



Single or Menu Contracting: A Game Theory Application of the Hersanyi Model to Mudaraba Financing

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ABSTRACT

In Islamic banking, the offering of a Mudaraba contract to a privately informed agent results in adverse selection. In incentive theory, a hypothesis is that the seller, in our case the Islamic bank, may offer different menu of contracts to separate non-efficient agents from the efficient ones. To test this hypothesis, we apply a game theory approach using an incomplete information model combined with an adverse selection index. From a rational point of view a bank would like to offer a higher type contract to an efficient agent to get higher rewards. Under an asymmetric case, however, we found evidence that in some cases offering a lower type contract can result in higher social value. Menu offering is found not to be the ultimate solution for agent's types' separation.

Keywords: Mudaraba, Self-Selection Mechanism, Adverse Selection Index, Frequency of Due Diligence, Islamic Venture Capitalist, Incomplete Information

JEL Classifications: C7, G02, G24, G17

1. INTRODUCTION

The central idea in the concept of Mudaraba is that two parties, one with capital and the other with know-how, get together to carry out a project. The financier provides the capital and plays no further part in the project. Specifically, he does not interfere in its execution, which is the exclusive province of the entrepreneur. If the project ends in profit they share the profit in a pre-arranged proportion. If it results in a loss the entire loss is borne by the financier, and the entrepreneur gains no benefit out of his effort, which was his part of the investment. There are many variations of this simple model but this is the basic concept (Gafoor, 2001). In this type of financing the Mudarib may possess private information that he can use to have some informational gains. This kind of problems is called adverse selection. Our motivation starts from the fact that Mudaraba financing, due to its risks, is of less practice despite its overwhelming profits in case of success.

The last point is behind our motivation to design a mechanism of contracting that can reduce the asymmetry of information between the financier (the bank in our case) and the agent who undertakes

the project. The Harsanyi model is introduced to take into account the different types of the agent and of the financier and to see how these different types interact so that the financier can choose an optimal contract.

This research will proceed as follows:

In Section 2 we provide a review of the literature with regard to models dealing with asymmetric information. Section 3 provides the theoretical framework of our work all along with the model under both asymmetric and symmetric information case. Section 4 provides the methodology. Section 5, provides the theoretical result using a game theory framework. Section 6 provides an experimental design. Finally Section 7 will conclude with suggestion for future directions.

2. LITERATURE REVIEW

In his book "the theory of corporate finance," Tirole (2006) describes an adverse selection model in which the agent has an

informational advantage over the investors about the prospects of their projects. He has introduced an adverse selection index but did not mention how it can be used to reduce adverse selection. We derived two indices of two contracts of different risk and returns to decide on the level of due diligence needed.

To overcome the adverse selection problem, the use of dissipative signals is of great importance. For example, collateral can be used by efficient agents as a signaling mechanism of their type. This signaling method is consistent with previous research (Besanko and Thakor, 1987; Bester 1985; 1987; Chan and Kanatas, 1985). Other research also claim that banks can use collateral in debt contracts to overcome information asymmetries, in particular arising from ex-ante adverse selection (Berger et al., 2011).

Collateral as a self-selection mechanism was studied in Bester (1985). The paper assumes that banks decide upon the rate of interest and the collateral of their creditors simultaneously rather than separately. Therefore, it becomes possible to use different contracts as a self-selection mechanism. In our research we are not using collateral as this is, from Shari'a perspective, not permissible under Mudaraba contracts. However, we have applied two different contracts as a self-selection mechanism. At the same time we have used different sharing ratios, instead of different interest rates in conventional system.

However, and inconsistent with these findings, the use of warranties (collateral, in the conventional system, seems to be no more than a limited control mechanism to overcome the agency problem (Manove and Padilla, 1999).

Low job protection can also be made similar to a high pledged collateral i.e. a confident manager will demand a high reward in case of success but also signs for a low job protection in case of failure. This is consistent with previous research as in Subramanian et al. (2002).

Short term maturities: Using short-term contracts may be used as signal for the quality of the entrepreneur. Diamond (1991; 1993) shows that efficient entrepreneurs use short term contracting to show that they are confident about the prospects of their projects. This is consistent with other research. In fact, Landier and Thesmar (2004) show that in a competitive credit market, optimistic (confident) entrepreneurs opt for shorter debt maturities than realistic entrepreneurs, to signal that they are unlikely to face difficult circumstances.

Underpricing: This occurs when the entrepreneur reduce their compensation in order to obtain financing. According to Tirole (2006), when the two types become more similar, the efficient borrower must underprice more (i.e., accept a lower compensation in order to make the issue unappealing to an inefficient borrower).

In other research other mechanisms were studied to overcome the problem of adverse selection:

Information sharing: Previous work has shown how information sharing promotes credit market efficiency with benefits for the

whole economy. In fact, credit bureaus have been shown to decrease adverse selection (Jappelli and Pagano, 1993) and increase efforts from borrowers (Padilla and Pagano, 1997; Padilla and Pagano, 2000). At the same time, information sharing may be used to reduce competition between banks (Gehrig and Stenbacka, 2007). Also, information sharing is more likely if borrower mobility is higher (Jappelli and Pagano, 1993) and if asymmetric information problems are more important (Brown and Zehnder, 2010). Empirical research has shown that, information sharing is correlated with higher access to credit (Jappelli and Pagano, 1993), especially in developing countries with inefficient creditor rights (Djankov et al. 2007; Brown et al., 2009), but lower lending to low-quality borrowers (Hertzberg et al., 2011).

However, some negative points arise from information sharing. Gehrig and Stenbacka (2007), show that information sharing reduces the returns from establishing banking relationships. This, therefore, weakens banks competition. Therefore, information sharing is a mechanism to redistribute surplus from talented entrepreneurs to banks but, due to the implied anti-competitive effects, reduce the social returns of information sharing.

In relation to Islamic venture capital, few models have been developed to solve adverse selection problem and information asymmetry (Jouahr and Mehr, 2012). One of the main reasons is the weakness of practice of Mudaraba in Islamic finance due to misreporting risk. According to Al-Suwailem (2006) misreporting of risk happens when the agent announces losses while the project is making profits. Based on a survey by Khalil et al. (2002), misreporting is the prime reason why Islamic banks are not applying Mudaraba on the asset side of their balance sheets.

Greening and Iqbal (2007) explain that the significant investment risk of Mudaraba is reflected in its small share in total assets.

Another study shows the agent has a tendency to overestimate the quality of his activities (Al-Jarhi and Iqbal, 2001). For example he can overestimate the probability of success of his project in order to get financing (Manove et al., 2001) or declare, ex-ante, a higher expected profit in order to induce the Islamic Venture Capitalist (IVC) to reduce its profit sharing ratio.

In order to solve the selection adverse problem, some researchers propose that a contract with a predetermined profit sharing ratio induces the entrepreneurs to behave honestly (Khan, 1985). This is due to the fact that the return of their project depends on their actions (Sarker, 1999). Sarker (1999) proposes the offering of performance based shares and/or reserves plans when profits are achieved. In fact one of the critical issues is to determine the sharing ratio that will solve the adverse selection. These methods are, therefore, considered as mechanisms of compensation rather than prevention methods against adverse selection.

Bacha (1997) proposes that a fair distribution of profit and risk can be achieved through some financing mechanisms like "Mezzanine" and "Vertical Strip Financing."

Since mezzanine financing is usually provided to the borrower very quickly with little due diligence on the part of the lender and little or no collateral on the part of the borrower, this type of financing is aggressively priced to the extent that it might put off efficient entrepreneurs from undertaking the project. Karim (2000) proposes that the entrepreneur’s participation in the capital and the submission of a warranty can resolve the adverse selection problem. In our case we are dealing with Mudaraba financing in which case there is no participation in the capital by the entrepreneur.

Also the delivery of a warranty against performance is not permissible under the Shari’a Law (AAOIFI, 2003). However, the recourse to a warranty is permissible if there is a proof of negligence or non-respect of the contract terms by the entrepreneur. This last point was made use of in this research under the misreporting penalty.

Shaikh (2011) argument is that the agency problem is based on an unfair distribution of returns if the project fails. Taking into consideration the risks related to a project, the IVC may demand a higher sharing ratio. This however may result in less motivation of the entrepreneur and therefore a lower project returns.

In dealing with moral hazards in Mudaraba financing, Ouidad (2013) suggested higher incentives in case the project is risky and lower compensation schemes in case the project is not risky. This concept is also tested for in our research.

Many researchers have tried to develop an optimal sharing ratio under symmetrical information.

There exist some differences between venture capital financing in Islamic finance and conventional finance.

First, the problem of adverse selection is more important in Islamic Mudaraba than in Musharaka or conventional banking as the entrepreneur does not contribute with a capital (Jouahr and Mehr, 2012). Chapra and Khan (2000) consider that Mudaraba is the most risky in comparison with the rest of modes of financing. Second, the Islamic bank does not intervene in the project and therefore assume all losses in case the project fails (Jouahr and Mehr, 2012).

Third, from, the above, we conclude that the entrepreneur is induced to undertake some decisions that serve only his own interests.

Fourth, the projects undertaken by the agent has to be in conformity with the Shari’a. This is, however, not a requirement in conventional finance.

Al-Suwailem (2006) argues that, there should be a higher due diligence from the part of the Islamic institutions as compared to conventional banks. In our case we propose the supplying of funds to be conditioned on the acceptance by the agent of a certain number of due diligence actions. We argue that the acceptance of such condition can signal which type of agents the bank is addressing to.

This research will proceed as follows:

In a previous paper (ELFakir and Tkiouat, 2015a) of ours we have proposed an incentive scheme to deal with moral hazards. This scheme allows for higher social value and more freedom to the agent in terms of negotiating the profit sharing. However, the model contrary to the current one, does not provide for two contracts type. In another paper, (ELFakir and Tkiouat, 2015b), we proposed the use of an effort based contract instead of an output based contract. We found evidence that an effort based contract offer better compensation to the agent in the form of lower sharing ratio to the financier. This result has two important Islamic implications. First it emphasizes the sentiment of altruism which the financier shows by taking a smaller sharing ratio. Second it emphasizes the sentiment of positive reciprocity which the agent exhibits by providing high effort.

3. THE MODEL

Consider an Islamic bank and a client (agent) who are about to engage in a Mudaraba Contract. The bank exists in a competitive market. Initially nature draws a profile (B;A) with prob $\{ (B;A) \} = \theta_{BA}$ of the type of the Islamic bank (high expertise or low expertise) $B \in \{H, L\}$ and the type of the agent (efficient type or non-efficient type) $A \in \{E, N\}$. Both participants are risk neutral. We can then provide the following probability matrix for the participant’s profiles.

$$\begin{matrix} & E & N \\ H & \theta_{HE} & \theta_{HN} \\ L & \theta_{LE} & \theta_{LN} \end{matrix}$$

The bank can offer a single type out of two contracts C_i such that $i \in \{1, 2\}$ at a time or offer a menu of two types of contracts where the agent is left to choose between the two contracts. Hereafter, we refer to C_1 and C_2 as the high type and low type contract with high expected returns R_1 and low expected returns R_2 respectively.

Each contract C_i has a probability of success $P(S_{C_i}|A)$ depending on the type of the agent “A” who undertakes it. We assume that an efficient agent has higher probability of success in both contracts compared to a non-efficient agent who can only perform well in a low type contract.

Both parties should agree on a sharing ratio “ t_i ” given to the agent where $i \in \{1, 2\}$. Put it in another way “ $1-t_i$ ” represents the price at which the bank negotiated to sell contract C_i in exchange for the agent work $W(A)$. The negotiated ratio should satisfy two constraints:

$$t_i R_i \geq W(A) \tag{1}$$

$$(1-t_i) \cdot R_i - D_i = U_i \tag{2}$$

Where U_i and D_i represents, respectively, the utility and the cost of expertise to the bank from undertaking a project “ i ”.

The last condition holds because the bank is assumed to exist in a competitive market and therefore its aim is to breakeven.

At date 1, the bank has three choices to offer depending on the bank's Type "B."

(1) $\{C_1(t_{(1/B)})\}$ or (2) $\{C_2(t_{(2/B)})\}$ (3) or a menu $\{C_1(t_{(1/B)}); C_2(t_{(2/B)})\}$

At date 2, the agent makes a decision of whether to accept the offer under the three scenarios.

All model parameters are common knowledge except the profile types. This issue is treated using the Harsanyi model as modified by Mertens–Zamir.

Mertens–Zamir modified the Harsanyi model by introducing the following notations:

N : As the number of players

K : The state of nature

Y : The set of states of the world (ω)

ω : The state of the world which combines the state of nature and the beliefs of each player about the state of nature.

Specifically let us consider two scenarios: Symmetric case and the asymmetric case. Under each case a social value (SV) is calculated as the sum of the expected profit of the bank and the expected profit of the agent:

$$SV = E\pi (B) + E\pi (A) \tag{3}$$

3.1. Scenario 1: The Symmetric Case

In the symmetric case all the bank and the agent are informed about the type of their opponents. i.e., $\text{prop}\{(B;A)\} = \theta_{BA} = 1$. In this case we can describe this game as a Harsanyi model by recognizing that the only state of nature k is a common knowledge. The Harsanyi model as modified by Mertens–Zamir is:

$N = \{B, A\}$

$K = k$

$Y = \{\omega\}$

$\omega = k, (1), (1)$

3.2. Scenario 2: The Asymmetric Case

In the asymmetric case both parties have private information about their types; i.e. both parties do not know the realization of $\{(B; A)\}$

$N = \{B, A\}$

$K = \{k_{HE}, k_{HN}, k_{LE}, k_{LN}\}$

$Y = \{\omega_{HE}, \omega_{HN}, \omega_{LE}, \omega_{LN}\}$

$\omega_{HE} = k_{HE} : (P(E/H), P(N/H), 0, 0) (P(H/E), 0, P(L/E), 0)$

$\omega_{HN} = k_{HN} : (P(E/H), P(N/H), 0, 0) (0, P(H/N), 0, P(L/N))$

$\omega_{LE} = k_{LE} : (0, 0, P(E/L), P(N/L)) (P(H/E), 0), (P(L/E), 0)$

$\omega_{LN} = k_{LN} : (0, 0, P(E/L), P(N/L)) (0, P(H/N), 0, P(L/N))$

Since the bank exists in a competitive market, it will set the sharing ratio to break even depending on its type.

4. METHODOLOGY

We analyze our model under two cases. The first case considered is when the bank is of high type and the second case when the bank is of low type. Under each case, we consider each contract

on its own. We provide a framework for the calculation of the sharing ratio of the Mudaraba contract. The calculation of such ratio is done via the calculation of a measure of adverse selection (MAS) and respecting the break-even-condition of the bank. The break-even of the bank means that the minimum return the project can give should be at least equal to the bank opportunities cost.

We make use of such ratios to calculate the expected profits of each contract to the agent and the bank under each bank's type (low or high).

We plug the expected profits of the agent and the bank in a payoff matrix using a game theory framework under each case and under each contract.

To understand the usefulness of the Harsanyi model, we provide a numerical demonstration. In this demonstration we look for the best response of the agent to the offering of the bank in terms of Contract 1, Contract 2, or a menu. Base on this expected best response, the bank can decide which contract to offer. If the bank is a socially oriented entity it should favor the contract that gives the highest SV. If the bank is profit oriented it should favor the contract that gives it the highest profit.

5. RESULTS

5.1. Sharing Ratio if the Bank is of Type H

We calculate the sharing ratio of the Mudaraba contract for Contract 1 and Contract 2 when the bank is of high type

5.1.1. Contract 1

In this case the expected profit from contract "1" to the bank is given as:

$$E\pi_1(B/H) = P(E/H).(P(S1|E).(1-t_{(1/H)}).R_1 - (1-P(S1|E)).I) + P(N/H). (P(S1|N).(1-t_{(1/H)}).R_1 - (1-P(S1|N)).(1-LV\%).I) - MAS_{1/H}.C.I$$

Where,

$$MAS_{1/H} = P(N / H). \frac{(P(S1|E) - P(S1|N))}{P(S1|E)} \tag{4}$$

Represent the adverse selection index for contract 1 when the bank is of high type.

$P(N/H)$: Probability that the agent is non-efficient given that the bank is of high type.

C : Represents the cost of expertise as a percentage of the capital. In our case then: $MAS_{1/H}.C.I$ represents the cost of the bank expertise when it is of the high type.

$LV\%$: Represents the estimated percentage liquidation value of the project capital in case of failure.

Modifying and having the condition (2) of the bank breaking even we have:

$$E\pi_1(B/H) = (P(S1|H).(1-t_{(1/H)}).R_1 - (1-P(S1|H)).I) - MAS_1.C.I = \bar{O}.r.I \tag{5}$$

Where \bar{O} a risk premium due to the high type of the Contract 1.

We can then extract the sharing ratio:

$$t_{(1/H)} = 1 - \frac{\bar{C}.r.I + (1 - P(S1|H)).(1 - LV\%).I + MAS1.C.I}{(P(S1|H)).R1} \quad (6)$$

The agent's profit from Contract 1 if he/she accepts the contract is:

$$E\pi_1(A/H) = t_{(1/H)}.R_1 - w(A)$$

5.1.2. Contract 2

Similarly we can model the expected profit for Contract 2:

$$E\pi_2(B/H) = \frac{(P(S2|H)).(1 - t_{(2/H)}).R1 - (1 - P(S2|H)).(1 - LV\%).I - \alpha}{I - MAS_{2/H}.C.I = r.I} \quad (7)$$

Where similarly to Contract 1:

$$MAS_{2/H} = P(N/H) \cdot \frac{(P(S2|E) - P(S2|N))}{P(S2|E)} \quad (8)$$

Represent the adverse selection index for Contract 2 when the bank is of high type.

$P(N/H)$: Probability that the agent is non-efficient given that the bank is of high type.

We can then extract the sharing ratio for Contract 2 as:

$$t_{(2/H)} = 1 - \frac{r.I + (1 - P(S2|H)).(1 - LV\%).I + MAS(2/H).C.I}{(P(S1|H)).R1} \quad (9)$$

The agent's profit if he/she accepts the contract is:

$$E\pi_2(A/H) = t_{(2/H)}.R_2 - w(A) \quad (10)$$

5.2. Sharing Ratio if the Bank is of Type L

Similarly to the high type case we can model the expected profit for each contract with the exception that we add a cost of expertise multiplier " α " to take into account the increased cost of expertise when the bank is of low type. For example the bank may have recourse to an external source of expertise.

5.2.1. Contract 1

$$E\pi_1(B/L) = \frac{(P(S1|L)).(1 - t_{(1/L)}).R1 - (1 - P(S1|L)).(1 - LV\%).I - \alpha}{MAS_{1/L}.C.I = \bar{C}.r.I} \quad (11)$$

Where,

$$MAS_{1/L} = P(N/L) \cdot \frac{(P(S1|E) - P(S1|N))}{P(S1|E)}$$

Represent the adverse selection index for Contract 1 when the bank is of low type.

$P(N/L)$: Probability that the agent is non-efficient given that the bank is of low type.

We can then extract the sharing ratio:

$$t_{(1/L)} = 1 - \frac{\bar{C}.r.I + (1 - P(S1|L)).(1 - LV\%).I + MAS(1/L).C.I}{(P(S1|1)).R1} \quad (12)$$

The agent's profit if he/she accepts the contract is:

$$E\pi_1(A/L) = t_{(1/L)}.R1 - w(A) \quad (13)$$

5.2.2. Contract 2

$$E\pi_2(B/L) = \frac{(P(S2|L)).(1 - t_{(2/L)}).R1 - (1 - P(S2|L)).(1 - LV\%).I - \alpha}{MAS_{2/L}.C.I = r.I} \quad (14)$$

Where:

$$MAS_{2/L} = P(N/L) \cdot \frac{(P(S2|E) - P(S2|N))}{P(S2|E)} \quad (15)$$

Represent the adverse selection index for Contract 2 when the bank is of low type.

$P(N/L)$: Probability that the agent is non-efficient given that the bank is of low type.

We can then extract the sharing ratio for Contract 2 as:

$$t_{(2/L)} = 1 - \frac{r.I + (1 - P(S2|L)).(1 - LV\%).I + \alpha.MAS(2/L).C.I}{(P(S1|L)).R1} \quad (16)$$

The agent's profit if he/she accepts the contract is:

$$E\pi_2(A/L) = t_{(2/L)}.R2 - w(A) \quad (17)$$

We can then provide a game theory approach where under each type of the bank, the bank has the choice between single contracting (one type of contracts is on offer) or menu contracting (agent can choose between Contract 1 or Contract 2).

5.3. Game Theory Approach

We provide a game theoretical framework under each type of the bank (either high or low type). The bank has three strategies: Offer Contract 1, Contract 2 or a menu (i.e. the agent has the right to choose between the contracts). The agent on the other hand, can either accept or reject the contracts. Each payoff cell has two payoffs. The first payoff in a given cell is that of the bank while the second is that of the agent. Each payoff considers whether we are in the case of a bank's high type case (H) or low type (L).

5.3.1. Game approach if the bank is of Type H

	Accept	Refuse
Contract 1	$(E\pi_1(B/H); E\pi_1(A/H))$	$(\sigma.r.I; W(A))$
Contract 2	$(E\pi_2(B/H); E\pi_2(A/H))$	$(r.I; W(A))$
Menu	$(E\pi_1(B/H) \text{ if } \text{Max}(E\pi_1(A/H); E\pi_2(A/H)) = E\pi_1(A/H))$	$(\sigma.r.I; W(A))$
	$(E\pi_2(B/H) \text{ if } \text{Max}(E\pi_1(A/H); E\pi_2(A/H)) = E\pi_2(A/H))$	

5.3.2. Game approach if the bank is of Type L

	Accept	Refuse
Contract 1	$(E\pi_1 (B/L); E\pi_1 (A/L))$	$(\sigma.r.I; W (A))$
Contract 2	$(E\pi_2 (B/L); E\pi_2(A/L))$	$(r.I; W (A))$
Menu	$(E\pi_1 (B/L) \text{ if } \text{Max} (E\pi_1 (A/L); E\pi_2 (A/L))=E\pi_1 (A/L)$ $(E\pi_2 (B/L) \text{ if } \text{Max} (E\pi_1 (A/L); E\pi_2 (A/L))=E\pi_2 (A/L))$	$(\sigma.r.I; W (A))$

6. EXPERIMENTAL DESIGN

At this point, to decide whether single contracting or menu contracting is better for the bank is not a one off decision but is a matter that takes into consideration all the parameters in the model which are not necessarily under the control of the bank. So we can provide a numerical demonstration for illustration.

Base data:

Consider the following:

$P (S1 E)$	80%	$P (S2 E)$	90%	$W (E)$	400	LV	30%
$P (S1 N)$	10%	$P (S2 N)$	60%	$W (N)$	200		
$R1$	6000	$R2$	4000	C	200%		
σ	1.5	r	500%	I	10000		

Where, as before:

$P(S1|E), P(S1|N)$: Probability of success of Contract 1 if undertaken by an efficient or non-efficient agent respectively.

$P(S2|E), P(S1|N)$: Probability of success of Contract 2 if undertaken by an efficient or non-efficient agent respectively.

$R1, R2$: Rate of return of Contract 1 and 2 respectively.

σ : Risk premium of Contract 1.

$W(E), W(N)$: Wage of the efficient and non-efficient agent if they do not undertake any of the contracts.

I : The required fund to undertake the project.

C : Represents the cost of expertise to the bank as a percentage of the capital.

6.1. Scenario 1: The Symmetric Case

Under the symmetric information both player have complete beliefs about each others, therefore, applying the Harsanyi Model, each profile at a time has a probability of 100%. i.e., the bank (agent) is 100% sure that the agent (the bank) is of a specific type.

6.1.1. State of nature K_{HE}

$$N = \{B, A\}$$

$$K = \{k_{HE}\}$$

$$Y = \{\omega_{HE}\}$$

$$\omega_{HE} = k_{HE}; (1) (1)$$

The profile probability matrix is:

	E	N
H	100%	0
L	0	0

The intermediate results of this case:

$Ps (1/H)$	80%	$Ps (2/H)$	90%
$t (1/H)$	55%	$t (2/H)$	67%
$MAS (1/H)$	0%	$MAS (2/H)$	0%

Where,

$Ps(1/H), Ps(2/H)$ are the calculated probabilities of success of Contract 1 and 2 respectively given that the bank is of high type.

The payoff matrix along with the best response of the agent, the resulting profits and the final SV are as follows:

Agent	Agent		Best action of action	Bank's profit	Agent's profit	Social value
	Refuse	Accept				
Bank C1	$(750; 2650)$	$(750; 400)$	Accept	750	2650	3400
C2	$(500; 2400)$	$(500; 400)$	Accept	500	2400	2900
Menu	$(750; 2650)$	$(750; 400)$	Accept	0	2650	2650

Whether, the bank is socially oriented or profit oriented, it should offer Contract 1 as it gives the highest profit to the bank and the highest SV.

6.1.2. State of nature K_{HN}

$$N = \{B, A\}$$

$$K = \{k_{HN}\}$$

$$Y = \{\omega_{HN}\}$$

$$\omega_{HE} = k_{HN}; (1) (1)$$

The profile probability matrix is:

	E	N
H	0	100%
L	0	0

The intermediate results of this case:

$Ps (1/H)$	10%	$Ps (2/H)$	60%
$t (1/H)$	-1104%	$t (2/H)$	-40%
$MAS (1/H)$	88%	$MAS (2/H)$	33%

The payoff matrix along with the best response of the agent, the resulting profits and the final SV are as follows:

Agent	Agent		Best action of action	Bank's profit	Agent's profit	Social value
	Refuse	Accept				
Bank C1	$(750; -6625)$	$(750; 200)$	Refuse	750	200	1500
C2	$(500; -967)$	$(500; 200)$	Refuse	500	200	700
Menu	$(750; -967)$	$(750; 200)$	Refuse	0	200	200

In this case when the bank is of high type and the agent is of non-efficient type, none of the contract on offer will be accepted by the agent as all give him a lower value than his opportunity cost.

6.1.3. State of nature K_{LE}

$$N = \{B, A\}$$

$$K = \{k_{LE}\}$$

$$Y = \{\omega_{LE}\}$$

$$\omega_{LE} = k_{LE} \quad (1) \quad (1)$$

The profile probability matrix is:

	E	N
H	0	0
L	100%	0

The intermediate results of this case:

MAS (1/L)	0%	MAS (2/l)	0%
Ps (1/L)	80%	Ps (2/L)	90%
t (1/L)	55%	t (2/L)	67%

The payoff matrix along with the best response of the agent, the resulting profits and the final SV are as follows:

Agent	Agent		Best action of action	Bank's profit	Agent's profit	Social value
	Refuse	Best action				
C1	(750;2250)	(750;400)	Accept	750	2250	3000
Bank C2	(500;2000)	(500;400)	Accept	500	2000	2500
Menu	(750;2250)	(750;400)	Accept	750	2250	3000

In this case the bank is indifferent between offering Contract 1 or a menu. The rationality principle, dictates that the agent will choose Contract 1 as it offers him the highest payoff.

6.1.4. State of nature K_{LN}

$$N = \{B, A\}$$

$$K = \{k_{LN}\}$$

$$Y = \{\omega_{LN}\}$$

$$\omega_{LN} = k_{LN} \quad (1) \quad (1)$$

The profile probability matrix is:

	E	N
H	0	0
L	0	100%

The intermediate results of this case:

MAS (1/L)	88%	MAS (2/l)	33%
Ps (1/L)	10%	Ps (2/L)	60%
t (1/L)	1133%	t (2/L)	-43%

The payoff matrix along with the best response of the agent, the resulting profits and the final SV are as follows:

Agent	Agent		Best action of action	Bank's profit	Agent's profit	Social value
	Refuse	Best action				
C1	(750;-7000)	(750;200)	Refuse	750	200	950
Bank C2	(500;-1234)	(500;200)	Refuse	500	200	700
Menu	(500;-1234)	(500;200)	Refuse	500	200	700

Under this contract no contract is to be offered as the agent is not gaining above his opportunity cost.

6.2. Scenario 2: The Asymmetric Case

Under this case both player do not have complete information about their opponents. Therefore each one assign a certain probability about the state of the world depending on his type and what he believes (with a certain probability) about the other opponents.

6.2.1. All profiles have the same probabilities of occurrence

The profiles' probability matrix is:

	E	N
H	25%	25%
L	25%	25%

The intermediate results of this case:

Ps (1/H)	45%	Ps (2/H)	75%	MAS (1/L)	44%	MAS (2/l)	17%
Ps (1/L)	45%	Ps (2/L)	75%	t (1/H)	-74%	t (2/H)	24%
MAS (1/H)	44%	MAS (2/H)	17%	t (1/L)	-77%	t (2/L)	23%

The payoff matrix along with the best response of the agent, the resulting profits and the final SV are as follows:

- If bank is of Type H:

Agent	Agent Efficient		Best action of agent
	Accept	Refuse	
C1	(6846;2650)	(-3533;400)	<input type="checkbox"/> Refuse
Bank C2	(2007;2400)	(860;400)	<input type="checkbox"/> Accept
Menu	(2007;2650)	(860;400)	<input type="checkbox"/> Accept

Agent	Agent Non – Efficient		Best action of agent
	Accept	Refuse	
C1	(-5346;-422)	(750;200)	Refuse
Bank C2	(-1007;574)	(500;200)	<input type="checkbox"/> Accept
Menu	(-1007;574)	(750;200)	<input type="checkbox"/> Accept

	Bank's profit	Agent's profit	Social value
C1	750	300	1050
C2	500	716	1216
Menu	0	716	716

In this case, a selfish behavior of the bank dictates offering Contract 1. Yet a more socially concerned bank can offer Contract 2. The offering of a menu in this case is not one of the options as it results in the lowest SV and in no profits to the bank.

- If bank is of Type L

Agent	Agent Efficient		Best action of agent
	Accept	Refuse	
C1	(6914;-4089)	(750;400)	Refuse
Bank C2	(2013;420)	(500;400)	Accept
Menu	(2013;420)	(750;400)	Accept

		Agent Non-Efficient		
		Accept	Refuse	Best action of agent
Bank	C1	(-5514; -661)	(750; 200)	Refuse
	C2	(-1013; 347)	(500; 200)	Accept
	Menu	(-1013; 347)	(500; 200)	Accept

	Bank's profit	Agent's profit	Social value
C1	750	300	1050
C2	500	383	883
Menu	500	383	883

Under, this case, the offering of Contract 1 results in the highest possible payoff to the bank, it also results in the highest possible SV.

6.2.2. Only high type bank exist with higher probability of efficient agent existing

The profiles' probability matrix is:

	E	N
H	60%	40%
L	0%	0%

Ps (1/H)	52%	Ps (2/H)	78%
t (1/H)	-34%	t (2/H)	34%
MAS (1/H)	35%	MAS (2/H)	13%

The final result of this case:

		Agent Efficient		
		Accept	Refuse	Best action of agent
Bank	C1	(4961; -1631)	(750; 400)	Refuse
	C2	(1658; 1215)	(500; 400)	Accept
	Menu	(1658; 1215)	(750; 400)	Accept

		Agent Non-Efficient		
		Accept	Refuse	Best action of agent
Bank	C1	(-5566; -204)	(750; 200)	Refuse
	C2	(-1237; 810)	(500; 200)	Accept
	Menu	(-1237; 810)	(750; 200)	Accept

	Bank's profit	Agent's profit	Social value
C1	750	300	1070
C2	500	716	1533
Menu	0	716	1053

Under this case, a profit oriented bank will offer Contract 1 as it gives it the highest profits. A socially oriented bank will offer Contract 2 as it offers the highest SV.

6.2.3. Only high type bank exist with higher probability of non-efficient agent existing

The profiles' probability matrix is:

	E	N
H	40%	60%
L	0%	0%

Ps (1/H)	38%	Ps (2/H)	72%
t (1/H)	-128%	t (2/H)	13%
MAS (1/H)	53%	MAS (2/H)	20%

The final result of this case:

		Agent Efficient		
		Accept	Refuse	Best action of agent
Bank	C1	(9432; -6137)	(750; 400)	Refuse
	C2	(2385; 475)	(500; 400)	Accept
	Menu	(2385; 475)	(750; 400)	Accept

		Agent Non-Efficient		
		Accept	Refuse	Best action of agent
Bank	C1	(-5083; -767)	(750; 200)	Refuse
	C2	(-757; 317)	(500; 200)	Accept
	Menu	(-757; 317)	(750; 200)	Accept

	Bank's profit	Agent's profit	Social value
C1	750	280	1030
C2	500	380	880
Menu	0	380	380

In this case, the offering of Contract 1 is refused by the agent whether he is efficient or non-efficient. The bank can make a compromise by offering Contract 2 giving the bank an equivalent value of its opportunity cost. This also results in a better payoff to the agent.

6.2.4. Only low type bank exists with higher probability of efficient agent existing

The profiles' probability matrix is:

	E	N
H	0%	0%
L	60%	40%

MAS (1/L)	35%	MAS (2/L)	13%
Ps (1/L)	52%	Ps (2/L)	78%
t (1/L)	-36%	t (2/L)	33%

The payoff matrix along with the best response of the agent, the resulting profits and the final SV are as follows:

		Agent Efficient		
		Accept	Refuse	Best action of agent
Bank	C1	(4999; -2138)	(750; 400)	Refuse
	C2	(1662; 785)	(500; 400)	Accept
	Menu	(1662; 785)	(750; 400)	Accept

7. CONCLUDING REMARKS

In this paper we have tried to apply the Hersanyi model to Mudaraba mode of financing. We have identified two types of banks and two types of clients. We found evidence that menu contracting might not be the best solution to achieve a higher SV. We found evidence that, even under a higher probability of efficient agent existing, the bank is not better off offering a higher type contract. The use of the Hersanyi model is proved to be very useful in our case as the banker can decide which type of contract to offer given its type and its beliefs about its clients (agents).

An extension of this model can involve the use of multiple agents rather than two. One such additional agent can involve another bank which can enter as competitor against the existing bank in our case. We can then infer how such additional competing bank can influence the decision of our existing bank in terms of single or menu contracting.

Another point we have mentioned is the compromise between banks profitability and SV, the latter of which one of the supposedly characteristics of an Islamic bank. We have found cases that a bank can offer a contract which is profit optimal, yet socially suboptimal. An empirical research is needed to assess to what extent Islamic banks have favored socially optimal projects over purely profit optimal projects.

Our model is used as a one stage game which if a contract is offered can go for a one period of time. The model can be extended as a repeated game over two or multiple period of times. The extended model can test if one type of contracting is optimal over a one period of time can still be feasible or optimal over multiple period of times.

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		Agent Non-Efficient		
		Accept	Refuse	Best action of agent
Bank	C1	(-5623;-417)	(750;200)	Refuse
	C2	(-1243;590)	(500;200)	Accept
	Menu	(-1243;590)	(750;200)	Accept
		Bank's profit	Agent's profit	Social value
C1	750	320	1070	
C2	500	707	1207	
Menu	500	707	1207	

In this case the offering of Contract 1 is to be refused by the agent as it results in getting lower than his opportunity cost. In this case the bank is better off offering Contract 2. The offering of a menu in this case is meaningless since a rational agent would not choose Contract 1.

6.2.5. Only low type bank exists with higher probability of non-efficient agent existing

The profiles' probability matrix is:

	E	N
H	0%	0%
L	40%	60%

MAS (1/L)	53%	MAS (2/l)	20%
Ps (1/L)	38%	Ps (2/L)	72%
t (1/L)	-132%	t (2/L)	12%

The payoff matrix along with the best response of the agent, the resulting profits and the final SV are as follows:

		Agent Efficient		
		Accept	Refuse	Best action of agent
Bank	C1	(9548;6758)	(750;400)	Refuse
	C2	(2395;25)	(500;400)	Refuse
	Menu	(2395;25)	(500;400)	Refuse
		Agent Non-Efficient		
		Accept	Refuse	Best action of agent
Bank	C1	(-5115;-995)	(750;200)	Refuse
	C2	(-763;83)	(500;200)	Refuse
	Menu	(-763;83)	(500;200)	Refuse
		Bank's profit	Agent's profit	Social value
C1	750	280	1030	
C2	500	280	780	
Menu	500	280	780	

In this case, none of the contracts on offer is to be accepted by the agent as they result in a lower value than the agent opportunity cost.

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