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The Price Distribution of Consumer Goods in Retail Markets

Joachim Kaldasch*

International School of Management, Hauptstraße 27, 10827, Berlin, Germany.
Email: joachim.kaldasch@international-business-school.de

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ABSTRACT

Derived is an analytic dynamic model of the price distribution of consumer goods in retail markets. The stationary price distribution is established from the conservation equation of offered units and two simplifying assumptions. Under the condition that independent traders make small random price variations around a nearly constant supply price, the stationary price distribution of a good must have the form of a fat-tailed Laplace distribution. The standard deviation of the distribution is determined by the price volatility of the goods. Also, the price distribution of an ensemble of goods is established and applied to empirical data with good quantitative agreement.

Keywords: Price Dispersion, Consumer Goods, Retail Markets, Market Dynamics, Laplace Distribution
JEL Classifications: D0, E3, F0

1. INTRODUCTION

A consumer good is an item that serves as a solution to a specific consumer problem having the same features. The neo-classic microeconomic theory states that a product must sell for the same price, known as “the law of one price” (Hens and Rieger, 2010). However, many empirical investigations show the existence of a price dispersion (Lach, 2002; Berardi et al., 2017; González and Miles-Touya, 2018). Economists give four popular explanations for the existence of a price dispersion: amenities, heterogeneous costs, intertemporal price discrimination and search costs. The first explanation suggests that identical products sell at different prices because they are bundled with different amenities in different transactions (Sorensen, 2000). The second states that firms at different locations have different costs causing prices to vary for similar goods (Golosov and Lucas, 2007). Also, time dependent price fluctuations occur to satisfy different consumer groups (Conlisk et al., 1984; Sobel, 1984; Albrecht et al., 2013). And finally, the limited ability of buyers to search the entire market allows traders to vary the price (Butters, 1977; Burdett and Judd, 1983; Seifert et al., 2020). Several theoretical attempts are made explaining the price dispersion based on the seller-buyer relationship (see for example, Donna et al., 2020; Myatt and Ronayne, 2019; Pennerstorfer et al., 2020).

It was shown that the empirical price distribution of consumer goods in retail markets can be described by a Laplace distribution (Kaldasch and Koursovitis, 2021). This paper aims at deriving this price distribution from the dynamics of supply and demand in retail markets. A retail market consists of a supplier selling product units of a good for a supply price $p_s(t)$ to retailers.[1] They sell them in a second step to consumers. The key idea of the model is to describe the dynamics of supply and demand on the one hand with the conservation equation of offered units.[2] On the other hand, price fluctuations are taken into account by two simplifying assumptions:

1 The supply price p_s is a list price containing some mean profit for the retailers.

2 The approach is based on the theory of complex systems. This theory suggests that the main dynamics of a complex system is governed by so-called slow modes. Among others, slow modes are given by conservation relations (Sayama, 2015).

(i) For the considered time interval, the magnitude of price variations caused by the supplier are small compared to price variations made by retailers.
(ii) Traders are treated as independent in their price decision behavior. To satisfy demand, price changes of offered units are small random variations in time.[3]

It is shown in the next section that in market equilibrium, based on (i) and (ii), the stationary price distribution must be a fat-tailed Laplace distribution. Furter established is the price distribution of an ensemble of goods. After comparison with empirical data of an aggregated good (Kaplan and Menzio, 2015), the paper ends with a conclusion.

2. THEORY OF THE PRICE DISTRIBUTION OF CONSUMER PRODUCTS

2.1. General Framework

The model is established for a good supplied by a supplier in a free retail market. In a finite time-interval Δt the supplier delivers product units for a supply price $p_s(t)$ to traders. They sell them for a price $p(t)$ to consumers. The probability density function (pdf) of the price of sold units to consumers is defined by:

$$P_y(t,p) = \frac{y(t,p)}{\tilde{y}(t)} \tag{1}$$

where $y(t,p)$ is the number of sold units per unit time at time step t in a price interval p and $p+dp$. The total unit sales of the good at time step t is:[4]

$$\tilde{y}(t) = \int_0^\infty y(t,p)dp \tag{2}$$

The key idea of the model is that the main dynamics of the market can be given by the conservation equation of the number of offered units to consumers $z(t,p)$ at time step t in the price interval p and $p+dp$. It has the form:

$$\frac{\partial z(t,p)}{\partial t} = s(t,p) - y(t,p) - \frac{\partial j(t,p)}{\partial p} \tag{3}$$

The time-dependent evolution of the number of available units is determined by three processes expressed by the three terms on the right-hand side of this relation. The number of units entering the market per unit time at time step t and price p is defined by the supply rate $s(t,p)$. The number of available units decreases by the sales process with the unit sales rate $y(t,p)$.[5] The last term takes price variations of offered units by traders into account. It can be considered as a flow of available units on the price scale by a current $j(t,p)$ (see Appendix A).

3 This assumption implies that the price evolution of offered units can be treated as an arithmetic Brownian motion on the price scale.

4 A tilde over variables indicates total numbers.

5 Also included to $y(t,p)$ are units withdrawn from the market. This happens for example for non-durable consumer goods, if they increase the expiry date. However, traders try to reduce this contribution. It is considered here sufficiently small to be neglected.

The total number of offered units at timestep t is:

$$\tilde{z}(t) = \int_0^\infty z(t,p)dp \tag{4}$$

and the total supply rate:

$$\tilde{s}(t) = \int_0^\infty s(t,p)dp \tag{5}$$

From (3) follows for the total number of offered units with (4) and (5):

$$\frac{d\tilde{z}(t)}{dt} = \tilde{s}(t) - \tilde{y}(t) \tag{6}$$

In market equilibrium total supply equals total demand, $\tilde{s}(t) = \tilde{y}(t)$. This condition leads to:

$$\frac{d\tilde{z}(t)}{dt} = 0 \tag{7}$$

For the applicability of the model, the considered time interval Δt must be chosen such, that (7) is satisfied. For an evaluation of (3) we take advantage from (i) and (ii).

Based on assumption (i), the supply price p_s can be approximated as constant for the considered time interval Δt. The supply rate can then be written with the help of a Dirac δ-function as:[6]

$$s(t,p) = \tilde{s}\delta(p - p_s) \tag{8}$$

where the total supply rate over Δt is:

$$\tilde{s} = \int_{\Delta t} \tilde{s}(t)dt \tag{9}$$

The application of assumption (ii) implies that the price evolution of offered units can be approximated as a random walk on the price scale. The flow rate $j(t,p)$ can therefore be modeled as:

$$j(t,p) = -D\frac{\partial z(t,p)}{\partial p} \tag{10}$$

The parameter D expresses mean square deviations of the price during the time units are offered by retailers. Hence:[7]

$$D = \frac{\overline{v^2}}{\tau} \tag{11}$$

where τ expresses the mean time that available units are offered by retailers before they are purchased. It is termed mean offering time. Mean square price deviations have the form:

$$\overline{v^2} = \overline{(p - \overline{p})^2} \tag{12}$$

while $\overline{p}$ is the mean price of the good.

6 Dirac-delta function: δ(p-ps)= 1 for p=ps and 0 for p≠ps

7 Note that D is equivalent to a diffusion coefficient in particle physics.

2.2. Derivation of the Price Distribution of a Good

To derive the price distribution of a good, the unit sales rate is written as:

$$y(t,p)=\frac{1}{\tau}z(t,p) \tag{13}$$

It suggests that the unit sales rate at a given price and time is proportional to the number of available units and a mean rate $1/\tau$.[8] With this relation, equation (3) turns with (8) and (10) into:[9]

$$\frac{\partial z(t,p)}{\partial t}=\left(\tilde{s}\delta(p-p_s)-\frac{1}{\tau}\right)z(t,p)+D\frac{\partial^2 z(t,p)}{\partial p^2} \tag{14}$$

The price distribution of offered units is defined by:

$$P_z(t,p)=\frac{z(t,p)}{\tilde{z}(t)} \tag{15}$$

Taking the time derivative, we obtain:

$$\frac{\partial P_z(t,p)}{\partial t}=\frac{1}{\tilde{z}(t)}\frac{\partial z(t,p)}{\partial t}-\frac{z(t,p)}{\tilde{z}(t)}\frac{d\tilde{z}(t)}{dt} \tag{16}$$

In market equilibrium, eq.(7) implies that the second term on the right-hand side disappears. Hence, (16) turns with (14) into:

$$\frac{\partial P_z(t,p)}{\partial t}=\left(\tilde{s}\delta(p-p_s)-\frac{1}{\tau}\right)P_z(t,p)+D\frac{\partial^2 P_z(t,p)}{\partial p^2} \tag{17}$$

For the stationary state, we demand:

$$\frac{\partial P_z(t,p)}{\partial t}=0 \tag{18}$$

Hence, the stationary price distribution becomes with (17):

$$D\frac{d^2 P_z(p)}{dp^2}=\left(\frac{1}{\tau}-\tilde{s}\delta(p-p_s)\right)P_z(p) \tag{19}$$

For $p<p_s$, (19) reads:

$$\frac{d^2 P_z(p)}{d^2 p}=k^2 P_z(p) \tag{20}$$

with

$$k^2=\frac{1}{D\tau} \tag{21}$$

The solution of (20) is given by:

$$P_z(p)=Ae^{k(p-p_s)}+Be^{-k(p-p_s)} \tag{22}$$

with the free parameters A and B. For $p<p_s$ the distribution can be normalized only if $B=0$ and thus:

$$P_z(p)=Ae^{k(p-p_s)} \tag{23}$$

For $p>p_s$, the solution of (20) becomes:

$$P_z(p)=Fe^{k(p-p_s)}+Ge^{-k(p-p_s)} \tag{24}$$

with the free parameters F and G. For $p>p_s$ the distribution can be normalized only if $F=0$. Hence:

$$P_z(p)=Ge^{-k(p-p_s)} \tag{25}$$

We demand that the distribution $P_z(p)$ must be a continuous function everywhere on the price scale and $dP_z(p)/dp$ must be continuous, except at p_s. This condition implies that:

$$A=G \tag{26}$$

Form (23) and (25) follows therefore:

$$P_z(p)=Ae^{-k|p-p_s|} \tag{27}$$

For the normalization of the price distribution, we further demand that $P_z(p)$ is sufficiently localized around p_s, such that the normalization integral can be extended to minus infinity with a negligible error. For $\sigma/p_s<<1$, where σ is the standard deviation of the price distribution, the normalization condition reads:

$$\int_0^\infty P_z(p)dp\cong\int_{-\infty}^{\infty}Ae^{-k|p-p_s|}dp=1 \tag{28}$$

which yields $A=k/2$. The price distribution of available units can therefore be approximated by a Laplace distribution:

$$P_z(p)\cong\frac{1}{2\sigma}\exp\left(-\frac{|p-p_s|}{\sigma}\right) \tag{29}$$

with the standard deviation:

$$\sigma=\frac{1}{k}=\sqrt{D\tau}=\sqrt{v^2} \tag{30}$$

where we used (11). The standard deviation expresses the price volatility of the good.

With (13) we further obtain for the stationary state:

$$y(p)=\frac{1}{\tau}z(p) \tag{31}$$

From (2) and (4) follows then for their total numbers in the considered time interval:

$$\tilde{y}=\frac{1}{\tau}\tilde{z} \tag{32}$$

8 In other words, the more units are offered the more are purchased, proportional to the rate $1/\tau$. Deviations from this relation are contained in a time and price dependent offering time $\tau(t,p)$. As a first approximation $\tau(t,p)$ is replaced in this model by its mean value τ.

9 Note that relation (14) is known as a convection-diffusion equation. It takes the in-and outflow of available units and price variations by the retailers into account.

where:

$$\tilde{y} = \int_{\Delta t} \tilde{y}(t)dt; \tilde{z} = \int_{\Delta t} \tilde{z}(t)dt \;; \quad (33)$$

Thus, we obtain form (1) and (15) by applying (31) and (32):

$$P_y(p) = P_z(p) \quad (34)$$

It means, in market equilibrium is the stationary price distribution of sold units equivalent to the stationary price distribution of offered units. The price distribution of sold units has therefore with (29) the form:

$$P_y(p) = \frac{1}{2\sigma} \exp\left(-\frac{|p-\bar{p}|}{\sigma}\right) \quad (35)$$

while the standard deviation is given by (30). We used in this relation that the Laplace distribution is symmetric and therefore:

$$\bar{p} = p_s \quad (36)$$

The main result of this chapter is that, under the conditions (i) and (ii), the price distribution of sold units of a good in a retail market must have in general the form of a Laplace distribution. The standard deviation determines the price volatility of the good.

3. COMPARISON WITH INVESTIGATIONS

3.1. Price Distribution of an Aggregated Good

To compare the model with empirical data a price distribution $P_y(p)$ of an ensemble of goods is established.[10] For this purpose it is assumed that an aggregated good consists of two categories of goods. There are goods with fixed prices over the considered time interval. Their total unit sales rate is denoted $\tilde{y}_f$. The other goods are grouped to a combined good with variable prices governed by the market dynamics of the presented model. Their total unit sales rate is denoted $\tilde{y}_v$. The total unit sales rate of the aggregated good becomes:

$$\tilde{y} = \tilde{y}_v + \tilde{y}_f \quad (37)$$

The total price distribution of sold units of the aggregated good reads:

$$P_y(p) = (1-a)P_y^f(p) + aP_y^v(p) \quad (38)$$

with the free parameter:

$$a = \tilde{y}_v / \tilde{y} \quad (39)$$

10 The price distribution of an aggregated good can be established by two different procedures. One is to scale the price data of the individual goods by their mean price before aggregation. In this case the evolution of the supply price is eliminated, and the price distribution of the goods have according to this model the form of a Laplace distribution. The other is to aggregate all price data of the goods without scaling. Then the price distribution is dominated by distribution of the supply prices , which has the form of a lognormal distribution. This case is not considered here.

Since $\sigma \approx 0$ for the goods with rigid prices, their price distribution can be modelled as:

$$P_y^f(\mu) \cong \delta(\mu - 1) \quad (40)$$

where we introduced a scaled price:

$$\mu = p / \bar{p} \quad (41)$$

The scaled price distribution of the combined good with variable prices becomes with (35):[11]

$$P_y^v(\mu) = \frac{1}{2\sigma_\mu} \exp\left(-\frac{|\mu-1|}{\sigma_\mu}\right) \quad (42)$$

using a scaled standard deviation:

$$\sigma_\mu = \sigma / \bar{p} \quad (43)$$

Thus, the scaled price distribution of the aggregated good (38) has with (40) and (42) the form:

$$P_y(\mu) = (1-a)\delta(\mu-1) + \frac{a}{2\sigma_\mu} \exp\left(-\frac{|\mu-1|}{\sigma_\mu}\right) \quad (44)$$

It consists of a central peak at $\mu=1$ surrounded by a Laplace distribution. Note that the scaled aggregated price distribution depends only on the two unknown parameters a and σ_μ.

For a fit with an empirical price distribution the probability density function (pdf) can be transformed into the corresponding cumulative distribution function (cdf). The cdf of the aggregated good is defined by:

$$F_y(\mu) = \int_0^\mu P_y(\mu')d\mu' \quad (45)$$

The cdf of (44) reads:

$$F_y(\mu) = \begin{cases} \frac{a}{2}\exp\left(\frac{\mu-1}{\sigma_\mu}\right) & for \quad 0 < \mu < 1 \\ 1 - a/2 & for \quad \mu = 1 \\ 1 - \frac{a}{2}\exp\left(-\frac{\mu-1}{\sigma_\mu}\right) & for \quad \mu > 1 \end{cases} \quad (46)$$

The fit procedure can be performed by applying a least square fit of the cumulative distribution (46) using the two free parameters a and σ_μ.

3.2. Comparison with Empirical Data

The model is compared with empirical data of a comprehensive investigation of prices of consumer goods performed by Kaplan

11 The combined good is treated as a single good governed by the market dynamics of a retail market. As a consequence, the combined good must have a price distribution that has the form of a Laplace distribution with a standard deviation generated by the price volatility of the ensemble of goods.

and Menzio (2015). They studied data from the Kilts-Nielsen Consumer Panel Dataset (KNCP) and investigated price and quantity information for over 300 million transactions by 50,000 households. The panel covers over 1.4 million goods in 54 geographical markets for 2004-2009 (Δt=5 years). The investigators aggregated the data of products into four different categories of a good:

1. UPC: A good is the set of products with the same barcode (Universal Product Code: UPC).
2. Generic Brand Aggregation: According to this definition, a good is the set of products that share the same features, the same size, and the same brand, but may have different UPC's. Since the KNCP assigns the same brand code to all generic brands (regardless of the retailer), this definition collects all generic brand products that are otherwise identical.
3. Brand Aggregation: According to this definition, a good is the set of products that share the same features and the same size but may have different brands and different UPCs.
4. Brand and Size Aggregation: In this case a good is the set that share the same features but may have different sizes, different brands and different UPCs.

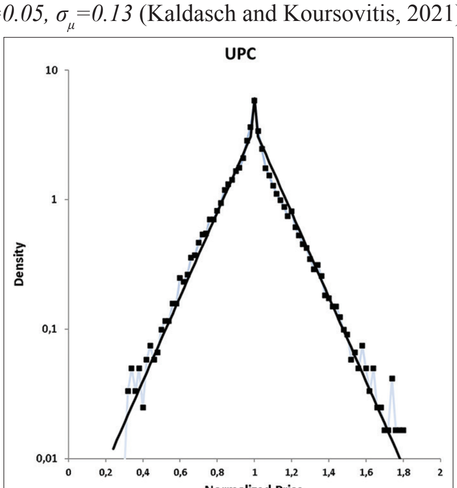
After scaling the price data of the goods by their mean price, they combined the data to an aggregated price distribution of the corresponding definitions of a good. Since the first definition of a good is closest to the assumptions (i) and (ii) used in this model, we confine here to a comparison with empirical data of the UPC definition.

The application of the theory to the empirical data is displayed in Figure 1. The squares indicate the empirical pdf, while the solid line is the outcome of the two-parameter fit procedure. A good quantitative coincidence with the empirical data can be obtained with $a= 0.05$ and a scaled standard deviation $\sigma_\mu=0.13$.[12]

12 The relative error is less than 10%.

13 It corresponds with the maximum of the distribution of standard deviations of relative prices of the considered goods (Kaplan and Menzio, 2015).

Figure 1: Displayed is the empirical price distribution (squares) for the first definition of a good, by plotting the distribution of normalized prices across all investigated markets, goods, and quarters (Kaplan and Menzio, 2015). The solid line indicates a least square fit of (46) with $a=0.05$, $\sigma_\mu=0.13$ (Kaldasch and Koursovitis, 2021).[13]



The data suggest that only a small part of 5% of the total sales rate are related to goods with fixed prices.

4. CONCLUSION

The paper establishes a model for the price distribution of goods in free retail markets. The main goal was to derive the price distribution of a good from the dynamics of demand and supply modelled by the conservation equation of offered units. Based on the assumptions (i) and (ii), the stationary price distribution is not a normal, but must have the form of a Laplace distribution. The standard deviation σ of the price distribution is determined by the price volatility of the good. To compare with aggregated price data an appropriate price distribution of an aggregated good was established. The presented theory suggests that the price distribution consists of central peak due to goods with rigid prices surrounded by a Laplace distribution. A comparison with an empirical investigation yields a good quantitative agreement with the two free parameters of the model.

The model suggests that the price evolution of goods in retail markets is governed on the one hand by the price of the supplier. The supply price evolution can be approximated as a geometric Brownian motion, generating to a lognormal price distribution. Retailers, on the other hand, make small random price variations to satisfy demand. Their price variations can be treated as an arithmetic Brownian motion generating a Laplace price distribution. Since for short time periods the supply price can be viewed as constant, the Laplace distribution can be also considered as a short-term price distribution.[14]

The presented theory yields a general form of the price distribution in retail markets. But assumption (ii) implies that the model is incapable to analyze specific origins of the price variations. To relate this approach to other models, for example to seller-buyer models as proposed by Varian (1980), further investigations are necessary.

14 An example for a short-term price distribution is electricity is a good. The price distribution has the form of eq.(35) (Sapio, 2004), not further discussed here.

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APPENDIX A

Derived is the last term of eq.(3). The idea is to consider the in- and outflow of available units $z(t,p)$ in a price interval $[p_2,p_1]$. The inflow of the absolute number of offered units at price p_1 is determined by:

$$dn_{in} = z(t,p_1)q(t,p_1)dt \tag{A.1}$$

and the outflow at p_2 *by:*

$$dn_{out} = z(t,p_2)q(t,p_2)dt \tag{A.2}$$

while $q(t,p)$ indicates the velocity of price changes of available units at time step t and price interval p and $p+dp$. The change in the number of offered units in the price interval dp during the time interval $\Delta t=t_2-t_1$ reads:

$$dn = (z(t_2,p)-z(t_1,p))dp \tag{A.3}$$

The change of the absolute number of available units in the considered interval can be obtained on the one hand by:

$$\Delta \mathrm{n} = \int_{\mathrm{p}_1}^{\mathrm{p}_2} \mathrm{dn} = \int_{\mathrm{p}_1}^{\mathrm{p}_2} (z(t_2,p)-z(t_1,p))dp \tag{A.4}$$

and on the other hand:

$$\Delta \mathrm{n} = \int_{\mathrm{t}_1}^{\mathrm{t}_2} \mathrm{dn_{in}} - \mathrm{dn_{out}} = \int_{\mathrm{t}_1}^{\mathrm{t}_2} (z(t,p_1)v(t,p_1)-z(t,p_2)q(t,p_2))dt \tag{A.5}$$

Since:

$$z(t_2,p)-z(t_1,p) = \int_{\mathrm{t}_1}^{\mathrm{t}_2} \frac{\partial z(t,p)}{\partial t}dt \tag{A.6}$$

and

$$z(t,p_1)q(t,p_1)-z(t,p_2)q(t,p_2) = -\int_{\mathrm{p}_1}^{\mathrm{p}_2} \frac{\partial z(t,p)q(t,p)}{\partial p}dp \tag{A.7}$$

we get for the difference between eq.(A.4) and (A.5):

$$\int_{p_1}^{p_2} \int_{\mathrm{t}_1}^{\mathrm{t}_2} \left(\frac{\partial z(t,p)}{\partial t} + \frac{\partial z(t,p)q(t,p)}{\partial p} \right) dtdp = 0 \tag{A.8}$$

while we have changed the integration order. Since this relation is valid for every price and time interval, we finally obtain for the last contribution to eq.(3):

$$\frac{\partial z(t,p)}{\partial t} = -\frac{\partial j(t,p)}{\partial p} \quad \text{(A.9)}$$

with the flow rate of offered units on the price scale:

$$j(t,p) = z(t,p)q(t,p) \quad \text{(A.10)}$$