a fair comparison between the three models. We obtain structural parameters estimated through option prices for the *ad-hoc* BS model (implied volatility), CEV(0, δ) and Kou's jump-diffusion $\{\sigma, \lambda, \eta_1, \eta_2\}$.

5.2. Results of the Estimation

Estimates of the structural parameters of the CEV and the Kou jump-diffusion models are included in Table 4 (SP 500), Table 5 (Nasdaq 100) and Table 6 (Russell 2000).

For the CEV model, estimates show a poor negative correlation between the level of the SP 500 index and its volatility. Indeed, as θ tends to 1, the CEV model tends to the BS model. Such a result is quite logical since the SP 500 index representing the U.S. equity market has strongly rebounded after the technology bubble and volatility indices have stabilized afterwards. This may explain the poor negative correlation generated by the nonlinear least squares, which only reflect the renewed confidence of market participants.

For the Kou jump-diffusion model, estimates are quite reasonable for a fairly diversified stock index, such as the SP 500, especially during stable times. According to the estimation results, the market participants anticipate to achieve an average of 3.514 jumps per year with average amplitude of −3.51% per jump. The overall average volatility that is measured by the variance represents 1.44% (being a standard deviation of 11.98%), distributed between 1.05% to the "diffusion" component (or standard deviation 10.24%) and 0.39% for the "jumps" component (being a standard deviation of 6.23%). In other words, the diffusion process contributes to 73% in the overall risk of the underlying asset against only 23% for the "jumps" component. The same procedure was repeated for options on Nasdaq 100 and Russell 2000.

For the Nasdaq 100 options, daily estimates are close to each other as shown by the standard deviation of the estimated parameters, equal to 0.0216, with an average of 0.9509. The same observation is valid for δ with a standard deviation of 0.0528 and an average of 0.2252 estimates. However, the daily calibration of the structural parameters of the model using nonlinear least squares remains useful given the sensitivity of the option price to forecast volatility parameters used in the calculation of theoretical prices.

Estimates of Kou model implicit parameters from Nasdaq 100 options show that there is an average of 5.796 jumps per year. The average amplitude of the jump is equal to −3.925%, where the average amplitude is calculated using the following formula: $E(Y)=\frac{p}{\eta_1}-\frac{q}{\eta_2}$. Y is the amplitude of the jump on the index return. p and q are respectively the probabilities of upward and downward jumps. $p=q=0.5$. The Nasdaq 100 has an overall instantaneous variance equal to 3.68% (an overall standard deviation of 19.18%). Jumps contribute to 14.4% on the actual index volatility against 85.6% for the component "diffusion" process. However, it is worth noting that compared to the SP 500 options, the estimated parameters for the Nasdaq 100 options show a significantly higher overall volatility and negative amplitude of jumps three times larger than the SP 500 options. Such a result stems from the different characteristics of the two indexes. While the SP 500 contains the 500 largest market capitalization of the U.S. economy, the Nasdaq 100 is limited to the 100 most highly capitalized technology companies naturally belonging to one of the riskiest sectors of the American economy.

Table 4: Estimation of structural parameters for CEV and Kou models – SP 500 options

Parameters	Mean±Standard deviation	
	CEV	Kou
θ	0.8963±0.0429	
δ	0.2036±0.0481	
σ		0.1024±0.011
λ		3.514±1.174
η_1		379.179±180.957
η_2		13.746±3.112

For each day of the sample, the structural parameters of the CEV Cox and jump-diffusion Kou models are estimated by minimizing the sum of squared errors between the observed option price and its theoretical price determined by each of the two models. These estimates will be realized for each day of the sample using the technique of nonlinear least squares. The table brings forward the estimates and the corresponding standard deviation for each parameter. CEV refers to Cox's CEV model (1976) whereas Kou refers to the Kou's jump-diffusion model (2002). θ and δ are the structural parameters of the CEV model and correspond to the elasticity of volatility and to a positive scalar. σ, λ, η_1, η_2 are the structural parameters to be estimated for the model of Kou. σ designates the portion of the volatility generated by the diffusion process component. λ refers to the average number of jumps per year. η_1 and η_2 respectively control the amplitude of upward jumps (η_1) and downward jumps (η_2). The average amplitude is equal to $p/\eta_1-q/\eta_2$. p and q denote the probabilities of an upward or a downward jumps. $p=q=0.5$. Source: Authors. CEV: Constant elasticity variance, SP 500: Standard and Poor 500

Table 5: Estimation of structural parameters for CEV and Kou models – Nasdaq 100 options

	Mean±Standard deviation	
Parameters	CEV	Kou
θ	0.9509±0.0216	
δ	0.2252±0.0528	
σ		0.1776±0.3313
λ		5.7906±1.9921
η_1		243.3514±91.365
η_2		12.104±3.001

For each day of the sample, the structural parameters of the CEV Cox and jump-diffusion Kou models are estimated by minimizing the sum of squared errors between the observed option price and its theoretical price determined by each of the two models. These estimates will be realized for each day of the sample using the technique of nonlinear least squares. The table brings forward the estimates and the corresponding standard deviation for each parameter. CEV refers to Cox's CEV model (1976) whereas Kou refers to the Kou's jump-diffusion model (2002). θ and δ are the structural parameters of the CEV model and correspond to the elasticity of volatility and to a positive scalar. σ, λ, η_1, η_2 are the structural parameters to be estimated for the model of Kou. σ designates the portion of the volatility generated by the diffusion process component. λ refers to the average number of jumps per year. η_1 and η_2 respectively control the amplitude of upward jumps (η_1) and downward jumps (η_2). The average amplitude is equal to $p/\eta_1-q/\eta_2$. p and q denote the probabilities of an upward or a downward jumps. $p=q=0.5$. Source: Authors. CEV: Constant elasticity variance

Table 6: Estimation of structural parameters for CEV and Kou models – Russell 2000 options

	Mean±Standard deviation	
Parameters	CEV	Kou
Θ	0.9555±0.0214	
Δ	0.1982±0.0334	
Σ		0.1654±0.0233
Λ		6.221±2.66
η_1		153.6922±86.0209
η_2		15.6833±4.0418

CEV: Constant elasticity variance

Compared to SP 500 and Nasdaq 100 options, the estimated parameters for the Russell 2000 options show a global volatility that is much higher than that of SP 500 options but lower than that Nasdaq 100 options. Although the Russell 2000 is the most diversified index grouping 2000 American firms, the “size” effect still works in favor of the SP 500 Index. Whereas, the Nasdaq 100 index continues to be penalized by both low diversification and more volatile stocks even though it contains firms with significantly larger market capitalization than those of the Russell 2000.

For each day of the sample, the structural parameters of the CEV Cox and jump-diffusion Kou models are estimated by minimizing the sum of squared errors between the observed option price and its theoretical price determined by each of the two models. These estimates will be realized for each day of the sample using the technique of nonlinear least squares. The table brings forward the estimates and the corresponding standard deviation for each parameter. CEV refers to Cox’s CEV model (1976) whereas Kou refers to the Kou’s jump-diffusion model (2002). θ and δ are the structural parameters of the $\eta_2$are the structural parameters to be estimated for the model of Kou. σ designates the portion of the volatility generated by the diffusion process component. λ refers to the average number of jumps per year. η_1 and η_2 respectively control the amplitude of upward jumps (η_1) and downward jumps (η_2). The average amplitude is equal to $p/\eta_1-q/\eta_2$. p and q denote the probabilities of an upward or a downward jumps. $p=q=0.5$.

Daily estimates of the two parameters θ and δ release respective averages of 0.9555 for the parameter θ, and 0.1982 for the parameter δ. These estimates show an almost zero correlation between the level of the Russell 2000 index and its volatility, as evidenced by the estimate of θ which is much closer to 1, the threshold for which the CEV model coincides with the BS model.

For the Kou jump-diffusion model, estimates show that there is an average of 6.22 jumps per year - the highest number of jumps compared to SP 500 and Nasdaq 100 indexes. The average amplitude of the jump, equal to −3.925%, is identical to that of the options on Nasdaq 100. The Russell 2000 index has an instantaneous variance equal 3.065% (being a total standard deviation of 17.51%). Jumps contribute to 10.73% on the actual volatility of the index against 89.27% for the component “diffusion” process.

5.3. Performance Models

Three criteria were used to conduct a comparative analysis between the three models:

- The mean squared errors: This is the average of the squared differences between the observed price of the option and its theoretical price calculated using each of the three models. This measure gives more weight to ITM calls compared to other options in the sample.
- The mean absolute error: At first, the absolute value of the difference is calculated for each option between the observed mid-price of the option and its theoretical price. Then, the average difference was reported at the observed mid-price. This will calculate the percentage of the estimation error for each model. Such a measure tends to give greater weight to evaluation errors related to the calls OTM at the expense of other options.
- The frequency: This is the number of times where each model has led to the estimation error (mean absolute error) that is lowest compared to the other two models.

This comparative analysis will be conducted by sub-sample “moneyness/time-to-expiration” instead of testing the performance of the three models for the entire sample as a single compact component. Such an approach should allow a better understanding of the elements that may represent sources of estimation bias for our theoretical models. The results are summarized in Tables 7-9 respectively for the SP 500, Nasdaq 100 and Russell 2000 indexes.

Table 7 summarizes these criteria divided into nine sub-samples that are usually based on the moneyness and time-to-expiration. The jump-diffusion model of Kou largely outperforms the CEV and *ad-hoc* BS models for all subcategories of the table. The superiority of the model becomes more evident as one moves away from the ATM calls and notably for ITM ones where the average relative error records its lowest level throughout the sample.

This result was predictable since the Kou model was the only one of the three models studied to take into account the aspect of the leptokurtic distribution of the underlying asset. Thus, excess kurtosis can be integrated via the amplitude and frequency of jumps while the negative skewness was present throughout the anticipated jump sign (negative jumps).

We also note that the performance of the model does not seem to suffer a lot from moneyness or time-to-expiration changes. The performance of the Kou model can be explained by several factors. First, the choice of a double exponential distribution, which characterizes the jumps, has improved the quality of estimates since it is more likely to reflect the extreme. Then, the technique of the structural parameters estimation of the model based on the nonlinear least squares has identified estimates based on option prices rather than time series of returns of the SP 500 index, as Bates (2000) showed that only sporadic jumps with large amplitude are able to achieve results that deviate significantly from the BS model. But such jumps are difficult to detect from the time series of the underlying asset.

Once we examine the performance of the CEV model, we note that even though we have applied the same method to estimate the Kou model (The estimate of the model structural parameters is made with nonlinear least squares technique using cross sectional SP 500 options prices) and have made a daily calibration of the structural parameters, the performance of the CEV model remains widely below those of the Kou model.

The CEV model provides the worst results for ITM options where he concedes the second position to the *ad-hoc* BS model for both short-term options, medium and long-term options. We can explain this result by the poor negative correlation between the index level and its volatility especially that we know that 2007 was a relatively “quiet” year. The inverse relationship between the price

Table 7: In-sample performance evaluation models for SP 500 options

Moneyness (%)	Time-to-expiration (days)								
	6-30			31-60			61-100		
	Kou	CEV	BS	Kou	CEV	BS	Kou	CEV	BS
OTM									
−10, −6	0.101	0.339	0.451	0.045	0.568	4.654	0.09	1.22	14.189
	46.5%	88.1%	71.3%	18.8%	70.1%	146.8%	11.04%	55.2%	142.3%
	(15)	(0)	(18)	(322)	(4)	(37)	(556)	(31)	(12)
−6, −3	0.058	0.332	3.856	0.095	0.646	12.731	0.147	1.236	26.815
	18.6%	46.2%	129.8%	8.87%	24.5%	103.4%	3.83%	14.9%	55.4%
	(456)	(66)	(49)	(642)	(189)	(12)	(431)	(88)	(2)
ATM									
−3, 0	0.123	2.731	8.254	0.114	3.668	18.894	0.188	3.297	26.268
	6.52%	26.3%	62.7%	2.45%	14.1%	30.5%	1.72%	7.6%	17.6%
	(1102)	(108)	(11)	(1040)	(42)	(7)	(493)	(50)	(38)
0, 3	0.169	2.231	5.09	0.096	2.032	13.89	0.232	1.862	24.173
	1.57%	6.6%	8.3%	0.82%	4.6%	7.9%	0.97%	3.1%	7.1%
	(881)	(148)	(118)	(824)	(68)	(106)	(380)	(78)	(91)
ITM									
3, 6	0.191	2.669	1.573	0.14	8.489	5.785	0.255	16.318	14.107
	0.55%	2.8%	1.9%	0.49%	5.1%	2.8%	0.64%	6.4%	3.9%
	(630)	(94)	(107)	(644)	(4)	(107)	(401)	(0)	(58)
6, 10	0.311	1.167	0.921	0.309	5.116	2.884	0.2	19.862	7.501
	0.47%	1%	0.9%	0.5%	2.4%	1.5%	0.39%	4.7%	2.3%
	(449)	(118)	(113)	(530)	(75)	(125)	(464)	(4)	(34)

Table 7 shows the three criteria used to assess the quality of the estimate of each of the three valuation models. These three criteria are, in order of appearance in the table, the mean squared errors, the mean absolute error and the frequency (in parentheses). The mean squared error is calculated from the squared deviations and calculated for each of the options in the sample using the theoretical price and the observed price. The mean absolute error is the average of the absolute values of differences between the theoretical option price and observed price, divided by the observed prices. Frequency reports the number of calls for which each of the three theoretical models released the lowest mean absolute error compared to the two other models. These three criteria are calculated for each moneyness/time-to-expiration sub-sample. The moneyness is equal to $(S-K.e^{-r.t})/K.e^{-r.t}$. *S* means the spot level of the SP 500 index. *K* stands for the strike price while (*r*) stands for the risk-free interest rate corresponding to the maturity of the call and (*t*) the time-to-expiration. OTM, ATM and ITM calls denote the out-of -the-money, at-the-money and in-the-money. Source: Authors. CEV: Constant elasticity variance, BS: Black–Scholes, SP 500: Standard and Poor 500

Table 8: In-sample performance evaluation models for Nasdaq 100 options

Moneyness (%)	Time-to-expiration (days)								
	6-30			31-60			61-100		
	Kou	CEV	BS	Kou	CEV	BS	Kou	CEV	BS
OTM									
−10, −6	0.155	0.504	5.058	0.386	2.06	11.906	0.43	6.6	11.793
	18.8%	47.6%	129.4%	12%	27.7%	78.7%	2.9%	20.1%	28.8%
	(176)	(23)	(12)	(520)	(139)	(6)	(644)	(35)	(17)
−6, −3	0.399	2.836	9.295	0.883	5.913	11.121	1.471	5.362	4.772
	11.3%	27.4%	73.4%	4.2%	12.2%	23.5%	1.9%	5.8%	7.3%
	(276)	(58)	(5)	(403)	(74)	(9)	(302)	(60)	(54)
ATM									
−3, 0	1.216	23.685	7.791	1.451	21.17	3.52	2.641	10.917	4.389
	5%	33.5%	22.3%	1.9%	14%	5.3%	1.5%	6%	3.5%
	(330)	(4)	(22)	(375)	(1)	(79)	(315)	(43)	(43)
0, 3	2.492	12.797	1.984	3.468	9.06	5.395	4.072	5.007	21.202
	2.2%	9.4%	3%	1.4%	4.6%	3.2%	1.2%	2.4%	5.7%
	(224)	(19)	(93)	(322)	(51)	(66)	(289)	(79)	(9)
ITM									
3, 6	1.086	6.988	9.138	3.226	16.31	19.484	3.519	30.643	48.983
	1%	2.7%	3.8%	0.9%	3.9%	4.8%	0.7%	5.1%	6.6%
	(192)	(59)	(25)	(340)	(32)	(10)	(251)	(3)	(4)
6, 10	1.419	16.446	20.176	3.858	38.755	35.235	5.164	79.557	73.52
	0.7%	2.9%	3.7%	0.8%	4.4%	4.6%	0.6%	6.4%	6.2%
	(216)	(29)	(27)	(372)	(31)	(15)	(247)	(1)	(4)

Table 8 shows the three criteria used to assess the quality of the estimate of each of the three valuation models. These three criteria are, in order of appearance in the table, the mean squared errors, the mean absolute error and the frequency (in parentheses). The mean squared error is calculated from the squared deviations and calculated for each of the options in the sample using the theoretical price and the observed price. The mean absolute error is the average of the absolute values of differences between the theoretical option price and observed price, divided by the observed prices. Frequency reports the number of calls for which each of the three theoretical models released the lowest mean absolute error compared to the two other models. These three criteria are calculated for each moneyness/time-to-expiration sub-sample. The moneyness is equal to $(S-K.e^{-r.t})/K.e^{-r.t}$. *S* means the spot level of the Nasdaq 100 index. K stands for the strike price while (r) stands for the risk-free interest rate corresponding to the maturity of the call and (*t*) the time-to-expiration. OTM, ATM and ITM calls denote the out-of-the-money, at-the-money and in-the-money. Source: Authors, CEV: Constant elasticity variance, BS: Black–Scholes, SP 500: Standard and Poor 500

Table 9: In-sample evaluation performance models for Russell 2000 options

Moneyness (%)	Time-to-expiration (days)								
	6-30			31-60			61-100		
	Kou	CEV	BS	Kou	CEV	BS	Kou	CEV	BS
OTM									
−10	0.027	0.53	0.133	0.035	0.129	0.314	0.14	1.722	0.468
−6	0.118	0.64	0.327	0.054	0.462	0.24	0.033	0.266	0.122
	(94)	(2)	(26)	(495)	(0)	(46)	(516)	(17)	(35)
−6,	0.061	0.198	0.555	0.043	0.353	0.491	0.08	0.489	0.278
−3	0.107	0.233	0.425	0.025	0.104	0.132	0.013	0.058	0.041
	(211)	(64)	(9)	(376)	(69)	(25)	(347)	(57)	(79)
ATM									
−3,	0.103	2.025	1.167	0.057	1.923	0.409	0.1	1.799	0.241
0	0.051	0.256	0.244	0.012	0.114	0.048	0.011	0.074	0.015
	(326)	(17)	(7)	(385)	(0)	(72)	(286)	(2)	(175)
0,	0.164	2.147	0.933	0.056	1.282	0.509	0.136	0.975	0.810
3	0.018	0.114	0.068	0.007	0.054	0.018	0.009	0.035	0.024
	(289)	(14)	(27)	(321)	(27)	(92)	(372)	(54)	(39)
ITM									
3,	0.237	0.666	0.892	0.066	1.053	0.793	0.125	1.63	1.651
6	0.011	0.024	0.025	0.005	0.026	0.016	0.006	0.03	0.03
	(204)	(60)	(26)	(323)	(30)	(58)	(388)	(26)	(9)
6,	0.356	1.204	0.904	0.172	2.722	0.972	0.124	6.052	1.992
10	0.009	0.022	0.176	0.006	0.031	0.135	0.004	0.048	0.024
	(193)	(30)	(33)	(348)	(41)	(108)	(440)	(2)	(22)

Table 9 shows the three criteria used to assess the quality of the estimate of each of the three valuation models. These three criteria are, in order of appearance in the table, the mean squared errors, the mean absolute error and the frequency (in parentheses). The mean squared error is calculated from the squared deviations and calculated for each of the options in the sample using the theoretical price and the observed price. The mean absolute error is the average of the absolute values of differences between the theoretical option price and observed price, divided by the observed prices. Frequency reports the number of calls for which each of the three theoretical models released the lowest mean absolute error compared to the two other models. These three criteria are calculated for each moneyness/time-to-expiration sub-sample. The moneyness is equal to $(S{-}K.e^{-r.t})/K._{e}^{-r.t}$. S means the spot level of the Russell 2000 index. *K* stands for the strike price while (*r*) stands for the risk-free interest rate corresponding to the maturity of the call and (*t*) the time-to-expiration. OTM, ATM and ITM calls denote the out-of-the-money, at-the-money and in-the-money. Source: Authors. CEV: Constant elasticity variance, BS: Black–Scholes, SP 500: Standard and Poor 500

of the underlying asset and its volatility was first introduced to the options with the argument of the leverage effect. In the absence of such an effect for indexes, the only plausible argument to explain this inverse relationship would be the panic effect in the presence of downside market movements. This could be explained by the inability of the CEV model to figure kurtosis excess that has always characterized the performance of indexes.

Finally, the *ad-hoc* BS model has usually a good performance for at-the-short term options. For ITM options it provides the best performance ahead of the CEV model and rivaling with the Kou model which is another less expected result. Once more, this result confirms that the short term ITM options are less sensitive to the choice of models and structural parameters. However, as one moves towards OTM options, the generated results are deteriorating in a spectacular way including a time-to-expiration above 30 days. Unfortunately, the daily calibration of volatilities was not able to offset these biases due to the unrealistic assumption of the lognormal asset price. Such a hypothesis is unable to integrate the phenomena that are empirically proved of high kurtosis and non-zero skewness.

The results for the Nasdaq 100 options in Table 8 confirm those already presented in Table 7 for the SP 500 options. The main conclusions are maintained for Russell 2000 options in Table 9 except for the CEV model which was less efficient than it was for the SP 500 and Nasdaq 100 options. Also, the *ad-hoc* BS model was outperformed in most sub-samples, especially for ATM and ITM options. This result was expected as long as the estimation of the structural parameters of the Cox model showed a $\theta = 0.9555$ against 0.8963 for the SP500 and $\theta = 0.9509$ options for Nasdaq100 options. Nonetheless, let's remember that the more θ is close to 1, the greater the CEV model coincides with the BS model.

5.4. Estimation Errors and Regression

We conduct a regression analysis to identify the factors responsible of the estimation errors for all the three models. By estimation error, we mean the mean absolute error $\vartheta_i(t)$ which is a function of $moneyness_i(t)$ the degree of moneyness, $\tau_i(t)$ the time to expiration, and of $spread_i(t)$ the spread relative to the i^{th} call observed at date (t). This regression performed using the technique of ordinary least squares will be applied to each of our three indexes. It will cover all 12,499 SP 500 calls, the 7151 Nasdaq 100 calls and the 7318 Russell 2000 calls. The regression equation is of the following form:

$$\vartheta_i(t) = \beta_0 + \beta_1 moneyness_i(t) + \beta_2 \cdot \tau_i(t) + \beta_3 \cdot spread_i(t) + \varepsilon_i(t) \qquad (17)$$

Regression for SP 500 options (Table 10) shows that, regardless of valuation models, all variables have a significant explanatory power at the confidence level of 1% estimation errors. In other words, the estimation errors of the three valuation models are, in part, due to moneyness, time-to-expiration or spread bias.

The magnitude of this bias differs, however, from one model to the other. The moneyness bias is the highest for the *ad-hoc* BS model. The percentage of the estimation error for this model is expected to increase by 5.347 points every time the moneyness decreases by one point. Yet, the bias of the moneyness decreases with the CEV model. Hence, the error estimation should increase by 1.517 points every time the moneyness decreases by one point. That is to say that the CEV model offers a better diffusion process than the *ad-hoc* BS model. Finally, the bias of moneyness is lower for the Kou jump- diffusion model. The error estimation of this model should increase by only 0.319 points every time the moneyness decreases by one point. This improvement is due to a better process modeling of the underlying asset, thus a better volatility estimate thanks to the introduction of jumps in addition to the diffusion process.

The bias of the residual time is much more discreet than the moneyness for all of the three models. This is due to the daily calibration which allows updating the structural parameters. Such a result is consistent with other studies that have shown that with such a calibration, the BS model was able to mimic, in an acceptable manner, the stochastic volatility models, Bakshi et al. (1997), Bates (2003), Berkowitz (2001).

The same observation is valid for the CEV model that has been able best to mitigate the time-to-expiration bias factor. This performance is due to two factors. First, the daily calibration of the model parameters has been updated daily. Then, using the process proposed by the CEV model, the volatility does not change in a purely stochastic manner but is inversely related to the price of the underlying asset, as demonstrated by several empirical studies, Heston and Nandi (2000), Jones (2003), Nandi (1998). Regression releases the same trends for Nasdaq 100 options as shown in Table 11.

Unlike the SP 500 and Nasdaq 100 options, the *ad-hoc* BS model for Russell 2000 options no longer suffers from the moneyness bias with an estimated coefficient close to zero (−0.01) and in addition insignificant at the 1% and 5% levels. This result is surprising when compared to estimates for the SP 500 (−5.347) and Nasdaq 100 (−1.778) moneyness coefficients. Also, the coefficient on the bias of the residual time (−0.292) is significantly lower, in absolute value, than the estimates found for the Nasdaq100 options (−1.208).

These results give an idea of the progress made by the *ad-hoc* BS and clearly visible in Table 12 where the model has claimed the second place to CEV model of Cox. The Kou jump-diffusion model continues meanwhile to outperform the *ad-hoc* BS and CEV models with estimated coefficients very close to zero.

6. CONCLUSION

The present work was interested in empirically validating three evaluation options models, the *ad-hoc* BS model, the Cox CEV model and the Kou jump-diffusion model using call options, negotiated during the year 2007, on the SP 500 index, the Nasdaq 100 index and the Russell 2000 index. The CEV model uses a diffusion process with the volatility which is a deterministic and inverse function of the underlying asset price. The Kou model offers meanwhile a hybrid model with a hybrid jump-diffusion process where volatility evolves in a stochastic manner. In order to perform these calculations, we must first estimate the structural parameters for all of the three models. To do so, we choose the nonlinear least squares econometric technique on cross sectional option prices.

Table 10: Regression results for the SP 500 options

Models	Parameters				$R^2_{adjusted}$
	Constant	Moneyness	Time	Spread	
Ad-hoc BS	0.233***	−5.347***	−0.207**	1.879***	0.306
	(0.023)	(0.272)	(0.098)	(0.155)	
CEV	0.085***	−1.517***	−0.113***	0.848***	0.604
	(0.004)	(0.055)	(0.019)	(0.027)	
Kou	0.023***	−0.319***	−0.128***	0.325***	0.438
	(0.002)	(0.025)	(0.009)	(0.015)	

Table 10 shows the regression results for equation (17). The endogenous variable, designated by $\vartheta_i(t)$, is the mean absolute error calculated at date (t) equal to the absolute value of the difference between the theoretical and observed prices of the i^{th} option price divided by the observed price. $moneyness_i(t)$, $\tau_i(t)$, $spread_i(t)$ are respectively the degree of moneyness, time-to-expiration and the spread relative to the i^{th} call observed at the date (t). The moneyness is equal to $(S-K.e^{-r.t})/K.e^{-r.t}$. The spread will be equal to the difference between the bid and ask price of the option divided by its mid-price. ε is the residual term. *Ad0hoc* BS is the Black-Scholes model with a daily implied volatility calibration. CEV model is the Cox CEV with a daily calibration of elasticity. KOU is the jump-diffusion model of Kou. The regression is performed for each of the 3 models using all 12,499 SP 500 calls which constitute our sample. The sample period spans 2007. The coefficient estimates appear in the first line for all three models. Figures in parentheses are standard deviations of the estimates. ***to mean that the estimate is significant to the 1% error. **to indicate that it is significant to the 5% error. Source: Authors. CEV: Constant elasticity variance, BS: Black–Scholes, SP 500: Standard and Poor 500

Table 11: Regression results for the Nasdaq 100 options

Models	Parameters				$R^2_{adjusted}$
	Constant	Moneyness	Time	Spread	
Ad-hoc BS	0.0357**	−1.778***	−1.208***	3.162***	0.522
	(0.0169)	(0.107)	(0.053)	(0.137)	
CEV	0.131***	−1.124***	−0.658***	0.774***	0.419
	(0.008)	(0.048)	(0.028)	(0.056)	
KOU	0.016***	−0.220***	−0.232***	0.469***	0.269
	(0.004)	(0.025)	(0.013)	(0.034)	

Table 11 shows the regression results for Equation (17). The endogenous variable, designated by $\vartheta_i(t)$, is the mean absolute error calculated at date (t) equal to the absolute value of the difference between the theoretical and observed prices of the i^{th} option price divided by the observed price. $moneyness_i(t)$, $\tau_i(t)$, $spread_i(t)$ are respectively the degree of moneyness, time-to-expiration and the spread relative to the i^{th} call observed at the date (t). The moneyness is equal to $(S-K.e^{-r.t})/K.e^{-r.t}$. The spread will be equal to the difference between the bid and ask price of the option divided by its mid-price. ε is the residual term. *Ad-hoc* BS is the Black-Scholes model with a daily implied volatility calibration. CEV model is the Cox CEV with a daily calibration of elasticity. KOU is the jump-diffusion model of Kou. The regression is performed for each of the three models using all 7, 151 Nasdaq 100 calls which constitute our sample. The sample period spans 2007. The coefficient estimates appear in the first line for all three models. Figures in parentheses are standard deviations of the estimates. ***To mean that the estimate is significant to the 1% error. **To indicate that it is significant to the 5% error. Source: Authors. CEV: Constant elasticity variance, BS: Black–Scholes, SP 500: Standard and Poor 500

A comparative analysis between the three models, based on the evaluation of the theoretical price of 12, 499 options on the SP 500; 7, 151 options on the Nasdaq 100 and 7, 318 options on the Russell 2000, shows a clear superiority of the Kou jump-diffusion model which vastly outperforms the two other models for the entire sample. This result shows that the implied distribution over the underlying asset is generally different from the objective distribution. The first is determined by the mood of market participants and their expectations for the future, while the second is simply based on the history of the underlying asset price without considering the psychological aspect of market participants.

Table 12: Regression results for the Russell 2000 options

Models	Parameters				$R^2_{adjusted}$
	Constant	Moneyness	Time	Spread	
Ad-hoc BS	0.029***	−0.01	−0.292***	1.472***	0.575
	(0.004)	(0.028)	(0.018)	(0.023)	
CEV	0.053***	−0.734***	−0.154***	1.418***	0.634
	(0.004)	(0.03)	(0.018)	(0.024)	
KOU	0	0.039***	−0.047***	0.436***	0.327
	(0.003)	(0.013)	(0.008)	(0.01)	

The table shows the regression results for Equation (17). The endogenous variable, designated by $\vartheta_i(t)$, is the mean absolute error calculated at date (t) equal to the absolute value of the difference between the theoretical and observed prices of the i^{th} option price divided by the observed price. $moneyness_i(t)$, $\tau_i(t)$, $spread_i(t)$ are respectively the degree of moneyness, time-to-expiration and the spread relative to the i^{th} call observed at the date (t). The moneyness is equal to $(S-K.e^{r.t})/K.e^{-r.t}$. The spread will be equal to the difference between the bid and ask price of the option divided by its mid-price. ε is the residual term. *Ad-hoc* BS is the Black-Scholes model with a daily implied volatility calibration. CEV model is the Cox CEV with a daily calibration of elasticity. KOU is the jump-diffusion model of Kou. The regression is performed for each of the 3 models using all 7, 318 Russell 2000 calls which constitute our sample. The sample period spans 2007. The coefficient estimates appear in the first line for all three models. Figures in parentheses are standard deviations of the estimates, where ***To mean that the estimate is significant to the 1% error and **To indicate that it is significant to the 5% error. Source: Authors. CEV: Constant elasticity variance, BS: Black–Scholes, SP 500: Standard and Poor 500

We also notice that the Kou model provided better results for the Nasdaq 100 and Russell 2000 index options than for the SP 500 ones, despite the fact that empirical analysis has focused on the same period. This improvement has its origin in the very nature of each of the three indexes. Thus the Nasdaq 100 is much less diversified than the SP 500 while the Russell 2000 is composed of small cap stocks. These effects of both "diversification" and "size" mean that the Nasdaq 100 and Russell 2000 have a much more unstable volatility than that of the SP 500. In other words, these two indexes have a greater probability for the realization of extreme values that is consistent with the assumptions of jump-diffusion models.

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