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Through-the-cycle to Point-in-time Probabilities of Default Conversion: Inconsistencies in the Vasicek Approach

L. J. Basson, Gary Van Vuuren*

School of Economics and Finance, University of the Witwatersrand, 1 Jan Smuts Ave, Braamfontein, Johannesburg, 2000, South Africa. *Email: vvgary@hotmail.com

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ABSTRACT

While regulators generate and advocate the use of through the cycle (TtC) probabilities of default (PDs) for regulatory capital calculations, accounting standards (such as IFRs9) require organisations to use point in time (PiT) PDs. TtC PDs are based on long-term average conditions and do not adequately capture current credit risk conditions, underestimating credit losses during economic downturns or periods of financial stress. PiT PDs reflect the specific risk conditions prevailing at a given moment in time and provide a more granular assessment of credit risk. While many techniques measure PiT PDs directly, mathematical approaches also exist which convert TtC PDs into PiT PDs. PiT PDs are also routinely forecasted, projected into the future to allow estimation of the present value of future possible credit-related losses. Vasicek's (1987) model is in common use for this purpose. Using a stylistic range of possible input values for Vasicek's model, loan credit quality is found to be differentially affected (improving for some and deteriorating for others) for some of these values. This is counterintuitive and reflects a functional flaw in the model.

Keywords: Vasicek, Point-in-time, Through-the-cycle, Probability of default, IFRS9
JEL Classifications: C3, C5, G1, M41.

1. INTRODUCTION

The International Financial Reporting Standards 9 (IFRS 9) accounting standards, introduced by the International Accounting Standards Board (IASB) in 2014 and effective from 2018, require banks to account for provisions based on the principle of anticipated or expected credit losses (ECL). This principle is both forward-looking and point-in-time (PiT), mandating that financial institutions use all available information on obligor exposures, including data on current and expected macroeconomic conditions to manage the provision procyclicality. With the installation of IFRS 9, measuring the ECL has become the basis for determining the amount that a financial institution must hold to act as a buffer to protect against potential impairments (IASB, 2003; 2014).

Loan exposures are allocated to one of three stages. Stage 1 comprises performing loans (low-risk exposures at valuation which have not experienced material credit quality deterioration since their origination), Stage 2 exposures have experienced a (loosely defined) significant increase in credit risk (SICR) since origination, and Stage 3 loans include all non-performing exposures. For Stage 1 exposures, the ECL is calculated over a 1-year horizon, while Stage 2 and 3 exposure ECLs are measured over the remaining lifetime of the loan. Allocating loans to stages requires an understanding of the evolution of macroeconomic variables and economic conditions for the full life of the loan which may extend significantly beyond 1 year. This forward-looking information (FLI), sometimes incorporated through judgment and expert credit assessments, must thus be included in the modelling methodology (Rhys et al., 2016). IFRS 9 also mandates that financial institutions consider two additional scenarios (in addition to the baseline, or expected, scenario); one pessimistic and one optimistic, with the final ECL calculated as a probability-weighted average of the ECLs determined under each scenario.

Measuring ECL requires an assessment of loan credit quality. This is accomplished through the estimation of a loss given default (LGD), an exposure at default (EAD), a suitable discount rate and the loan's probability of default (PD) at origination, at the reporting or valuation time, and its expected path over the loan's lifetime, to measure the present value of the ECL. Among the approaches to estimate not only the 1-year but also lifetime PDs include survival analysis (Witzany, 2017) and rating transition matrix modelling (Miu and Ozdemir, 2017) with the latter regarded as being more practical as it accounts for not only outright defaults but also rating transitions (upgrades or downgrades) and hence permitted loan allocation into the various IFRS 9 stages.

IFRS 9 encourages the use of both historical and forward-looking information to estimate PDs, but the specific method used varies depending on the availability and reliability of data, the complexity of the financial instrument, and the institution's risk management practices. IFRS 9 also requires regular monitoring and updates to PD estimates based on new information and changes in circumstances.

The IFRS 9 framework deliberately avoids a prescriptive determination of PDs, but common approaches estimate these based on historical data on credit losses and default rates associated with similar financial instruments. These are used to derive credit rating transition matrices that quantify the likelihood of default for different categories of borrowers or financial instruments. Such analyses are typically also regularly conducted by credit rating agencies (CRAs) such as Fitch Ratings and Moody's and lending institutions are allowed to make use of these historical PDs. The PDs thus determined take a broad, and more stable, view of credit risk and are intended to represent the average credit risk of a borrower or financial instrument over a complete economic cycle, comprising both upturns and downturns. TtC PDs are less influenced by short-term fluctuations and focus on capturing the long-term credit performance of the borrower. They provide a more smoothed-out estimate of credit risk and are less sensitive to immediate changes in economic conditions. As such, these are not suitable for use in the IFRS 9 framework which requires the incorporation of FLI (macroeconomic indicators, industry trends, and specific borrower circumstances) into the PD estimation, to assess the potential for default *in the future*. PDs which include FLI are forward-looking estimates that reflect the current credit quality of a borrower or financial instrument known as PiT PDs. They embrace the unique macroeconomic conditions at points in time by incorporating both historical data and FLI and as such are more sensitive to short-term fluctuations and changes in economic conditions.

Mathematical techniques exist for transforming TtC PDs into PiT PDs: a common approach is that introduced by Vasicek (1987). The methodology converts a TtC PD into a PiT PD by considering the current level of TtC PD (i.e., at the valuation date), the forecasted macroeconomic conditions at relevant future intervals and the degree of correlation between loan credit quality (TtC PD) and the prevailing macroeconomic circumstances. The technique is operationally relatively straightforward to implement, and it has been widely adopted in the industry. Using the Vasicek (1987) approach, this article explores the transformation of TtC to PiT PDs under both different macroeconomic conditions, different correlations between TtC PD levels and these macroeconomic values.

This remainder of this article proceeds as follows: Section 2 provides insight into previous work on the transformation of TtC PDs to PiT PDs as well as a background of the Vasicek model approach. The data used and the methodology employed (including the mathematical structure of the Vasicek model approach) are set out in Section 3, while Section 4 presents the results and provides a discussion of consequences for-inter alia-provision estimation. Section 5 concludes.

2. LITERATURE REVIEW

The methodology for calculating expected ECL is based on a decomposition of expected losses into three components: the PD, the LGD, and EAD. The PD component is driven by internal ratings and plays a crucial role. Several approaches exist for estimating not only the 1-year PD but also the lifetime PD, including survival analysis and the use of rating transition matrices (Witzany, 2017; Miu and Ozdemir, 2017). The latter approach, which involves modelling the probabilities of transitions between IFRS 9 stages, is more practical because it estimates lifetime PDs in addition to the probabilities of transitions between stages (Witzany, 2022).

Organisations generally estimate a TtC transition matrix (usually 1-year) based on historical data that records migrations between internal rating grades, including the state of default. The objective is then to adjust the TtC transition matrix such that it reflects the relevant prevailing (or future) macroeconomic scenario-not the long run average. This involves adjusting for future periods to obtain a sequence of conditional transition matrices, driven by the nature of historical data reporting frequency. If the rating transition process satisfies the Markov chain property or "memoryless property" (in which the probability of a future state (PD) depends only on the current PD state not on any previous observed PDs) matrix multiplication of the 1-year transition matrix by itself generates PDs over longer time horizons. This approach assumes that transition matrices are relatively stable (Malik and Thomas, 2012).

Vasicek's (1987) approach makes use of a single-factor Gaussian model, in which default is driven by a latent standard normal variable decomposed into systematic and idiosyncratic components (Belkin et al., 1998; Yang, 2015). Past systematic factors and their loadings (default correlations) are derived from historical default rates and then predicted using a macroeconomic model. FLIs are used to stress product-level PDs and associated transition probabilities.

TtC PDs (which are not procyclical[1]) are used (instead of PiT PDs, which are procyclical) for regulatory credit risk capital

1 During adverse economic times, financial institutions are compelled to raise capital requirements, resulting in reduced lending. Such cycles reinforce themselves leading to market procyclicality. Several approaches have been proposed to mitigate the effect, although none has (2023) garnered universal acceptance to 1987 (BCBS 2021).

requirements. These capital amounts provide a safeguard against unforeseen losses resulting from rare but potentially severe loss events. The magnitude of this safeguard is mandated by the BCBS to be the worst-case default rate with 99.9% confidence, (Vasicek, 2002). These unexpected losses are measured as the difference between losses at the 99.9th percentile worst loss scenario, and expected losses (defined as *PD LGD EAD*), as shown in Figure 1.

TtC PD estimates are calculated using long-term averages, resulting in a stable estimate that remains consistent throughout the business cycle and credit cycle (Novotny-Farkas, 2016; Rhys et al., 2016). In contrast PiT PD estimates are based on current economic conditions and incorporate all available information and forecasts. As a result, PiT estimates are more reflective of real-time conditions and are influenced by the business cycle and credit cycle (Novotny-Farkas, 2016; Rhys et al., 2016). Empirical findings suggest that PiT estimates provide a more precise method for estimating PD, LGD and EAD (Heitfeld, 2004). Nevertheless, one drawback of PiT compared to TtC is that PD, LGD, and EAD are more volatility prone (Maria, 2015; Rhys et al., 2016).

The BCBS strives to limit the influence of economic conditions on regulatory capital amounts for capital ratios (BCBS, 2006; Gordy and Howells, 2006), so volatility in this context is undesirable. Using PiT PD estimates during economic downturns result in lower net income for financial institutions and this effect is further exacerbated if the institution is required to hold more capital due to the use of other PiT variables, since PiT PD, PiT LGD, and PiT EAD all generally increase during such periods (Catarineu-Rabell et al., 2005). Existing literature has identified three primary techniques for incorporating macroeconomic factors to convert TtC estimates to forward-looking PiT estimates. These methods include macroeconomic adjusted Markov chains (Vanek et al., 2017), Vasicek's one-factor model (Carlehed and Petrov, 2012), and the KMV-Merton model (Bharath and Shumway, 2004).

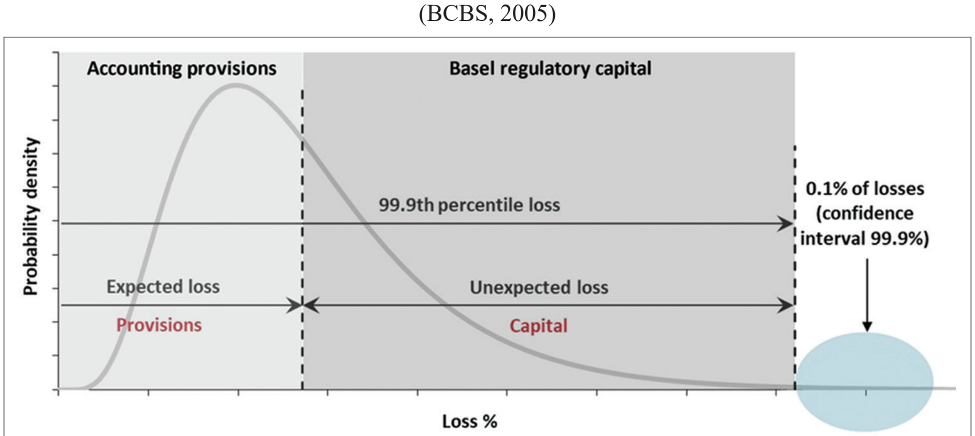
Vaněk and & Hampel, 2017 introduce a methodology for transforming TtC PD into PiT PD utilising Markov chains. The approach is grounded in the average characteristics of rating grades over time, which bear a close resemblance to TtC and may be expressed in the form of a migration matrix. The migration matrix is influenced by macroeconomic factors, thus including them in the migration probabilities. These probabilities reflect the chances of obtaining a certain rating at a specific time, given the influence of macroeconomic factors (Malik and Thomas, 2012).

Carlehed and Petrov (2012) utilise Vasicek's one-factor model to compute PiT PD using TtC PD, default rate, and correlation, if the default rate represents the current state of the economy. However, this model has a limitation in that it cannot determine PiT over multiple periods. Nonetheless, Vasicek's one-factor model is an analytical approach that can be repeated annually per rating, and thus, it is less computationally intensive compared to a Markov chain with macroeconomic adjustments, which is beneficial for model running time. Csaba (2017) capitalises on this benefit by adjusting the migration probabilities of a two-rating Markov chain with the results of Vasicek's one-factor model. In contrast, García-Céspedes and Moreno (2017) propose an extension of Vasicek's one-factor model to enable its use for multi-period purposes by giving a weight to the most recent default rate observation of the explanatory model and a certain weight to a random error term.

The KMV Corporation developed the KMV-Merton model, a default forecasting model based on Merton's debt pricing model (Merton, 1974). The model applies the debt pricing model to a company's balance sheet, in which the company's equity is viewed as a call option, and the strike price the face value of the company's debt (Bharath and Shumway, 2004). The PD is determined using the corporate debt face value, as well as its underlying value and volatility, neither of which are directly observable (Bharath and Shumway, 2004). However, the KMV-Merton model has the advantage of not requiring historical data for a specific company to determine a forward-looking PiT PD. Although the KMV-Merton model is not representative of a company's underlying value (since it assumes that a company's value follows Geometric Brownian motion) it is beneficial for specific company analysis. This article does not make use of this model further, as it focuses on a different method for PiT PD determination.

Frei and Wunsch (2018) present a new nonparametric estimator for credit risk modelling that corrects underestimated latent asset return correlations due to autocorrelation and short time series, resulting in more accurate correlation estimates. The method is based on convergence and approximation principles

Figure 1: The relationship between expected loss (EL), unexpected loss (UL) related to the capital requirements for financial institutions (BCBS, 2005)



for autocorrelated time series and offers an easily implementable solution.

Wunderer (2019) explores the impact of assuming homogeneity in exposure pools on asset correlations calculated from default time series data, finding that underestimation of asset correlations occurs when homogeneity with respect to the probability of default is incorrectly assumed. This underestimation is amplified when considering negative correlations between asset correlation and probability of default, providing a potential explanation for the tendency of asset correlations from default data to be lower than those from asset value data.

Pfeuffer et al. (2020) enhance the parameterisation of correlations in the Vasicek credit portfolio model by providing analytical approximations for value-at-risk and expected shortfall standard errors based on intra-cohort correlations, The authors introduce a new copula-based maximum likelihood estimator for inter-cohort correlations with derived standard errors, and demonstrate how these contributions address bias and improve uncertainty quantification (essential for regulatory requirements).

Jakob (2022) discusses the literature governing the inception of credit portfolio models in the late 20th century, (like CreditMetrics and CreditRisk+) in the context of correlation parameter estimation methods, many of which assume unrealistic infinite portfolio sizes and homogeneity. To address these limitations, Jakob (2022) introduces more flexible maximum likelihood estimation techniques capable of accommodating finite portfolios, limited default data, and time-varying, nonhomogeneous default probabilities, aligning with financial institutions' rating system philosophies and mitigating misspecifications and double-counting. Simulation results demonstrate that these new estimators often outperform established methods in practical applications, providing improved accuracy and adaptability for correlation parameter estimation.

Cho and Lee (2022) introduce a new time-varying credit risk model that captures cyclicality and asymmetry in asset correlations within credit portfolios, combining GJR-GARCH volatility modelling and copula-based conditional dependence. The authors find that the model's superiority over regulatory models in representing U.S. credit portfolios, showcasing its ability to address cyclical and asymmetric asset correlations and suggesting potential limitations of Basel's correlation criteria during economic downturns.

The primary objective of this work is to compare the implication of PiT PD to TtC PD conversion using the Vasicek single-factor model. Using Vasicek's mode, the impact of TtC PD to PiT PD conversion under a wide range of different macroeconomic conditions and correlations is explored. The consequences of the output on required IFRS 9 institutional provisions are also explored and reported.

3. DATA AND METHODOLOGY

Financial institutions typically employ a proprietary master rating scale (MRS) to facilitate the management of credit risk in its lending activities. The MRS is a standardised system for assigning credit ratings to borrowers based on their creditworthiness, and it provides a consistent and objective basis for assessing credit risk across the financial institution's lending portfolio. A well-designed MRS allows financial institutions to quantify credit risk and make informed credit decisions based on the level of risk associated with each borrower. This is essential for effective credit risk management, as it helps the financial institution to identify and mitigate potential losses in its lending activities. The MRS may be based on a combination of quantitative (e.g., financial ratios, industry trends) and qualitative factors (e.g., the borrower's financial history, extenuating circumstances). Ratings assigned to borrowers using the MRS reflect the financial institution's assessment of the borrower's creditworthiness, facilitating portfolio management by grouping obligors based on their assigned ratings, identifying credit risk concentrations, and triggering appropriate mitigation measures. In the absence of up-to-date CRA TtC PD data, the institution's MRS is frequently used instead.

3.1. Data

Because organisational borrower information is highly sensitive and proprietary, it is impossible and unethical to make use of empirical data for the purposes of this work. Instead-and without any loss of generalisation-input TtC PDs spanning a wide range of values may be used instead. This enjoys the benefit of encompassing the full range of possible TtC PDs, rather than (potentially) small TtC PD subranges.

Similar logic is applied for the single macroeconomic factor values. The Vasicek model requires the macroeconomic factor to be standard normal, so a range of ±4 (representing ±4 standard deviations from the average value of the macroeconomic factor) is considered appropriate to embrace all stylistic, realistic possibilities. In the Vasicek single-factor model, PDs are estimated based on a single common factor, typically a macroeconomic variable. The model assumes that the PD of an entity is influenced by the systematic factor and can be modelled as a function of that factor.

3.2. Methodology

Vasicek's one-factor model is characterised by its ability to transform a PiT PD into TtC PD. The model estimates the state of the economy by using a correlation-dependent weight to the long-term average (TtC) PD and another correlation-dependent weight to a macroeconomic factor, specifically the default rate. To obtain a forward-looking PD, an estimate of the future state of the economy is needed, which necessitates an estimation of future default rates. As the future default rate cannot be directly observed, it is estimated using a regression that relates the default rate to a macroeconomic factor.

It is posited that an economic cycle exists, whereby systemic factors exert an influence on all participants in a particular segment, such as an industry or country. Under this premise, the risk of default for an individual counterparty can be attributed to a combination of systemic factors and idiosyncratic risks that are specific to that counterparty (Figlewski et al., 2012).

The concept of PiT PD for a counterparty refers to the likelihood that the counterparty will experience default within the subsequent 12 months. It is evident that the PiT PD is contingent upon the information available at the time of estimation. The information can be bifurcated into two components theoretically: the status of systemic factors and any other pertinent details about the counterparty available. The TtC PD is the average PiT PD, average here is over all the systemic factors.

The model estimates the relationship between the systematic factor and the borrower's PD and uses this relationship to project the borrower's PD over the economic cycle. In summary, the Vasicek single factor model provides a useful framework for estimating TtC PD from PiT PD estimates. By incorporating the cyclical changes in the credit environment, the model provides a more accurate and stable measure of credit risk, which is crucial for making informed credit decisions and managing credit risk in a portfolio.

This paper undertakes a mathematical derivation of Vasicek's one-factor model. The model, as defined by Belkin et al. (1998), is the focus of this study. A key assumption of the single-factor Gaussian model is that an obligor *i*'s default event is driven by a standard normal variable:

$$Y_i = \sqrt{\rho} Z + \sqrt{1-\rho}\,\xi_i \tag{1}$$

Where $Z \sim N(0,1)$ is the systematic factor and $\xi_i \sim N(0,1)$ is an obligor-specific factor. The variable Y_i may be interpreted as the quantile-to-quantile transformed time-to-default variable or as the standardised asset return (Vasicek, 1987; Witzany, 2017). The parameter ρ represents the asset return correlation between the default factors of different obligors. The unconditional probability of default of the obligor over a given time horizon (e.g., 1 year) may in this context be expressed as $PD_i = Pr[Y_i \leq b_i] = \Phi(b_i)$ where b_i is a default threshold. Thus, if the probability of default PD_i is given, $b_i = \Phi^{-1}(PD_i)$. This model allows to condition the probability of default on the systematic factor value Z and obtain the well-known Vasicek's formula used by Basel II regulation (BCBS, 2006):

$$\begin{aligned} PD_i(Z) &= \Pr\left[\sqrt{\rho}Z + \sqrt{1-\rho}\,\xi_i Y_i \leq \Phi^{-1}(PD_i) \mid Z\right] \\ &= \Pr\left[\xi_i \leq \frac{\Phi^{-1}(PD_i) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right] \\ &= \Phi\left(\frac{\Phi^{-1}(PD_i) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right) \end{aligned} \tag{2}$$

The conditional probability of default can be used as a proxy of the future default rate driven by the unknown systematic factor on a large homogenous portfolio, i.e., on a portfolio with many exposures with the same unconditional *PD* and the same correlation parameter ρ. In practice, this may apply to a retail product portfolio of exposures with the same rating grade, and so with approximately the same probabilities of default. By stressing the latent systemic variable $Z \sim N(0,1)$ one may obtain quantile estimates of the future possible default rate, which is the goal of the regulatory capital calculations.

If a realised default rate p on a specific homogenous portfolio with the known parameters *PD* and ρ is given, then the latent (not observable) systemic factor Z may be derived from (2) as

$$Z = \frac{\Phi^{-1}(PD) - \sqrt{1-\rho}\,\Phi^{-1}(p)}{\sqrt{\rho}} \tag{3}$$

If a time series of historical default rates, p_t for $t = 1\ldots T$, is observed, the parameters *PD* and ρ can be estimated, even if they are unknown. To achieve this, the model (2) may be formulated as $p_t = \Phi(\alpha - \beta Z_t)$, where $Z_t = (\alpha - \Phi^{-1}(p_t))/\beta$ represents the latent systemic factor that has been realised. Assuming that the *Z*-factors are independently drawn from the standard normal distribution, the unknown parameters can be estimated by maximising the likelihood function. The maximum likelihood estimates can be obtained from the mean (α) and standard deviation ($-\beta$) of the transformed series $\Phi^{-1}(p_t)$, for $t = 1,\ldots,$ (Yang, 2013). The correlation and long-term *PD* can be calculated using the following methods:

$$\rho = \frac{1}{1+\beta^2},\ PD = \Phi\left(\alpha\sqrt{\rho}\right) \tag{4}$$

The assumption of the Gaussian independent realisations of z_t is questionable, and it is important to observe the default rates over non-overlapping time intervals to address the issue of independence. However, even in such cases, the autocorrelation of the series of systematic factors may be present, necessitating a more general specification as discussed in Witzany (2011).

To address the probability distribution of the systematic and idiosyncratic factors, a generalised model can be proposed where $Z \sim F_1$, $\xi_i \sim F_2$, and $Y_i \sim F_3$, with F_3 being a mixture of the independent distributions F_1 and F_2, given by the parameter ρ in (1). The resulting formula (2) can be expressed as follows:

$$PD_i(Z) = F_2\left(\frac{F_3^{-1}(PD_i) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right) \tag{5}$$

For a time series of historical default rates, the parameters in the model $p_t = F_2(\alpha - \beta Z_t)$ can be estimated through maximum likelihood estimation (MLE) from the transformed latent factors:

$$Z_t = \frac{(\alpha - F_2^{-1}(p_t))}{\beta}$$

The likelihood of the single default rate p_t observation is dependent on the assumption that z_t are independently drawn from F_1:

$$L(P_t) = \frac{f_2(Z_t)}{\beta f_2(F_2^{-1}(p_t))}$$

And so, the total log-likelihood is:

$$LL(\alpha,\beta) = \sum_t \ln f_2(Z_t) - \ln f_2\left(F_2^{-1}(p_t)\right) - \ln\beta \tag{6}$$

After calculating the correlation parameter ρ as in (4), the resulting mixed distribution F_3 can be utilised to obtain the unconditional probability of default, $PD = F_3\left(\alpha\sqrt{\rho}\right)$. Witzany (2013) suggests using the logistic distribution as it has become an industry standard for credit risk modelling through the Logit model (logistic regression). Empirical studies have revealed that the impact of the logistic distributional assumptions, compared to the Gaussian model, can be dramatic when estimating unexpected default rates on high probability levels (such as 99.9% as used in the Basel regulation). Alternatively, the Student t-distribution, proposed by Perederiy (2017), is another candidate distribution.

To stress rating migration matrices conditionally on economic scenarios, it is necessary to further generalise the single factor model described above, as stated in the introduction. Assuming there are K rating grades, denoted as $u = 1,\ldots,K$, where K represents the absorbing state of default, the objective is to estimate the probabilities of future rating transitions.

In (7), $rat(i,t) = u$ represents the rating grade assigned to exposure i at time t. If the transition probabilities are modelled on a homogeneous portfolio, the index i can be omitted. Assuming there are historical default and rating transition observations for periods $t = 1,\ldots,T$, the objective is to estimate the forward-looking transition probabilities $PD_{uv}(t|X_t)$ conditional on a scenario defined by a vector series of macroeconomic variables X_t for $t = T + 1,\ldots,T + M$ (where M is the maximum maturity for which the transition probabilities must be modelled).

$$PD_{uv}(i,t) = Pr[rat(i,t) = v[rat(i,t–1) = u] \tag{7}$$

Initially, we describe an uncomplicated generalisation of the single-factor model, which is frequently employed in the banking sector for IFRS 9 modelling purposes (Rubtsov and Petrov, 2016). The primary assumption of the generalised single-factor model is that, in addition to the event of default, transitions are also influenced by the factors (1). Removing the time parameter t, the unconditional probability of transitioning from u to v is considered.

$$PD_{uv} = \Pr[b_{u,v+1} < Y_i \leq b_{u,v}] \tag{8}$$

The decreasing transition thresholds for $b_{u,v+1} = 1,\ldots,K$ are defined, with b_u, $K + 1 = –\infty$. To express the thresholds from the probabilities, cumulative transition probabilities are introduced:

$$\begin{aligned} PD_{uv+} &= \Pr\left[rat\left(i,t\right) \geq v|rat\left(i,t-1\right) = u\right] \\ &= \Pr\left[Y_i \leq b_{u,v}\right] \\ &= PD_{uv} + PD_{uv+1} + PD_{uK} \end{aligned} \tag{9}$$

Assuming the standard normal distribution of Yi, the decreasing transition thresholds can be expressed as $b_{u,v+1} = \Phi^{-1}(PD_{uv+})$. Conditional transition probabilities may be represented similar to (2):

$$PD_{uv+}\left(Z\right) = \Phi\left(\frac{\Phi^{-1}\left(PD_{uv+}\right) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right) \tag{10}$$

Knowing conditional cumulative transition probabilities, obtaining the standard transition probabilities is a straightforward task. Standard transition probabilities are defined using (10) which, in an ECL context, may be rewritten as:

$$PD^{PIT}(Z_t) = \Phi\left(\frac{\Phi^{-1}\left(PD^{TTC}\right) - \sqrt{\rho}\cdot Z_t}{\sqrt{1-\rho}}\right)$$

Where Z_t is a representation of the status of the economic conditions as well as thereafter.

To estimate expected credit losses under different scenarios, a five-step process can be followed. First, a time series of historical default rates is used to estimate the correlation ρ and calculate the historical Z-factors Z_t. Next, the historical rating transitions are used to estimate a TtC transition matrix. Then, a macroeconomic model is built by selecting appropriate predictors that link Z_t and the predictors. For stress-testing or IFRS9 scenarios, the model is used to estimate the forward-looking values of economic indicators X_t^{scen} and the implied factors Z_t^{scen} for $t = T + 1,\ldots,T + M$. The TtC PD transition matrix is then adjusted conditional on Z_t^{scen} using (10) for $t = T + 1,\ldots,T + M$. Finally, the adjusted transition matrices are multiplied through to estimate the migration probabilities over longer periods, which allows for the estimation of overall expected credit losses conditional upon the scenario.

In this paper, the limitations of a simple yet effective approach are investigated. Specifically, several weaknesses in the single-factor default model with the correlation parameter ρ are identified. Firstly, the model assumes credit risk homogeneity in the portfolio, which is not the case for portfolios aimed at modelling rating transition probabilities (such as mortgage, consumer, or corporate loans). Exposures with varying ratings violate the theoretical credit risk homogeneity assumption, and the estimation of a single correlation for all initial ratings in Step 1 is unsuitable. To address this, a possible solution is to estimate correlations ρ_u based on time series of default rates conditioned on the initial rating u, or by using observations of transitions from u to another rating v.

While IFRS9 does not mandate the use of specific formulae, it requires the provisions to be equivalent to the ECL. The ECL is calculated using the variables PD, LGD, and EAD, which are not known by a financial institution and therefore require estimation. The ECL for a loan is:

$$ECL_T = \sum_{t=1}^{T} \frac{PD_t \cdot LGD_t \cdot EAD_t}{(1+EIR)^t} \tag{11}$$

In the context of loan portfolio risk assessment, the PD between time $t – 1$ and t is denoted as PD_t. Additionally, LGD_t represents the percentage of loss given that a default occurs within the same time interval, and EAD_t signifies the exposure in the event of default between time $t – 1$ and t. The effective interest rate (EIR) is the discount rate used to evaluate the present value of future cash flows over the expected life of the loan, relative to the principal of the loan (IASB, 2003). T denotes the remaining time until loan maturity. For stage 1 loans (Performing loans), $T = 1$ always. However, if a stage 2 (distressed) or stage 3 (defaulted loans) loan is set to mature in, for example, 10 years, the financial institution will estimate ECL_{10}. The portfolio ECL is the aggregate ECL over all loans:

$$ECL_T = \sum_{n=1}^{N}\sum_{t=1}^{T}\frac{PD_{t,n}\cdot LGD_{t,n}\cdot EAD_{t,n}}{(1+EIR)^t} \quad (12)$$

In the loan portfolio analysis, the PD of loan n between time $t-1$ and time t is denoted as $PD_{t,n}$. Furthermore, $LGD_{t,n}$ represents the loss percentage if loan n defaults between the same period, while $EAD_{t,n}$ signifies the exposure given a default for loan n between time $t-1$ and t. N represents the total number of loans within the portfolio. (11) and (12) are TtC models and thus are formally incorrect as per the requirements of IFRS9. Banks must estimate PD, LGD, and EAD on a PiT basis (for IFRS 9 requirements). To determine the PiT PDs, PiT LGDs, and PiT EADs, a financial institution must consider all available information. Accounting for all relevant information, (12) can be restated as:

$$ECL_{T|i} = \sum_{n=1}^{N}\sum_{t=1}^{T}\frac{PD_{t|i,n}\cdot LGD_{t|i,n}\cdot EAD_{t|i,n}}{\left(1+EIR\right)^t} \quad (13)$$

$$= \sum_{i=1}^{N}\sum_{t=1}^{T}\frac{PD_{t,n}^{PIT}\cdot LGD_{t,n}^{PIT}\cdot EAD_{t,n}^{PIT}}{\left(1+EIR\right)^t} \quad (14)$$

The availability of all information at time t is denoted as i, which facilitates the calculation of PD, LGD, and EAD on a PiT basis, thus making them compliant with IFRS9 standards. IFRS9 does not provide a mathematical framework for computing the ECL. In practice, the ECL is often estimated using (14) as other regulatory frameworks may require unadjusted PiT values for PD, LGD, and EAD.

The Vasicek model provides a framework for estimating the PD component of ECLs by modelling the borrower's credit risk as a function of a systematic factor that follows a mean-reverting process (Perederiy, 2015). The model estimates the relationship between the systematic factor (11) and the borrower's PD and uses this relationship to project the borrower's PD over the economic cycle. The projected PDs are then used to calculate the expected credit losses for each borrower in the portfolio.

The Vasicek model also provides a robust and flexible approach for estimating credit risk and calculating ECLs under IFRS9. It enables financial institutions to make informed credit decisions, quantify credit risk, and manage their lending portfolios to minimise potential losses. By incorporating macroeconomic factors and projecting credit risk over the economic cycle, the Vasicek model provides a more accurate and stable measure of credit risk, which is crucial for complying with IFRS9 requirements.

4. RESULTS AND DISCUSSION

It is not unreasonable to expect *all* TtC PDs to be transformed into *worse* PiT PDs *if macroeconomic conditions are unfavourable* and correlations are high and *vice versa*. Under certain circumstances the Vasicek approach suggests that favourable (low) TtC PDs transform into *worse* PiT PDs while poor quality (high) TtC PDs transform into *better* PiT PDs *for the same macroeconomic conditions and correlations*. This is counterintuitive: unfavourable market conditions should always produce *worse* PiT PDs (i.e., worse than the TtC PDs from which they originate) across all TtC PDs, and *vice versa*.

These empirical results are shown in Table 1 for a fixed $\sqrt{\rho} = +0.34$. Columns show the systematic factor (Z_t) over a range of stylistically possible values (in this case, $-0.45 \leq Z_t \leq -0.20$). The rows represent TtC PDs, again over a stylistically possible range of "good" obligor PDs (in this $0.01\% \leq PD_{TtC} \leq 3.62\%$). Note here that the lowest TtC PD is assigned a value of

Table 1: PD_{PiT}s calculated over a range of PD_{TtC}s and systematic factors, Z_t using (11) at a fixed $\sqrt{\rho} = +0.34$. Shaded cells indicate $PD_{PiT} \leq PD_{TtC}$ and white cells $PD_{PiT} \geq PD_{TtC}$

PD_{TTC} Z_t (%)	−0.45 (%)	−0.40 (%)	−0.35 (%)	−0.30 (%)	−0.25 (%)	−0.20 (%)
0.010	0.010	0.010	0.009	0.009	0.009	0.008
0.014	0.015	0.014	0.013	0.013	0.012	0.012
0.020	0.021	0.020	0.019	0.018	0.018	0.017
0.028	0.030	0.028	0.027	0.026	0.025	0.024
0.040	0.043	0.041	0.039	0.038	0.036	0.035
0.057	0.061	0.059	0.056	0.054	0.052	0.050
0.080	0.086	0.083	0.080	0.077	0.074	0.071
0.113	0.123	0.118	0.114	0.110	0.106	0.102
0.160	0.175	0.169	0.163	0.157	0.151	0.146
0.226	0.248	0.240	0.231	0.223	0.216	0.208
0.320	0.353	0.342	0.330	0.319	0.308	0.298
0.453	0.503	0.487	0.471	0.456	0.441	0.427
0.640	0.714	0.692	0.670	0.649	0.629	0.609
0.905	1.013	0.983	0.954	0.925	0.898	0.871
1.280	1.438	1.397	1.357	1.319	1.281	1.244
1.810	2.039	1.984	1.930	1.878	1.827	1.777
2.560	2.890	2.816	2.744	2.673	2.605	2.537
3.620	4.091	3.993	3.897	3.803	3.710	3.620

Figure 2: (a) PiT_{PD} versus PD_{TtC} for 0.01% ≤PD_{TtC} ≤0.16% and (b) PiT_{PD} versus PD_{TtC} for 0.20% ≤PD_{TtC} ≤25.00%. The dashed line is the line on which $PiT_{PD} = PD_{TtC}$ and the two graphs are necessary to illustrate different PD scales.

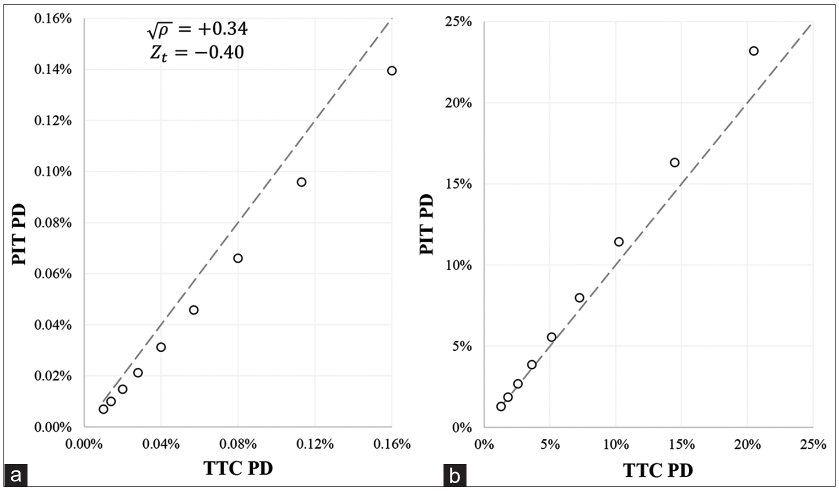
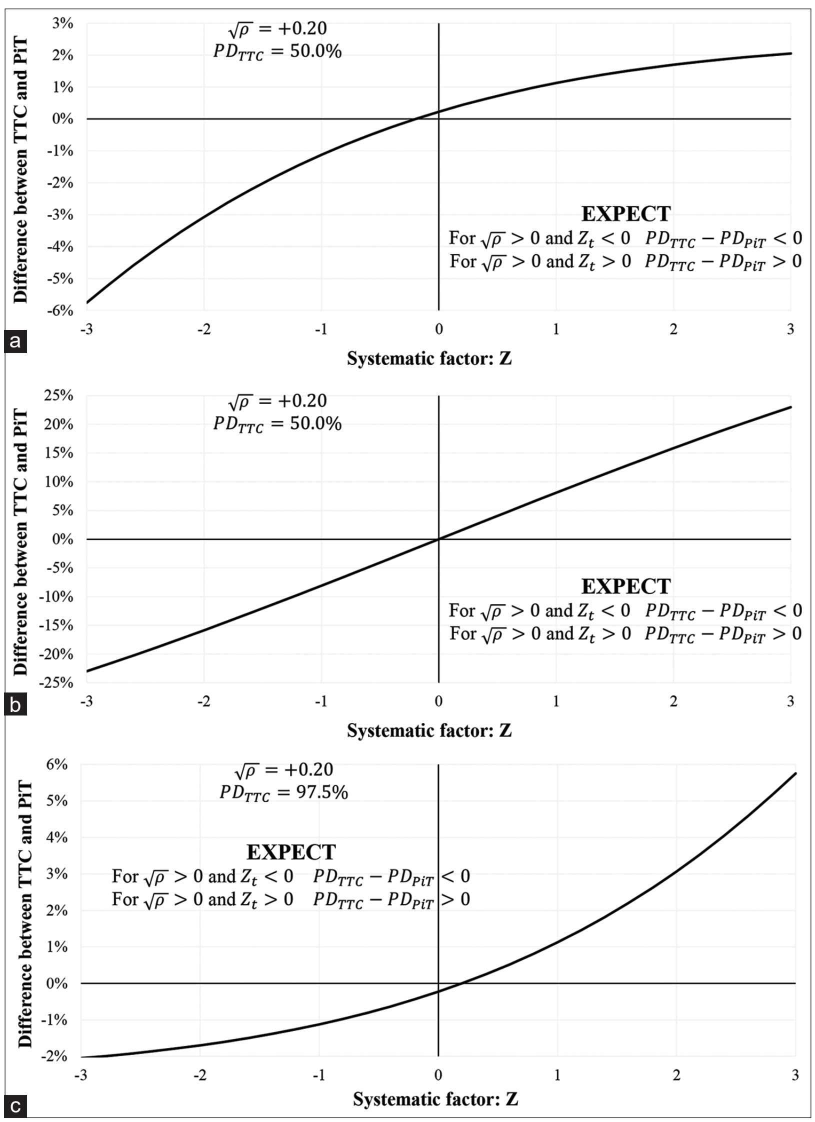


Figure 3: (a) Dependence of the difference between PD_{TtC} and PD_{PiT} as a function of the systematic factor, Z for $\sqrt{\rho} = +0.20$ and constant PD_{TtC}=2.5%. (b) Dependence of the difference between PD_{TtC} and PD_{PiT} as a function of the systematic factor, Z for $\sqrt{\rho} = +0.20$ and constant PD_{TtC}= 50.0%. (c) Dependence of the difference between PD_{TtC} and PD_{PiT} as a function of the systematic factor, Z for $\sqrt{\rho} = +0.20$ and constant PD_{TtC} = 95.5%



0.01% and these then scale by $\sqrt{2}$ for every subsequent TtC PD. Hence, the next grade's TtC PD is $0.01\% \times \sqrt{2} = 0.014\%$ and the next $0.01\% \times \sqrt{2}^2 = 0.020\%$ and so on. The elements which populate Table 1 are the calculated (transformed) PiT PDs using (11). Cells shaded in light grey reflect the situation where $PD_{PiT} \leq PD_{TtC}$ while white cells indicate $PD_{PiT} \geq PD_{TtC}$. Thus, for $Z_t \leq -0.45$ all $PD_{PiT} \geq PD_{TtC}$ s and for $Z_t \geq -0.20$ all $PD_{PiT} \geq PD_{TtC}$ s for a constant $\sqrt{\rho} = +0.34$. For $-0.40 \leq Z_t \leq -0.25$, however, PD_{PiT}s are lower for good (low) PD_{TtC}s but higher for bad (higher) PD_{TtC}s for the same $\sqrt{\rho}$. This means, for any fixed macroeconomic condition, Z_t, such that $-0.40 \leq Z_t \leq -0.25$ and for a fixed dependency (given by the fixed $\sqrt{\rho}$) of PD_{PiT}s on this systematic factor-the resulting PD_{PiT}s can deteriorate or improve relative to the associated PD_{TtC} from which it was derived.

The situation presented in Table 1 is illustrated in Figure 2. The dashed line indicates the line on which $PD_{PiT} = PD_{TtC}$ so markers below this line in (Figure 2a) indicate better PD_{PiT}s for 0.01% $\leq PD_{TtC} \leq$0.16% and markers above the line in (Figure 2b) indicate worse PD_{PiT}s for 0.20% $\leq PD_{TtC} \leq$25.00%.

(Figure 3a-c) shows three scenarios of differences between PD_{TtC} and PD_{PiT} as a function of the systematic factor, Z, spanning the full range of possible PD_{TtC} (2.5%, 50% and 97.5%) for a positive $\sqrt{\rho} = +0.2$. It is reasonable to expect that for bad economic conditions ($Z < 0$), $PD_{TtC} - PD_{PiT} < 0$ and vice versa for good economic conditions: i.e., for $Z > 0$, $PD_{TtC} - PD_{PiT} > 0$ as shown in (Figure 3a) for low PD_{TtC} (=2.5%), (Figure 3b) for PD_{TtC} (=0) and for high PD_{TtC} (=97.5%).

(Figure 4a-c) shows three scenarios of differences between PD_{TtC} and PD_{PiT} as a function of PD_{TtC}, spanning a wide range of possible

Figure 4: (a) Dependence of the difference between PD_{TtC} and PD_{PiT} as a function of PD_{TtC} for $\sqrt{\rho} = +0.20$ and constant $Z_t << 0$. (b) Dependence of the difference between PD_{TtC} and PD_{PiT} as a function of PD_{TtC} for $\sqrt{\rho} = +0.20$ and constant $Z_t \approx 0$. (c) Dependence of the difference between PD_{TtC} and PD_{PiT} as a function of PD_{TtC} for $\sqrt{\rho} = +0.20$ and constant $Z_t = +3.0$

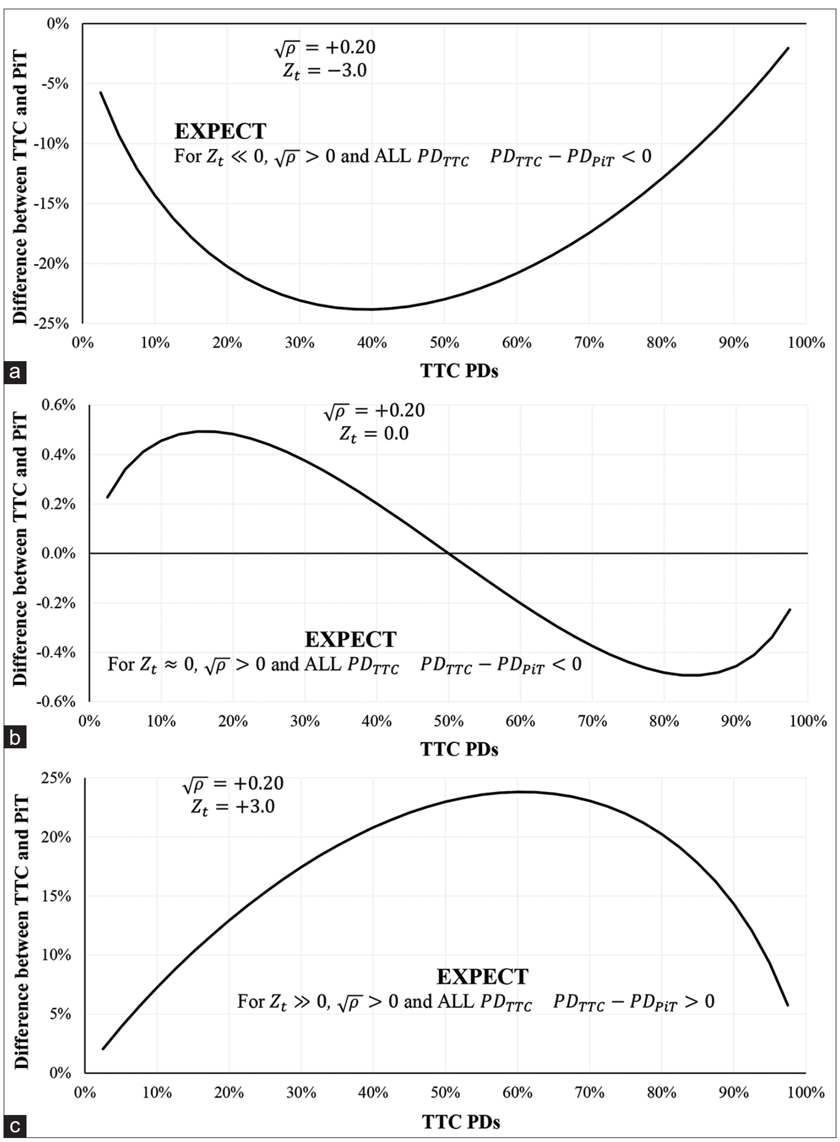
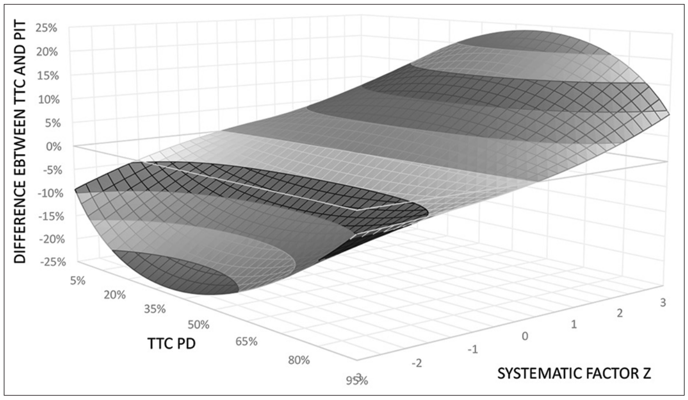


Figure 5: 3D view of difference between PD_{TtC} and PD_{PiT} over the full range of Z and PD_{TtC} for a constant $\sqrt{\rho} = +0.2$ showing the inversion at $Z \approx 0$



economic conditions (Z = –3.0,0.0 and +3.0) for again a positive $\sqrt{\rho} = +0.2$. It is again reasonable to expect that for bad economic conditions ($Z < 0$), $PD_{TtC}–PD_{PiT} < 0$ and vice versa for good economic conditions: i.e., for $Z > 0$, $PD_{TtC}–PD_{PiT} > 0$ as shown in (Figure 4a and c) for good and bad economic conditions respectively. However, for "average" economic conditions ($Z \approx 0.0$), the difference between PD_{TtC} and PD_{PiT} counterintuitively changes sign.

Figure 5 shows the difference between PD_{TtC} and PD_{PiT} over the full range of Z and PD_{TtC} for a constant $\sqrt{\rho} = +0.2$.

5. CONCLUSION AND RECOMMENDATIONS

This article explores Vasicek's approaches to convert TtC PDs into PiT PDs by considering the relationship between TtC PDs and a relevant macroeconomic factor (or factors in the multifactor model). This model for forecasting PiT PDs exhibits a functional flaw possibly in its assumptions of stationary, a linear relationship between TtC and PiT PDs, and the constancy of underlying factors driving credit risk over time. By employing a range of input values for Vasicek's model, the study demonstrates empirically counterintuitive results, with PiT loan credit quality improving for some TtC PDs and deteriorating for others (for the same macroeconomic conditions and the same dependency on that macroeconomic factor of the TtC PDs).The study's empirical analysis, which incorporates a range of input values for Vasicek's model, reveals counterintuitive results whereby PiT loan credit quality improves for some TtC PDs while deteriorating for others, despite identical macroeconomic conditions and dependencies on the macroeconomic factor of the TtC PDs.

The research underscores the need for improved models and methodologies to accurately estimate PiT PDs and assess credit risk. Addressing the functional flaws in existing models will enable organisations to make more informed decisions about credit-related losses and ensure regulatory compliance while adhering to accounting standards. Improved models and methodologies will provide greater transparency to regulators, stakeholders, and investors, instilling confidence in the credit risk assessment processes employed by banks.

Future work could focus on the flaw in the relationship between TtC PD and PiT PD using Vasicek's approach. This research could involve using advanced statistical techniques, such as machine learning algorithms or nonparametric regression models, to capture the complex interactions between TtC PDs and macroeconomic variables. By identifying and incorporating nonlinear relationships, more accurate PiT PD estimates can be obtained, leading to improved credit risk assessment and allocation of provisions.

REFERENCES

BCBS. (2005), An Explanatory Note on the Basel II IRB Risk Weight Functions. Switzerland: BIS.

BCBS. (2006), Basel II: Part 2: The First Pillar-Minimum Capital Requirements. Switzerland: BIS.

Belkin, B., Suchower, S., Forest L Jr. (1998), A one-parameter representation of credit risk and transition matrices. CreditMetrics Monitor, 1(3), 46-56.

Bharath, S.T., Shumway, T. (2004), Forecasting Default with the KMV-Merton Model. Michigan: University of Michigan, Unpublished Paper.

Carlehed, M., Petrov, A. (2012), A methodology for point-in-time through-the-cycle probability of default decomposition in risk classification systems. The Journal of Risk Model Validation, 6(3), 3-25.

Catarineu-Rabell, E., Jackson, P., Tsomocos, D.P. (2005), Procyclicality and the new Basel Accord-banks' choice of loan rating system. Economic Theory, 26(3), 537-557.

Cho, Y., Lee, Y. (2022), Asymmetric asset correlation in credit portfolios. Finance Research Letters, 49, 103037.

Csaba, K. (2017), Modelling of Provision Under New International Financial and Reporting Standard (IFRS 9). In: Proceedings-31st European Conference on Modelling and Simulation, ECMS. p153-157.

Figlewski, S., Frydman, H., Liang, W. (2012), Modeling the effect of macroeconomic factors on corporate default and credit rating transitions. International Review of Economics and Finance, 21(1), 87-105.

Frei, C., Wunsch, M. (2018), Moment estimators for autocorrelated time series and their application to default correlations. Journal of Credit Risk, 14(1), 1-29.

García-Céspedes, R., Moreno, M. (2017), An approximate multi-period Vasicek credit risk model. Journal of Banking and Finance, 81, 105-113.

Gordy, M.B., Howells, B. (2006), Procyclicality in basel II: Can we treat the disease without killing the patient? Journal of Financial Intermediation, 15(3), 395-417.

Heitfeld, E. (2004), Rating System Dynamics and Bank-Reported Default Probabilities Under the New Basel Capital Accord. Research Paper. United States: Board of Governors of the Federal Reserve System.

IASB. (2003), International Accounting Standard 39-Financial Instruments: Recognition and Measurement. United Kingdom: IASB.

IASB. (2014), IFRS 9: Financial Instruments, International Accounting Standards Board. United Kingdom: IASB.

Jakob, K. (2022), Estimating Correlation Parameters in Credit Portfolio Models Under Time-Varying and Nonhomogeneous Default Probabilities. Journal of Credit Risk, 18(4), 29-63.

Malik, M., Thomas, L.C. (2012), Transition matrix models of consumer credit ratings. International Journal of Forecasting, 28(1), 261-272.

Maria, H. (2015). The IFRS 9 Standard: Assessment of the impacts on the European banking industry. Ovidius University Annals, Economic Sciences Series, 10(1), 930-935.
Merton, R.C. (1974), On the pricing of corporate debt: The risk structure of interest rates. The Journal of Finance, 29(2), 449-470.
Miu, P., Ozdemir, B. (2017), Adapting the Basel II advanced internal-ratings-based models for international financial reporting standard 9. Journal of Credit Risk, 13(2), 53-83.
Novotny-Farkas, Z. (2016), The interaction of the IFRS 9 expected loss approach with supervisory rules and implications for financial stability. Accounting in Europe, 13(2), 197-227.
Perederiy, V. (2015), Endogenous Derivation and Forecast of Lifetime PDs. Deutsche Bank Group. In Munich Personal RePEc Archive. MPRA Paper No. 65679. p1-16.
Perederiy, V. (2017), Sparse Structural Approach for Rating Transitions. Available at SSRN: https://ssrn.com/abstract=3009979 or http://dx.doi.org/10.2139/ssrn.3009979 [Last accessed on 2020 Jan 20].
Pfeuffer, M., Nagl, M., Fischer, M., Rösch, D. (2020), Parameter estimation, bias correction and uncertainty quantification in the Vasicek credit portfolio model. Journal of Risk, 22(4), 1-29.
Rhys, M., Wilson, I., Hales, D., Clifford, T., Hughes, S., Beighton, A. (2016), A Drain on Resources? The Impact of IFRS 9 on Banking Sector Regulatory Capital. England: Deloitte.
Rubtsov, M., Petrov, A. (2016), A point-in-time-through-the-cycle approach to rating assignment and probability of default calibration. Journal of Risk Model Validation, 10(2), 83-112.
Vasicek, OA. (1987), Probability of Loss on a Loan Portfolio, KMV Working Paper.
Vasicek, OA. (2002), The Distribution of loan portfolio value. Risk, 15(12), 160-162.
Witzany, J. (2011), A two-factor model for PD and LGD correlation. Bulletin of the Czech Econometric Society, 18(28), 1-25.
Witzany, J. (2013), A note on the Vasicek's model with the logistic distribution. Ekonomický Časopis, 61(10), 1053-1066.
Witzany, J. (2017), Credit Risk Management: Pricing, Measurement, and Modeling. Germany: Springer.
Witzany, J. (2022), Stressing of migration matrixes for International Financial Reporting Standard 9 and internal capital adequacy assessment process calculations. Journal of Credit Risk, 18(3), 49-76.
Wunderer, C. (2019), Asset correlation estimation for inhomogeneous exposure pools. Journal of Credit Risk, 15(3), 1-19.
Yang, B.H. (2015), Stress testing and modeling of rating migration under the Vasicek model framework. Journal of Risk Model Validation, 9(2), 33-47.