International Journal of Energy Economics and Policy

ISSN: 2146-4553

available at http: www.econjournals.com

International Journal of Energy Economics and Policy, 2020, 10(5), 164-182.

Does the Choice of the Multivariate GARCH Model on Volatility Spillovers Matter? Evidence from Oil Prices and Stock Markets in G7 Countries

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Received: 20 February 2020 **Accepted:** 03 June 2020 **DOI:** https://doi.org/10.32479/ijeep.9469

ABSTRACT

In this paper, we employ asymmetric multivariate GARCH approaches to examine their performance on the volatility interactions between global crude oil prices and seven major stock market indices. Insofar as volatility spillover across these markets is a crucial element for portfolio diversification and risk management, we also examine the optimal weights and hedge ratios for oil-stock portfolio holdings with respect to the results. Our findings highlight the superiority of the asymmetric BEKK model and the fact that the choice of the model is of crucial importance given the conflicting results we got. Finally, our results imply that oil assets should be a part of a diversified portfolio of stocks as they increase the risk-adjusted performance of the hedged portfolio.

Keywords: Asymmetry, Multivariate GARCH, Stock Market, Oil Price, Volatility Spillover
JEL Classifications: C32, F3, G15, Q4

1. INTRODUCTION

Over the past years, the stock markets and crude oil markets have developed a reciprocal relationship. Every production sector in the international economy depends on oil as an energy source. Based on such dependence, fluctuations in oil price and its volatility are likely to affect the production sector and the international economy in general. Mork (1989) and Hooker (1999) documented that there is a significant negative relationship between crude oil price increases and world economic growth. Given that negative relationship, one would expect that increases in crude oil market prices will affect the firms' earnings and hence their stock price levels. Subsequently, the linkage between crude oil price volatility and stock markets seems to be quite evident. Many relevant studies such as Sadorsky (1999; 2001; 2006), Papapetrou (2001), Ewing and Thompson (2007) and Aloui and Jammazi (2009) conclude that a change in oil prices of either sign may affect stock price behavior. For this reason, investors should be aware of how shocks and volatility are transmitted across markets over time. Also, the increased financial integration between countries and the financialization of oil markets can enhance the ways of diversification of investors' portfolios. In order to take advantage of these ways, investors require a better understanding of how financial and oil markets correlate. By modeling volatility, researchers can produce accurate estimates of correlation and volatility which are key elements in developing optimal hedging strategies (see, for example, Chang et al. (2011)). Supporters of investing in commodities (mostly in oil) claim that if commodities have low or even negative correlations with stocks then a portfolio that includes commodities should perform better than a portfolio that excludes commodities (Sadorsky, 2014). This suggests that adding oil to an equity portfolio may lead to higher returns and lower risk than just investing in equities.

Since the development of the univariate ARCH model by Engle (1982) and GARCH model by Bollerslev (1986), an important body of literature has focused on using these models to model the volatility of oil and stock market returns. Furthermore, in the last decade, with the generalization of the univariate into multivariate GARCH models, the literature has focused on the volatility spillovers between oil and stock markets.

This paper makes several important contributions to the literature. First, while existing papers investigate the volatility dynamics between stock prices and oil prices, most of this literature focuses on individually developed economies, the Gulf Cooperation Council (GCC) countries or the BRICS (see, for example, Malik and Hammoudeh (2007); Arouri et al. (2011b); Creti et al. (2013)). This paper is specifically focused on the volatility dynamics between the G7 stock market prices and the Brent which is the global oil benchmark for light, sweet crudes. The choice of these countries is based on their importance to the global economy. For example, in 2017, according to worldstopexports.com the U.S. accounted for 15.9% of total crude oil imports and summing these percentages, the G7 countries accounted for 36.9% of total crude oil imports. Moreover, among the G7 countries, Canada is considered as an oil-exporter, so a slight distinction between oil -importers and -exporters can be made, adding this paper to the limited studies which make that kind of distinction (see, for example, Park and Ratti (2008); Apergis and Miller (2009); Filis et al. (2011)). Second, this paper differs from previous studies by comparing the performance of three asymmetric multivariate GARCH models namely, the ABEKK model of Kroner and Ng (1998), the AVARMA-CCC-GARCH model of McAleer et al. (2009) and the AVARMA-DCC-GARCH model which is a combination of the AVARMA-GARCH model of McAleer et al. (2009) and the DCC model of Engle (2002) in order to study the volatility spillover effects between developed stock market prices and oil prices. These models can simultaneously estimate the volatility cross-effects for the stock market indices and oil prices under consideration. In addition, these models can capture the effect of own shocks and lagged volatility on the current volatility, as well as the volatility transmission and the cross-market shocks of other markets.

The aim of this paper is to investigate the joint evolution of conditional returns, the correlation and volatility spillovers between the crude oil returns, namely Brent and the stock index returns of the G7 countries, namely CAC40 (France), DAX (Germany), DJIA (U.S.), FTSE100 (U.K.), MIB (Italy), Nikkei225 (Japan) and TSX (Canada). The asymmetric bivariate GARCH models are estimated using weekly return data from January 14, 1998, to December 27, 2017. A complementary objective is to use the estimated results to compute the optimal weights and hedge ratios that minimize overall risk in portfolios of each G7 country. Our results are crucial for building an accurate asset pricing model and forecasting volatility in stock and oil market returns.

The remainder of the paper is organized as follows. Section 2 reviews the literature. Section 3 describes the three asymmetric multivariate GARCH models. Section 4 presents the data and descriptive statistics. Section 5 discusses the empirical results and provides the economic implications for optimal portfolios and optimal hedging strategies. Section 6 concludes the paper.

2. LITERATURE REVIEW

This section presents a short literature review of papers that focus directly on the volatility dynamics between oil prices and stock markets. Malik and Hammoudeh (2007) investigate the volatility transmission between the global oil market (WTI), the U.S. equity market (S&P 500) and the Gulf equity market of Kuwait, Bahrain and Saudi Arabia. They use daily data from 14 February 1994 to 25 December 2001 and find evidence of bidirectional volatility spillovers only in the case of Saudi Arabia. Malik and Ewing (2009) use bivariate BEKK models to estimate volatility transmission between oil prices and five U.S. sector indices (Financial, Industrials, Health Care, Technology, and Consumer Services). Their results suggest significant transmission of shocks and volatility between oil prices and some of the examined market sectors. Choi and Hammoudeh (2010) investigate the time-varying correlation between the S&P500 and oil prices (Brent and WTI), copper, gold, and silver. They find decreasing correlations between the commodities and the S&P500 index since the 2003 Iraq war. Vo (2011) examines the inter-dependence between crude oil price volatilities (WTI) and the S&P500 index over the period 1999-2008. The author supports that there is inter-market dependence in volatility. Arouri et al. (2011a) employ bivariate GARCH models using weekly data from 01 January 1998 to 31 December 2009 to examine volatility spillovers between oil prices and stock markets in Europe and United States at the sector-level. They find a bidirectional spillover effect between oil and U.S. stock market sectors and a univariate spillover effect from oil to stock markets in Europe. Arouri et al. (2011b) study the return and volatility transmission between oil prices and stock markets in the Gulf Cooperation Council (GCC) countries over the period 2005 and 2010. They use the VAR-GARCH approach to conclude that there are spillovers between these markets. Arouri et al. (2012) investigate volatility spillovers between oil and stock markets in Europe. They use weekly data from January 1998 to December 2009 and a bivariate GARCH model. They find evidence of volatility spillovers between oil prices and stock market prices. Chang et al. (2013) employ multivariate GARCH models to investigate conditional correlations and volatility spillovers between oil prices and the stock prices of the U.S. and U.K. Their findings provide little evidence of volatility spillovers between these markets. Mensi et al. (2013) use bivariate VAR-GARCH models to study volatility transmission between S&P500 and energy price indices (WTI and Brent), among other commodities, over the period 2000 and 2011. Their results suggest significant transmission among the S&P500 and commodity markets, while the highest conditional correlations are between S&P500 and gold index and between the S&P500 and WTI index. Bouri (2015) studies four MENA countries, namely Lebanon, Jordan, Tunisia, and Morocco over the period 2003-2013. His results suggest that in the pre-financial crisis period there is no volatility transmission between oil and stock markets of MENA countries. However, some evidence of linkages is revealed in the post-financial crisis period but not for all countries. Du and He (2015) examine the risk spillovers between oil (WTI) and stock (S&P500) markets using daily data from September 2004 to September 2012. Their findings suggest that in the pre-financial crisis period, there are positive risk spillovers from the stock market to the oil market and negative spillovers from oil to the stock market. In the post-financial crisis period, bidirectional positive risk spillovers are reported. Khalfaoui et al. (2015) is one of the extremely limited studies focusing on G7 countries. They investigate the linkage of the crude oil market (WTI) and stock markets of the G7 countries using a combination of multivariate GARCH models and wavelet analysis. They find strong volatility spillovers between oil and stock markets and that oil market volatility is leading stock market volatility. Phan et al. (2016) examine the price volatility interaction between the crude oil (WTI) and equity markets in the U.S. (S&P500 and NASDAQ) using intraday data over the period 2009 and 2012. They claim that even in the future markets there are cross-market volatility effects. Ewing and Malik (2016) use univariate and multivariate GARCH models to investigate the volatility of oil prices (WTI) and U.S. stock market prices (S&P500). They use daily data over the period from July 1996 to June 2013 and take into account structural breaks. Their results show no volatility spillover between these markets when structural breaks are ignored. However, after accounting for breaks, they find a significant volatility spillover between oil prices and the U.S. stock market.

The next few studies are focused on oil-exporting and oil-importing countries. Park and Ratti (2008) use monthly data for 13 European countries and the U.S. over the period 1986:1-2005:12. They find that positive oil price shocks cause positive returns for the stock market of the oil-exporting country (Norway), however, the opposite occurs for the rest of the European countries but not for the U.S. (oil-importers). Apergis and Miller (2009) use monthly data for the G7 countries and Australia to conclude that major stock market (independently of oil-exporting or oil-importing) returns do not respond in oil market shocks. Filis et al. (2011) employ multivariate DCC-GARCH-GJR models to investigate the time-varying correlation between oil prices and stock prices of oil-exporting (Brazil, Canada, and Mexico) and oil-importing (U.S.A., Germany, and Netherlands) countries. They find, among others, that the time-varying correlation does not differ between oil-importing and oil-exporting countries. Maghyereh et al. (2016) utilize 3 oil-exporting and 8 oil-importing countries over the period 2008-2015. Their findings support that oil price volatility is the significant transmitter of volatility shocks to stock market volatilities and that there is no difference between oil-importers and oil-exporters.

3. ECONOMETRIC METHODOLOGY

Since the objective of this paper is to investigate volatility interdependence and transmission mechanisms between stock and oil markets, multivariate frameworks such as the AVARMA-CCC-GARCH model of McAleer et al. (2009), the AVARMA-DCCGARCH and the ABEKK-GARCH model of Kroner and Ng (1998) are more relevant than univariate GARCH models. The first model assumes constant conditional correlations, while the last two accommodate dynamic conditional correlations. Combined with a vector autoregressive (VAR) model for the mean equation, they allow us to examine returns spillovers too. In what follows we present the bivariate framework of these three models.

The econometric specification has two components, a mean equation, and a variance equation. The first step in the bivariate GARCH methodology is to specify the mean equation. For each pair of stock and oil returns, we try to fit a bivariate VAR model. For example, a bivariate VAR(1) model has the following specification for the conditional mean[1]:

$$r_t = \mu + \Psi r_{t-1} + u_t \tag{1}$$

where $r_t = (r_{s,t}, r_{o,t})'$ is the vector of returns on the stock and oil price index, respectively. Ψ refers to a 2×2 matrix of parameters of the form $\Psi = \begin{pmatrix} \psi_{ss} & \psi_{so} \\ \psi_{os} & \psi_{oo} \end{pmatrix}$. $u_t = (u_{s,t}, u_{o,t})'$ is the vector of the error terms of the conditional mean equations for stock and oil returns, respectively.

The asymmetric BEKK model proposed by Kroner and Ng (1998) is an extension of the BEKK model of Engle and Kroner (1995). Their difference is one extra matrix that takes into account the asymmetries. Its equation has the following form:

$$H_t = C'C + A'u_{t-1}u'_{t-1}A + B'H_{t-1}B + D'v_{t-1}v'_{t-1}D \tag{2}$$

where $u_t = H_t^{1/2}\eta_t$, $\eta_t \sim iid\ N(0,1)$ and $H_t = \begin{bmatrix} h_{s,t} & h_{so,t} \\ h_{so,t} & h_{o,t} \end{bmatrix}$ is the conditional variance-covariance matrix. The individual elements for C, A, B and D matrices of equation (2) in the bivariate case are given as:

$$C = \begin{bmatrix} c_{ss} & c_{os} \\ 0 & c_{oo} \end{bmatrix} \quad A = \begin{bmatrix} a_{ss} & a_{so} \\ a_{os} & a_{oo} \end{bmatrix} \quad B = \begin{bmatrix} b_{ss} & b_{so} \\ b_{os} & b_{oo} \end{bmatrix}$$
$$D = \begin{bmatrix} d_{ss} & d_{so} \\ d_{os} & d_{oo} \end{bmatrix} \tag{3}$$

where C is a 2×2 upper triangular matrix, A is a 2×2 square matrix of coefficients and shows the extent to which conditional variances are correlated with past squared errors. B is also a 2×2 square matrix of coefficients and reveals how current levels of conditional variances are related to past conditional variances. D is a 2×2 matrix and v is defined as u if u is negative and zero otherwise. For example, a statistically significant coefficient on d_{ss} would indicate that the "bad" news of the first variable affects its variance more than the "good" news of the same magnitude. Moreover, it should be mentioned that if the D matrix is zero then the ABEKK model reduces to the simple BEKK model. The ABEKK model has the property that the conditional variance-covariance matrix is positive definite. However, this model suffers from the curse of dimensionality (for more details see McAleer et al. (2009)). The following likelihood function is maximized assuming normally distributing errors:

$$L(\theta) = -\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left(\log\left|H_t\right| + u'_t H_t^{-1} u_t\right) \tag{4}$$

1 The appropriate lag length of the VAR models was chosen on the basis of the Schwarz information criterion (SIC).

where T is the number of observations and θ refers to the parameter vector to be estimated. Numerical maximization techniques were employed to maximize this log-likelihood function. As recommended by Engle and Kroner (1995) several iterations were performed with the simplex algorithm to obtain the initial conditions. Then, the Broyden (1970), Fletcher (1970), Goldfarb (1970) and Shanno (1970) algorithm (BFGS) was employed to obtain the estimate of the variance-covariance matrix and the corresponding standard errors[2].

We now shift our attention to another class of GARCH specifications that model the conditional correlations rather than the conditional covariance matrix H_t. In order to take into account asymmetries and interdependencies of volatility across different markets, McAleer et al. (2009) proposed the AVARMA-CCC-GARCH(1,1) model which has the following specification in its bivariate form for the conditional variances-covariance:

$$h_{s,t} = c_{ss} + a_{ss}u_{s,t-1}^2 + b_{ss}h_{s,t-1} + a_{so}u_{o,t-1}^2 + b_{so}h_{o,t-1} + d_{ss}I_{t-1}u_{s,t-1}^2 \tag{5}$$

$$h_{o,t} = c_{oo} + a_{oo}u_{o,t-1}^2 + b_{oo}h_{o,t-1} + a_{os}u_{s,t-1}^2 + b_{os}h_{s,t-1} + d_{oo}I_{t-1}u_{o,t-1}^2 \tag{6}$$

$$h_{so,t} = \rho\sqrt{h_{s,t}}\sqrt{h_{o,t}} \tag{7}$$

where I_u is defined as follows:

$$\mathrm{I}_u = \begin{cases} 0, u_{i,t-1} > 0 \\ 1, u_{i,t-1} \le 0 \end{cases} \tag{8}$$

The volatility transmission between stock and oil markets over time is captured by the cross values of error terms ($u_{o,t-1}^2$ and $u_{s,t-1}^2$) and the lagged conditional volatilities ($h_{o,t-1}$) and ($h_{s,t-1}$). The error terms gauge the impact of direct effects of shock transmission, while the lagged conditional variances measure the direct effects of risk transmission across the markets. In other words, the conditional variance of the stock market depends not only on its own past values and its own innovations but also on those of the oil market and vice versa. Hence, this model allows shock and volatility transmission between the oil and stock markets under consideration. As it is clear if the d_{ii} are simultaneously zero, then the AVARMA-CCC model reduces to a VARMA-CCC model and if the elements a_{ij} and b_{ij} ($i \neq j$) are also zero then the model becomes the simple Constant Conditional Correlation (CCC). Ling and McAleer (2003) proposed the quasi-maximum likelihood estimation (QMLE) to obtain the parameters of the above bivariate model, which is appropriate when, η_t does not follow a joint multivariate normal distribution.

Our last model is a combination of the AVARMA-GARCH model of McAleer et al. (2009) and the DCC model of Engle (2002). This model is estimated in two steps simplifying the estimation of the time-varying correlation matrix. In the first step, the AVARMA-GARCH(1,1) parameters are estimated. In the second step, the conditional correlations are estimated. It has the same equation as the AVARMA-CCC-GARCH(1,1) model with an exception that the conditional covariance is not constant.

$$H_t = L_t R_t L_t \tag{9}$$

In the bivariate form, H_t is a 2 × 2 diagonal conditional covariance matrix, L_t is a diagonal matrix with time-varying standard deviations on the diagonal and R_t is the conditional correlation matrix.

$$L_t = diag\left(h_{s,t}^{1/2}, h_{o,t}^{1/2}\right) \tag{10}$$

$$R_t = diag\left(q_{s,t}^{-1/2}, q_{o,t}^{-1/2}\right) Q_t \, diag\left(q_{s,t}^{-1/2}, q_{o,t}^{-1/2}\right) \tag{11}$$

The expressions $h_{s,t}$ and $h_{o,t}$ are univariate GJR models of Glosten et al. (1993) with VARMA specification which is equal to an AVARMA specification (see, equations (5) and (6)). Q_t is a symmetric positive definite matrix.

$$Q_t = \left(1-\theta_1-\theta_2\right)\bar{Q} + \theta_1 z_{t-1}z_{t-1}' + \theta_2 Q_{t-1} \tag{12}$$

$\bar{Q}$ is a 2 × 2 unconditional correlation matrix of the standardized residuals $\eta_{i,t}$ ($\eta_{i,t} = u_{i,t}/\sqrt{h_{i,t}}$). The parameters θ_1 and θ_2 are non-negative. The model is mean-reverting as long as $\theta_1 + \theta_2 < 1$. The matrix Q_t does not replace H_t, its purpose is to provide conditional correlations $\rho_{so,t}$.

$$\rho_{so,t} = \frac{q_{so,t}}{\sqrt{q_{ss,t}q_{oo,t}}} \tag{13}$$

Hence, for the conditional covariance equation, we end up in the following expression

$$h_{so,t} = \rho_t\sqrt{h_{s,t}}\sqrt{h_{o,t}} \tag{14}$$

which is the only difference from the AVARMA-CCC-GARCH(1,1) model. The AVARMA specification on the CCC and DCC models allows for spillovers among the variances of the series, and also makes the DCC form almost identical to that used for the ABEKK model, allowing for direct comparisons of model performance (Efimova and Serletis, 2014). In addition, permitting for asymmetries in the models provides valuable information to policy-makers and financial market participants, on the existing differences between the impact of positive and negative news on stock and oil market price fluctuations. The fact that asymmetric effects are significant depicts potential misspecification if asymmetries are neglected.

4. DATA AND PRELIMINARY RESULTS

For this study, weekly data on the Wednesday closing prices for crude oil and stock indices were used. Crude oil includes one of the two global light benchmarks, namely the Europe

2 Quasi-maximum likelihood estimation was used and robust standard errors were calculated by the method given by Bollerslev and Wooldridge (1992).

Brent. The series for oil prices were obtained from the Energy Information Administration (EIA). The stock market indices are Dow Jones Industrial Average (United States), CAC40 (France), DAX (Germany), FTSE MIB (Italy), Nikkei225 (Japan), FTSE100 (United Kingdom) and S&P/TSX (Canada). This data[3] was obtained from Yahoo Finance. The data[4] range spans from 07 January 1998 to 27 December 2017 for a total of 1043 observations. Wednesday closing prices were used because in general there are fewer holidays on Wednesdays than on Fridays. Any missing data on Wednesday closes was replaced with closing prices from the most recent successful trading session. The use of weekly data significantly reduces any potential biases that may arise such as the non-trading days, bid-ask effect etc. Consistent with other studies, our analysis focuses on the returns as the price series were non-stationary in levels. Stock market and oil price returns are computed as the first log-difference, i.e. $r_t = 100 \times \ln(P_t/P_{t-1})$, where P_t is the weekly closing price. The summary for the corresponding returns, as well as the unit root tests and the Ljung and Box (1978) statistics, are shown in Table 1.

All the series have a positive mean except for MIB and for each series, the standard deviation is larger than the mean value. As measured by the standard deviation, equity market return unconditional volatility is highest in Italy, followed by Germany, Japan, France, U.K., Canada, and the U.S., while the oil price volatility is the highest among them all. In terms of skewness, each series displays negative skewness and a large amount of kurtosis, a fairly common occurrence in high-frequency financial data which implies that the GARCH model of Bollerslev (1986) is adequate. In addition, the null hypothesis of normality is rejected for all return series by the Jarque and Bera (1980) test statistic at 1% level of significance. The (squared) Q-statistic of Ljung and Box (1978) which is used for detection of (heteroskedasticity) autocorrelation is significant in all cases, implying that the past behavior of the market may be more relevant. The Augmented Dickey and Fuller (1979; 1981) unit root tests indicate that all the return series are stationary at the 1% level of significance. The unconditional correlations of all stock indices with the Brent crude oil are positive, yet not high. Figures A1 and A2 exhibits the evolution of the closing prices and the returns series during the period of the study. The oil series recorded sample high in 2008 and it is clear that it is the most volatile series.

5. EMPIRICAL RESULTS

This section reports on the empirical results obtained from the estimating bivariate GARCH models. Empirical results are presented for our three competitive models: ABEKK-GARCH(1,1), AVARMA-CCC-GARCH(1,1) and AVARMA-DCC-GARCH(1,1) in Tables A1-A7 (in Appendix). In order to compare their performance on the volatility spillover effects, we will interpret their estimates using Wald tests (Tables 2-4). We focus on statistical significance at the 5% level. Wald test is used to test the matrix elements of the volatility spillover effect, which is the joint test for the significance of the model coefficients (see, Beirne et al. (2010); Liu et al. (2017)). We test the following two set of hypotheses:

H_0: $a_{so} = b_{so} = 0$ or there is no volatility spillover from oil to stock (15)

H_1: $a_{so} \neq 0$ or $b_{so} \neq 0$ or there is volatility spillover from oil to stock (16)

H_0: $a_{os} = b_{os} = 0$ or there is no volatility spillover from stock to oil (17)

H_1: $a_{os} \neq 0$ or $b_{os} \neq 0$ or there is volatility spillover from stock to oil (18)

In addition, implications of the results on optimal weights and hedge ratios for oil-stock portfolio holdings are depicted in Table 5.

First, we have to determine the mean equations. As it is apparent from Table 6, the Schwarz information criterion indicates not to use a VAR framework. Hence, the mean equations for all pairs will consist of just a constant for each series. Therefore, we cannot seek for mean spillover effects among the markets.

Regarding the variance equations and the CAC40 index (Tables A1 and 2-4), we find that each model provides evidence of conditional GARCH (significant coefficients on b_{ss} and b_{oo}) effects in stock and oil's variance equations meaning that each current volatility

3 Indices' codes in the corresponding database, U.S.-^DJI, France-^FCHI, Germany-^GDAXI, Italy-FTSEMIB.MI, Japan-^N225, U.K.-^FTSE, Canada-^GSPTSE, Europe Brent spot price FOB-RBRTE.

4 Oil prices are measured in U.S. dollars per barrel, however stock prices are in national units.

Table 1: Descriptive statistics

Obs. 1042	CAC40	DAX	DJIA	FTSE100	MIB	Nikkei225	TSX	BRENT
Mean	0.056	0.106	0.110	0.036	−0.015	0.040	0.086	0.141
St. dev.	3.077	3.272	2.229	2.418	3.322	3.099	2.270	5.118
Skewness	−0.317	−0.632	−0.586	−0.310	−0.411	−0.466	−0.760	−0.117
Kurtosis	6.207	6.386	7.494	6.253	4.833	6.326	6.453	4.645
Jarque-Bera	467.47*	571.08*	942.27*	479.46*	176.8*	521.58*	621.82*	121.09*
$Q(24)$	49.10*	32.01	39.71**	38.01**	44.33*	50.30*	49.09*	27.65
$Q^2(24)$	318.13*	261.44*	245.44*	320.94*	286.09*	117.32*	355.85*	354.10*
ADF	−37.40*(0)	−35.43*(0)	−33.07*(0)	−35.74*(0)	−34.42*(0)	−33.06*(0)	−32.69*(0)	−31.81*(0)
Corr. with brent	0.214	0.195	0.185	0.222	0.241	0.183	0.353	1.000

*, ** indicate statistical significance at 1% and 5% respectively. The numbers within parentheses followed by ADF statistics represent the lag length of the dependent variable used to obtain white noise residuals. The lag lengths for ADF equations were selected using the Schwarz Information Criterion (SIC). MacKinnon (1996) critical values for rejection of the hypothesis of unit root applied. $Q(24)$ and Q^2 (24)are the Ljung and Box (1978) statistics for serial correlation and conditional heteroskedasticity of the series at 24th lag

Table 2: Wald tests for volatility spillover effects with the ABEKK model

	H_0	Wald	Sig.	Conclusion
CAC40	$a_{so}=b_{so}=0$	7.951	0.019	Spillover from Brent to CAC40
	$a_{os}=b_{os}=0$	11.086	0.004	Spillover from CAC40 to Brent
DAX	$a_{so}=b_{so}=0$	4.024	0.134	No spillover from Brent to DAX
	$a_{os}=b_{os}=0$	13.776	0.001	Spillover from DAX to Brent
DJIA	$a_{so}=b_{so}=0$	10.544	0.005	Spillover from Brent to DJIA
	$a_{os}=b_{os}=0$	29.538	0.000	Spillover from DJIA to Brent
FTSE100	$a_{so}=b_{so}=0$	11.084	0.004	Spillover from Brent to FTSE100
	$a_{os}=b_{os}=0$	13.146	0.001	Spillover from FTSE100 to Brent
MIB	$a_{so}=b_{so}=0$	9.813	0.007	Spillover from Brent to MIB
	$a_{os}=b_{os}=0$	25.814	0.000	Spillover from MIB to Brent
Nikkei225	$a_{so}=b_{so}=0$	6.211	0.045	Spillover from Brent to Nikkei225
	$a_{os}=b_{os}=0$	5.221	0.074	No spillover from Nikkei225 to Brent
TSX	$a_{so}=b_{so}=0$	6.680	0.035	Spillover from Brent to TSX
	$a_{os}=b_{os}=0$	13.887	0.001	Spillover from TSX to Brent

Table 3: Wald tests for volatility spillover effects with the AVARMA-CCC model

	H_0	Wald	Sig.	Conclusion
CAC40	$a_{so}=b_{so}=0$	2.481	0.289	No spillover from Brent to CAC40
	$a_{os}=b_{os}=0$	1.629	0.443	No spillover from CAC40 to Brent
DAX	$a_{so}=b_{so}=0$	2.444	0.295	No spillover from Brent to DAX
	$a_{os}=b_{os}=0$	2.022	0.364	No spillover from DAX to Brent
DJIA	$a_{so}=b_{so}=0$	2.314	0.314	No spillover from Brent to DJIA
	$a_{os}=b_{os}=0$	1.136	0.567	No spillover from DJIA to Brent
FTSE100	$a_{so}=b_{so}=0$	0.376	0.829	No spillover from Brent to FTSE100
	$a_{os}=b_{os}=0$	0.485	0.784	No spillover from FTSE100 to Brent
MIB	$a_{so}=b_{so}=0$	2.348	0.309	No spillover from Brent to MIB
	$a_{os}=b_{os}=0$	7.943	0.019	Spillover from MIB to Brent
Nikkei225	$a_{so}=b_{so}=0$	4.947	0.084	No spillover from Brent to Nikkei225
	$a_{os}=b_{os}=0$	0.404	0.817	No spillover from Nikkei225 to Brent
TSX	$a_{so}=b_{so}=0$	1.934	0.380	No spillover from Brent to TSX
	$a_{os}=b_{os}=0$	8.584	0.014	Spillover from TSX to Brent

Table 4: Wald tests for volatility spillover effects with the AVARMA-DCC model

	H_0	Wald	Sig.	Conclusion
CAC40	$a_{so}=b_{so}=0$	1.022	0.600	No spillover from Brent to CAC40
	$a_{os}=b_{os}=0$	1.403	0.496	No spillover from CAC40 to Brent
DAX	$a_{so}=b_{so}=0$	1.704	0.427	No spillover from Brent to DAX
	$a_{os}=b_{os}=0$	2.991	0.224	No spillover from DAX to Brent
DJIA	$a_{so}=b_{so}=0$	4.663	0.097	No spillover from Brent to DJIA
	$a_{os}=b_{os}=0$	3.650	0.161	No spillover from DJIA to Brent
FTSE100	$a_{so}=b_{so}=0$	1.243	0.537	No spillover from Brent to FTSE100
	$a_{os}=b_{os}=0$	2.344	0.310	No spillover from FTSE100 to Brent
MIB	$a_{so}=b_{so}=0$	3.308	0.191	No spillover from Brent to MIB
	$a_{os}=b_{os}=0$	9.121	0.010	Spillover from MIB to Brent
Nikkei225	$a_{so}=b_{so}=0$	8.446	0.015	Spillover from Brent to Nikkei225
	$a_{os}=b_{os}=0$	0.317	0.853	No spillover from Nikkei225 to Brent
TSX	$a_{so}=b_{so}=0$	0.028	0.986	No spillover from Brent to TSX
	$a_{os}=b_{os}=0$	2.864	0.239	No spillover from TSX to Brent

Table 5: Optimal portfolio weights and hedge ratios for pairs of oil and stock assets

Portfolio	ABEKK	AVARMA-CCC	AVARMA-DCC
CAC40/Brent			
$w_{so,t}$	0.2172	0.2160	0.2058
$\beta_{so,t}$	0.1311	0.1092	0.1327
DAX/Brent			
$w_{so,t}$	0.2518	0.2479	0.2368
$\beta_{so,t}$	0.1194	0.1078	0.1272
DJIA/Brent			
$w_{so,t}$	0.1165	0.1257	0.1121
$\beta_{so,t}$	0.0741	0.0538	0.0715
FTSE100/Brent			
$w_{so,t}$	0.1242	0.1269	0.1208
$\beta_{so,t}$	0.1117	0.0929	0.1061
MIB/Brent			
$w_{so,t}$	0.2742	0.2650	0.2655
$\beta_{so,t}$	0.1605	0.1301	0.1707
Nikkei225/Brent			
$w_{so,t}$	0.2561	0.2575	0.2494
$\beta_{so,t}$	0.0983	0.0967	0.1107
TSX/Brent			
$w_{so,t}$	0.0769	0.0599	0.0559
$\beta_{so,t}$	0.1461	0.1396	0.1521

The table reports average optimal weights of oil and hedge ratios for an oil-stock portfolio using the estimated conditional variances and covariance from the three models for each oil/stock pair: ABEKK-GARCH(1,1), AVARMA-CCC-GARCH(1,1) and AVARMA-DCC-GARCH(1,1)

is depending on its own past volatility. The same holds, only for the oil's variance equations for the ARCH effects (significant coefficients on a_{oo}) which means that the current volatility is affected by its own past shocks. In addition, bidirectional volatility

Table 6: Information criterion for VAR estimation

Lags	0	1	2	3	4	5
CAC40	11.1555*	11.1572	11.1767	11.1948	11.2165	11.2380
DAX	11.2865*	11.3003	11.3182	11.3371	11.3599	11.3786
DJIA	10.5235*	10.5447	10.5647	10.5816	10.6060	10.6285
FTSE100	10.6673*	10.6787	10.6974	10.7197	10.7378	10.7606
MIB	11.2955*	11.3149	11.3342	11.3486	11.3685	11.3907
Nikkei225	11.1757*	11.2007	11.2242	11.2436	11.2681	11.2914
TSX	10.4579*	10.4704	10.4951	10.5144	10.5348	10.5438

*indicates the optimal lag selected by the Schwarz information criterion for each pair of stock index and crude oil Brent returns

spillover between the French stock market and the Brent oil was found according to the Asymmetric BEKK model however, both the AVARMA models failed to detect any volatility transmissions between these markets.

In terms of the DAX index (Table A2), all three models show that the conditional variances of stock and oil markets are characterized by their own lagged conditional variances. The asymmetric BEKK model supports the presence of ARCH effects in both equations (significant coefficients on a_{ss} and a_{oo}) however, once again, the AVARMACCC and AVARMA-DCC models fail to provide evidence of own past shocks regarding the stock markets' equations (insignificant coefficients on a_{ss}), yet both models show that the current variance of the oil market is depending on its own past shocks. Furthermore, the ABEKK model reveals volatility spillover from DAX to Brent as indicated by the statistically significant coefficient of the Wald test (Table 2). In contrast, the results of the other two models agree on the absence of any volatility spillover effects between the two variables.

With regard to Table A3 and the American stock market, all three models present strong evidence of own short and long-term persistence (except for the AVARMA-CCC and AVARMA-DCC models in which the coefficients on a_{ss} are not significant). The ABEKK model uncovers bidirectional volatility transmission while the remaining models do not show any relation among the markets.

Turning our interest in the English index (Table A4) and regarding the ABEKK model, our findings show that the conditional variance of both indices is depending on its own past shocks and own past volatilities. The AVARMA-CCC model indicates that only the conditional variance of the Brent oil is affected by its own past volatility, while, the AVARMA-DCC model depicts that the stock market's variance is affected only by its own shocks and that the current volatility of the oil market depends on its own past shocks and past volatility. Once again, the AVARMA models validate the absence of any volatility transmission between the stock and oil markets. Nevertheless, the ABEKK model yields evidence of a two-way causality in the variance.

From the Italian stock market and Table A5, we ascertain that regardless of the model, the current volatility of the oil is affected by its own shocks and past volatility and that the current volatility of the MIB index is depending on its own past volatility. In addition, the ABEKK model depicts evidence of ARCH effects in the stock's equation. All three models reveal a unidirectional volatility transmission from the stock market to the oil market while the ABEKK model supports also the reverse direction of causality.

Particularly interesting results arise for the Japanese stock market (Table A6). First, while the ABEKK model indicates considerable evidence of own short persistence in the stock's equation, the rest of the models support that only the own past volatility has an effect on the current volatility for both indices. Second, the ABEKK framework provides evidence of unidirectional volatility spillover from the Brent oil to the Japanese stock market while, regarding the results of the AVARMA-CCC model, we find a lack of any volatility spillover. In contrast, the AVARMA-DCC model agrees with the asymmetric BEKK model on the one-way causality from the oil market to the stock market.

Finally from Table A7, the findings for the stock market of our only oil-exporting country-Canada note that for all models, the conditional variances are depending on their own lagged volatility. Moreover, the ABEKK model provides evidence of short-term persistence in the stock equation. In addition, according to ABEKK results, there is a feedback volatility spillover. The AVARMA-CCC reveals a unidirectional volatility transmission from the TSX to the Brent oil market. Instead, the AVARMA-DCC model supports that the two markets are independent.

For each pair of crude oil and stock assets, the estimated coefficients on the constant conditional correlations from the AVARMA-CCC models are very low and statistically significant. Moreover, the significant coefficients on d_{oo} and d_{ss}, in almost all cases, propose that the "bad" news tends to increase the volatility of the indices more than the "good" news of the same magnitude. In addition, given the significant coefficient on d_{os} only for the case of TSX, the results support that the past shocks of the Canadian stock market have an asymmetric effect on oil volatility.

The asymmetric BEKK model outperforms the rest of the models based on the Log-Likelihood value, with an exception of the DAX and Nikkei225 (AVARMA-DCC fits better), indicating its superiority. Diagnostics tests on the standardized residuals show that only in the Japanese stock market, the mean equations were not enough to deal with autocorrelation. Nevertheless, the Q-test statistics of Ljung and Box (1978) on the squared standardized residuals and the ARCH test of Engle (1982) are not statistically significant, implying that the MGARCH models were adequate to eliminate the ARCH effects.

Overall, the results from the ABEKK model reveal plenty of interactions among the markets while, both the AVARMA models are more parsimonious in the relations of the volatility. Figure 1 summarizes the results of the volatility spillover effects of the three competitive models. As it is apparent from Figure 1, in the case of the ABEKK model, all indices affect, or are affected by the oil market, yet this is not the case for the AVARMA models. The AVARMA-CCC model uncovers interactions only from the Italian and the Canadian stock markets to the oil market and the AVARMA-DCC model proposes that the Italian stock market is able to affect the oil market as well as that the Japanese stock market is depending on the Brent market. Moreover, for each asset, the estimated coefficient on own long-term persistence is greater than the estimated coefficient on own short-term persistence. Interestingly, we can conclude that volatility spillover effects are highly dependent on the choice of the multivariate GARCH model.

The conditional volatility estimates can be used to construct hedge ratios as proposed by Kroner and Sultan (1993). A long position in a stock asset can be hedged with a short position in an oil asset. The hedge ratio between stock and oil assets can be written as:

$$\beta_{so,t} = h_{so,t} / h_{oo,t} \tag{19}$$

where $h_{so,t}$ is the estimated covariance and $h_{oo,t}$ is the estimated variance of the crude oil market. We compute the hedge ratios from our three models (ABEKK, AVARMA-CCC and AVARMA-DCC). Their graphs are presented in Figures A3 and A4 and show considerable variability across the sample period indicating that hedging positions must be adjusted frequently.

Again, the estimated conditional volatilities from the three models can be used to construct optimal portfolio weights. The optimal holding weight of oil in a one-dollar portfolio of oil/stock asset at time t, according to Kroner and Ng (1998), can be expressed as:

$$w_{so,t} = \frac{h_{ss,t} - h_{so,t}}{h_{oo,t} - 2h_{so,t} + h_{ss,t}} \tag{20}$$

under the condition that

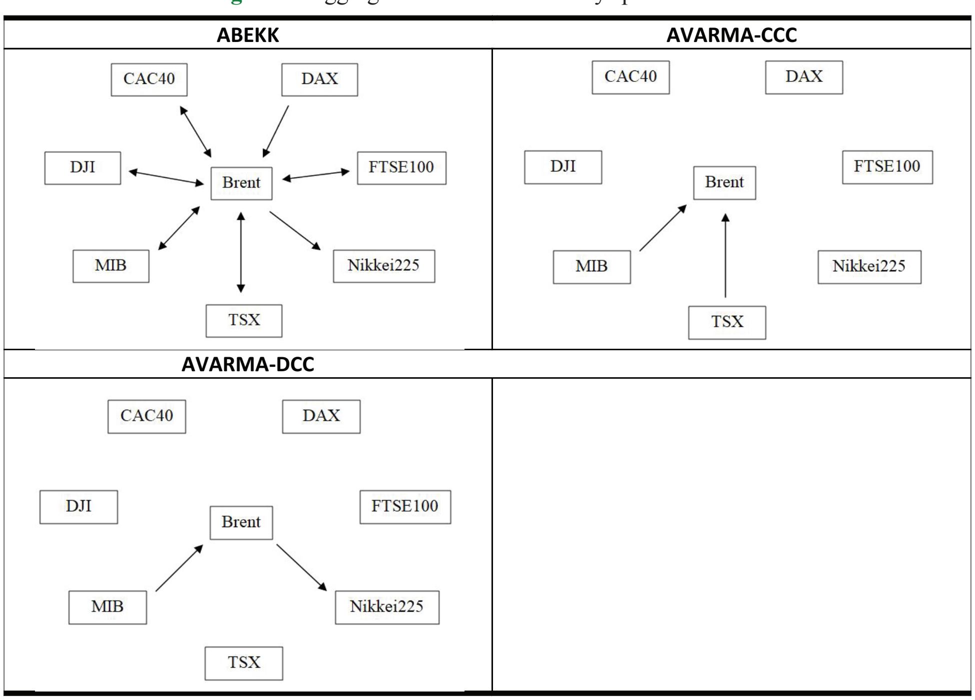
$$w_{so,t} = \begin{cases} 0 & w_{so,t} < 0 \\ w_{so,t} & 0 \le w_{so,t} \le 1 \\ 1 & w_{so,t} > 1 \end{cases} \tag{21}$$

Hence, the weight of the stock market index in the oil/stock portfolio is equal to $(1-w_{so,t})$. By using three multivariate GARCH models to compute the optimal portfolio weights and the hedge ratios enable us to discuss the results from a comparative perspective.

We report the average values of optimal weights $w_{so,t}$ and hedge ratios $\beta_{so,t}$ in Table 5. For example, the average value of the hedge ratio between CAC40 and Brent, according to the ABEKK model, is 0.1311 indicating that a 1$ long position in CAC40 can be hedged for 13.11 cents in the oil market. Similarly, the corresponding value of the hedge ratio under the AVARMA-CCC model is 0.1092 implying that a 1$ long position in CAC40 should be shorted by 10.92 cents of Brent oil. Overall, all models give suchlike results in each stock index that are low in values. Finally, we identify that investors operating in Italy, with relatively greater hedge ratios and thus higher hedging costs, require more oil assets than those operating in the other countries of the Group of Seven to minimize the risk.

Turning our interest in the optimal weights, Table 5 shows fairly similar results for all models in each stock index. The average

Figure 1: Aggregated results of volatility spillover effects



The diagrams are based on the Wald tests at 5% significance level. The arrows indicate the direction of the volatility spillover effects. When there are no arrows, it means that there are not any spillover effects between the indices weight for the CAC40/Brent portfolio, following the results of the ABEKK model, is 0.2172, implying that for a 1$ portfolio, 21.72 cents should be invested in the Brent oil and 78.28 cents invested in the stock index. In the same way, for the AVARMA-CCC model and the CAC40/Brent portfolio, the average portfolio weight is 0.2160, meaning that for a 1$ portfolio, 21.60 cents should be invested in Brent crude oil and the remaining 78.40 cents invested in French stock market index. On the whole, the average weights range from 0.0599 (TSX/Brent-AVARMA-DCC/CCC) to 0.2742 (MIB/Brent-ABEKK). This finding means that the oil risk is considerably greater for Canada than for Italy, and any fluctuation in the price of crude oil could lead to undesirable effects on the performance of hedged portfolios. Finally, given our results for optimal hedge ratios, oil assets should be a part of a diversified portfolio of stocks as they increase the risk-adjusted performance of the hedged portfolio.

6. CONCLUDING REMARKS

The main objective of this article was to investigate the performance of asymmetric multivariate GARCH models on the mean and volatility transmission between oil and the stock markets of the Group of Seven (G7). Employing asymmetric models such as the ABEKK, the AVARMA-CCC, and the AVARMA-DCC-GARCH, which permit volatility spillover; we find considerable volatility spillover effects among the markets according to the ABEKK results. However, based on the AVARMA models there are negligible interactions and mostly from the stock to the oil markets. This finding is crucial and implies that the results of the volatility spillovers are highly depending on the choice of the multivariate GARCH model. In addition, the consensus of our results shows that the ABEKK models outperform the rest of the models.

Our examination of optimal weights and hedge ratios implies that optimal portfolios in all countries of the Group of Seven should possess more stocks than oil assets and that stock investment risk can be hedged by taking a short position in the oil markets. Moreover, regardless of the multivariate GARCH model used, our findings indicate that optimally hedged oil/stock portfolios are performing better than portfolios containing only stocks.

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APPENDIX A. SUPPLEMENTARY MATERIAL

Figure A1: Plot of time-series

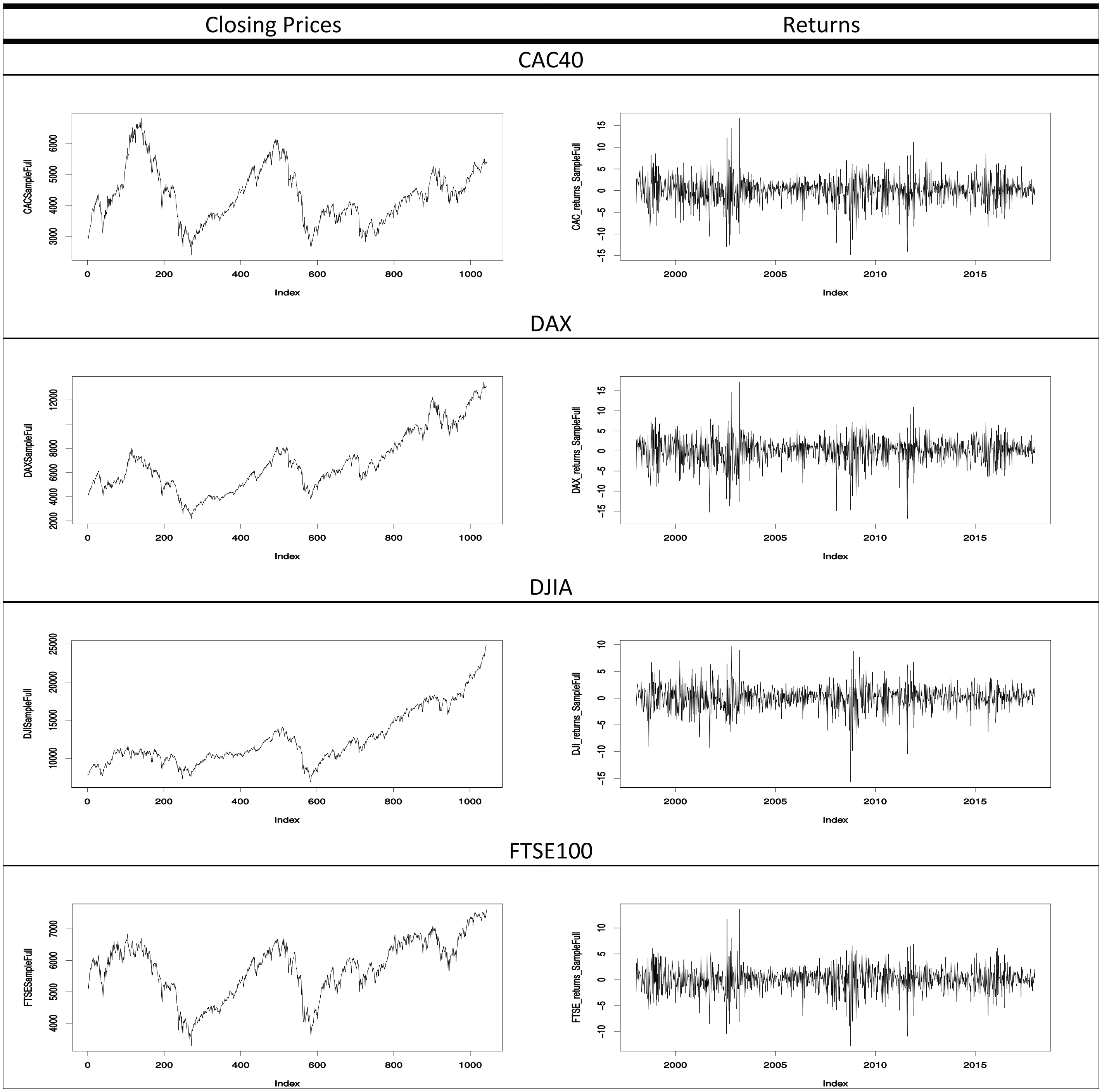

Figure A2: Plot of time-series

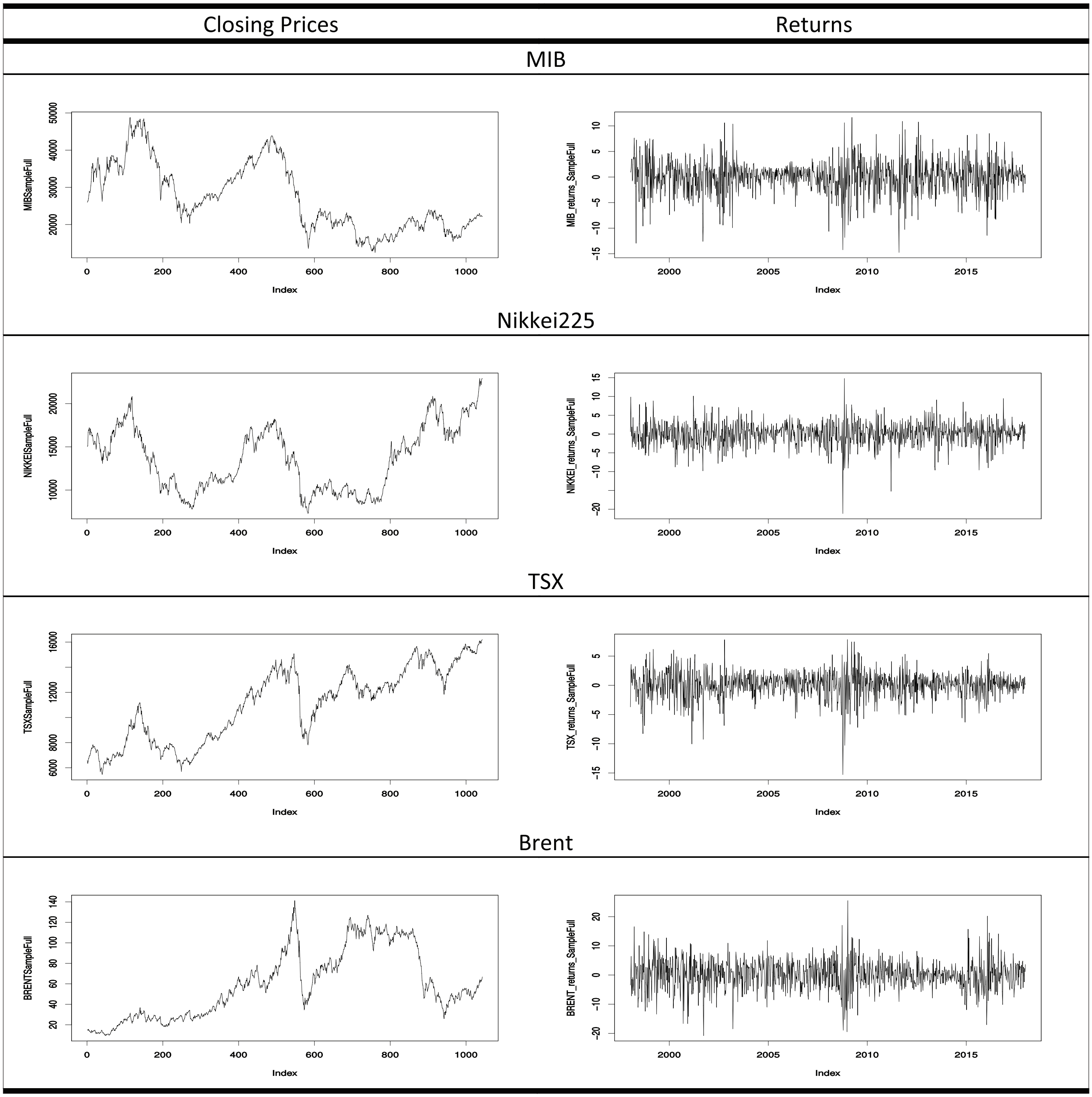

Figure A3: Hedge ratios from ABEKK, AVARMA-CCC and AVARMA-DCC models

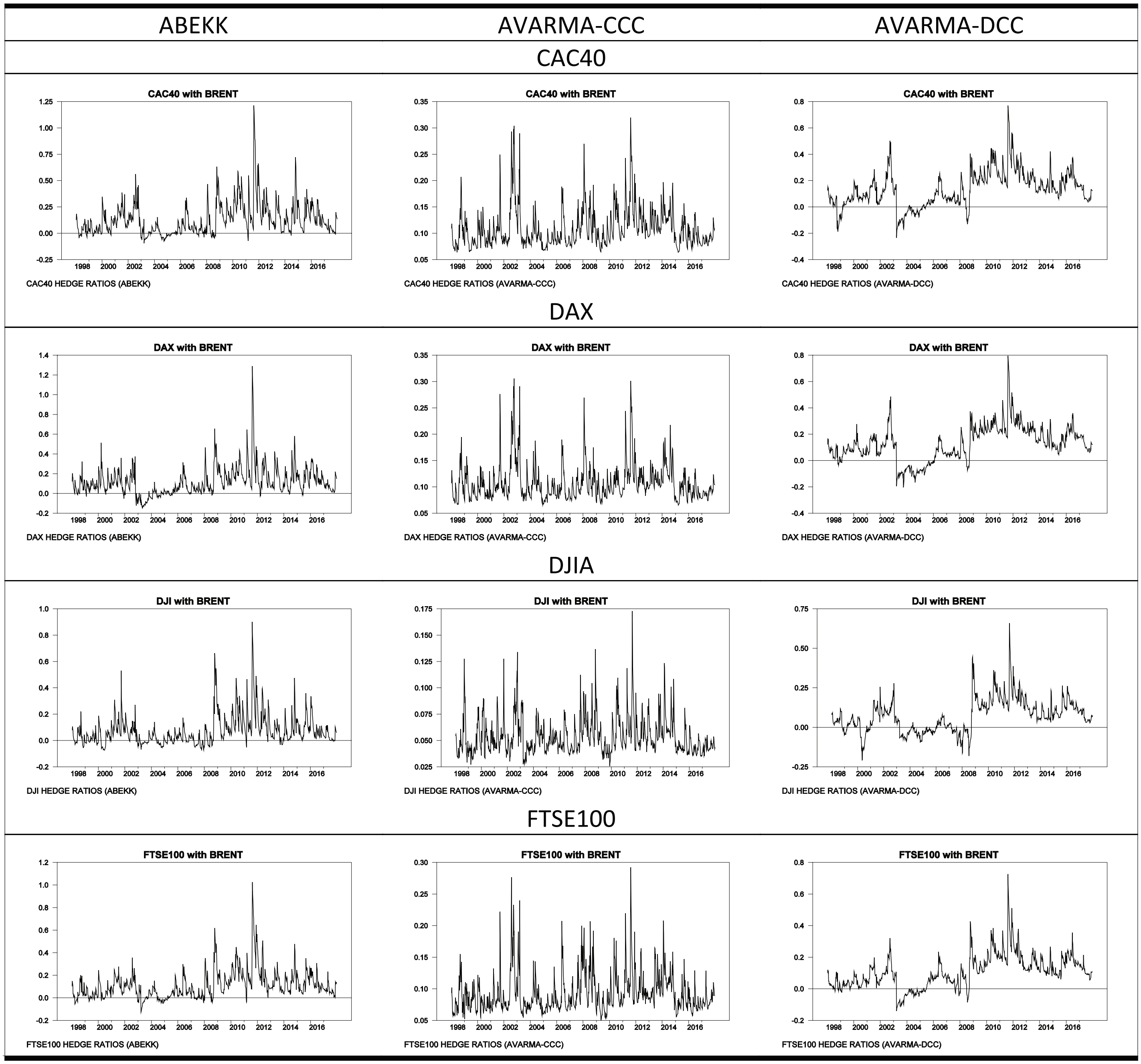

Figure A4: Hedge ratios from ABEKK, AVARMA-CCC and AVARMA-DCC models

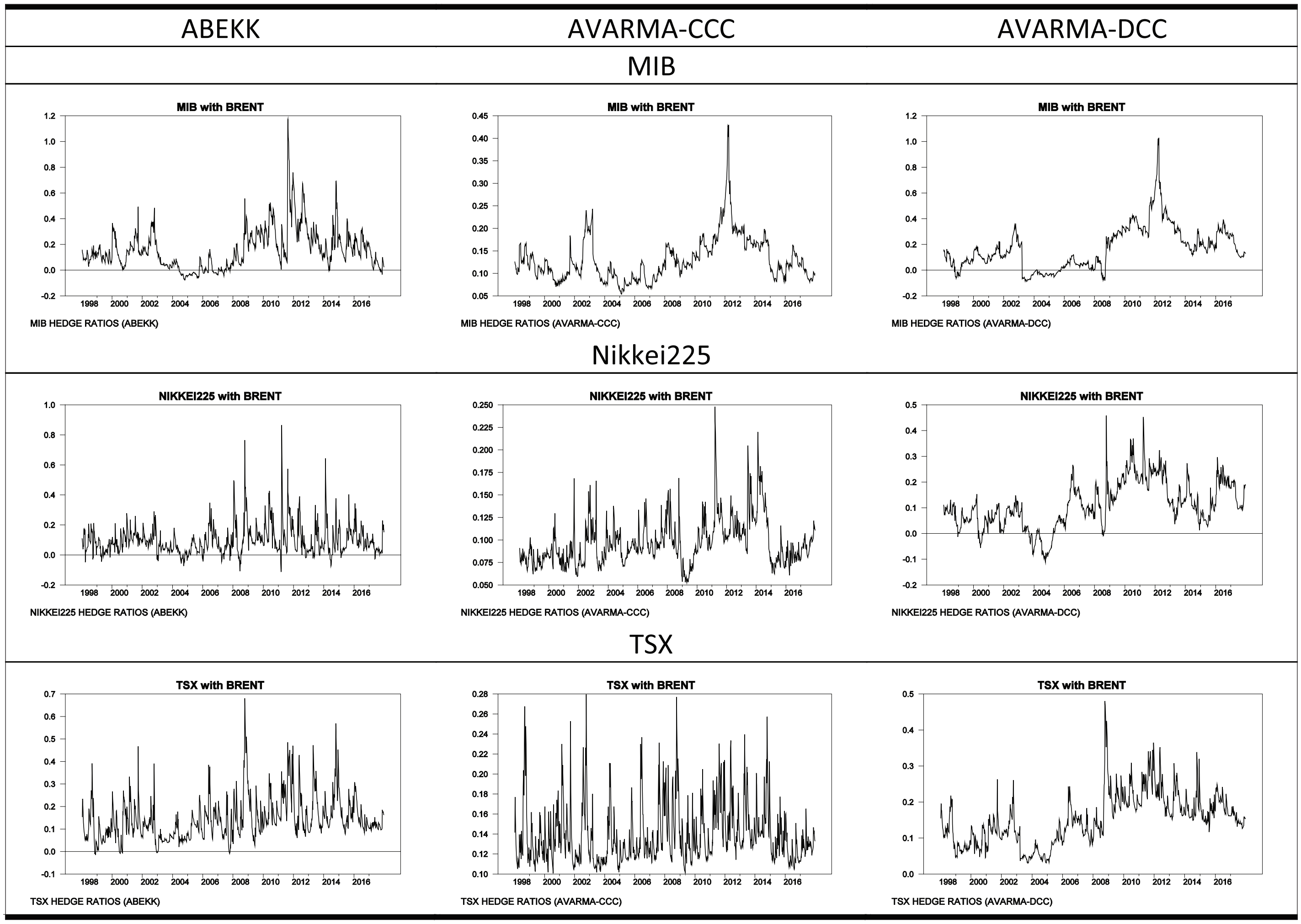

Table A1: Multivariate GARCH results for France CAC40

Parameters	ABEKK		AVARMA-CCC		AVARMA-DCC	
	Coeff.	Sig.	Coeff.	Sig.	Coeff.	Sig.
Mean equation						
con_s	0.044	0.515	0.040	0.554	0.064	0.324
con_o	0.016	0.884	0.061	0.641	0.087	0.519
Variance equation						
c_{ss}	0.606	0.000	0.421	0.005	0.399	0.051
c_{so}	0.109	0.475	-----	-----	-----	-----
c_{oo}	0.287	0.051	0.191	0.270	0.205	0.210
a_{ss}	0.052	0.684	0.006	0.830	0.010	0.780
a_{so}	−0.086	0.005	0.006	0.475	0.005	0.596
a_{os}	−0.148	0.091	0.045	0.262	0.046	0.274
a_{oo}	0.117	0.026	0.033	0.025	0.031	0.018
b_{ss}	0.895	0.000	0.723	0.000	0.746	0.000
b_{so}	0.007	0.381	0.014	0.262	0.012	0.392
b_{os}	−0.037	0.089	−0.027	0.543	−0.030	0.467
b_{oo}	0.968	0.000	0.917	0.000	0.917	0.000
d_{ss}	0.483	0.000	0.328	0.000	0.299	0.004
d_{so}	0.026	0.376	-----	-----	-----	-----
d_{os}	0.080	0.406	-----	-----	-----	-----
d_{oo}	0.303	0.000	0.073	0.007	0.076	0.001
ρ	-----	-----	0.183	0.000	-----	-----
θ_1	-----	-----	-----	-----	0.039	0.044
θ_2	-----	-----	-----	-----	0.946	0.000
Residual diagnostics for independent series						
	CAC40	BRENT	CAC40	BRENT	CAC40	BRENT
LogLik.	−5550.73		−5573.01		−5553.56	
Q (24)	28.341	19.725	29.384	18.703	29.098	22.114
Q^2 (24)	29.970	19.204	30.191	21.941	29.750	17.735
ARCH(10)	0.968	0.465	0.779	0.383	0.790	0.326

*, **, *** indicate statistical significance at 1%, 5% and 10% respectively. LogLik. is the value of the logarithmic likelihood. ARCH(10) represents the F-statistics of the ARCH test of Engle (1982) at 10th lag. Q(24) and Q^2 (24) are the Ljung and Box (1978) statistics for serial correlation and conditional heteroskedasticity of the series at 24th lag. con_s and con_o denote the constants in the mean equations of stock and oil, respectively

Table A2: Multivariate GARCH results for Germany DAX

Parameters	ABEKK		AVARMA-CCC		AVARMA-DCC	
	Coeff.	Sig.	Coeff.	Sig.	Coeff.	Sig.
Mean equation						
con_s	0.161	0.017	0.134	0.095	0.155	0.031
con_o	0.046	0.706	0.066	0.604	0.075	0.488
Variance equation						
c_{ss}	0.633	0.000	0.532	0.017	0.533	0.009
c_{so}	0.229	0.161	-----	-----	-----	-----
c_{oo}	0.280	0.062	0.234	0.184	0.261	0.121
a_{ss}	0.156	0.011	0.009	0.816	0.013	0.709
a_{so}	0.083	0.093	0.008	0.424	0.007	0.415
a_{os}	-0.112	0.086	0.057	0.164	0.058	0.084
a_{oo}	0.155	0.000	0.034	0.010	0.032	0.011
b_{ss}	0.902	0.000	0.700	0.000	0.715	0.000
b_{so}	−0.010	0.260	0.020	0.412	0.017	0.507
b_{os}	−0.028	0.212	−0.040	0.274	−0.044	0.148
b_{oo}	0.963	0.000	0.913	0.000	0.914	0.000
d_{ss}	0.452	0.000	0.324	0.016	0.303	0.023
d_{so}	−0.018	0.642	-----	-----	-----	-----
d_{os}	0.071	0.271	-----	-----	-----	-----

(*Contd...*)

Table A2: (*Continued*)

Parameters	ABEKK		AVARMA-CCC		AVARMA-DCC	
	Coeff.	Sig.	Coeff.	Sig.	Coeff.	Sig.
d_{oo}	0.299	0.000	0.077	0.002	0.080	0.002
ρ	-----	-----	0.169	0.000	-----	-----
θ_1	-----	-----	-----	-----	0.029	0.013
θ_2	-----	-----	-----	-----	0.963	0.000
Residual diagnostics for independent series						
	DAX	BRENT	DAX	BRENT	DAX	BRENT
LogLik.	−5640.91		−5650.37		−5630.98	
Q (24)	19.556	17.736	20.159	18.879	20.120	21.941
Q^2 (24)	28.767	18.177	28.009	22.026	27.426	19.296
ARCH(10)	0.778	0.392	0.806	0.398	0.775	0.347

Same as Table A1

Table A3: Multivariate GARCH results for U.S.A. DJIA

Parameters	ABEKK		AVARMA-CCC		AVARMA-DCC	
	Coeff	Sig.	Coeff	Sig.	Coeff	Sig.
Mean equation						
con_s	0.101	0.041	0.098	0.041	0.118	0.029
con_o	0.001	0.995	0.061	0.635	0.039	0.779
Variance Equation						
c_{ss}	0.457	0.000	0.200	0.033	0.187	0.018
c_{so}	0.217	0.307	-----	-----	-----	-----
c_{oo}	0.244	0.337	0.124	0.538	0.093	0.614
a_{ss}	-0.019	0.730	−0.080	0.001	−0.076	0.000
a_{so}	0.093	0.002	0.007	0.212	0.006	0.089
a_{os}	0.221	0.000	0.029	0.642	0.020	0.638
a_{oo}	−0.040	0.591	0.024	0.098	0.022	0.094
b_{ss}	0.871	0.000	0.761	0.000	0.787	0.000
b_{so}	0.002	0.814	0.006	0.548	0.005	0.442
b_{os}	−0.089	0.106	0.071	0.466	0.080	0.291
b_{oo}	0.970	0.000	0.913	0.000	0.914	0.000
d_{ss}	0.529	0.000	0.407	0.001	0.366	0.000
d_{so}	0.015	0.594	-----	-----	-----	-----
d_{os}	0.119	0.241	-----	-----	-----	-----
d_{oo}	0.322	0.000	0.080	0.001	0.087	0.001
ρ	-----	-----	0.126	0.000	-----	-----
θ_1	-----	-----	-----	-----	0.039	0.006
θ_2	-----	-----	-----	-----	0.950	0.000
Residual Diagnostics for Independent Series						
	DJIA	BRENT	DJIA	BRENT	DJIA	BRENT
LogLik.	−5215.86		−5240.68		−5216.26	
Q (24)	29.930	21.408	30.625	21.681	30.339	24.338
Q^2 (24)	21.102	13.652	23.141	17.872	22.287	12.889
ARCH(10)	0.667	0.283	0.570	0.299	0.548	0.264

Same as Table A1

Table A4: Multivariate GARCH results for U.K. FTSE100

Parameters	ABEKK		AVARMA-CCC		AVARMA-DCC	
	Coeff.	Sig.	Coeff.	Sig.	Coeff.	Sig.
Mean equation						
con_s	−0.005	0.932	−0.024	0.678	−0.002	0.962
con_o	−0.033	0.806	0.047	0.739	0.045	0.753
Variance equation						
c_{ss}	0.487	0.000	0.300	0.488	0.242	0.017
c_{so}	0.161	0.389	-----	-----	-----	-----
c_{oo}	0.341	0.005	0.140	0.591	0.100	0.597
a_{ss}	0.076	0.135	−0.057	0.417	−0.051	0.042
a_{so}	−0.069	0.001	0.003	0.580	0.001	0.708
a_{os}	−0.191	0.021	0.043	0.756	0.015	0.840
a_{oo}	0.136	0.003	0.030	0.099	0.026	0.064
b_{ss}	0.889	0.000	0.718	0.151	0.790	0.000
b_{so}	0.007	0.218	0.013	0.812	0.007	0.458
b_{os}	−0.034	0.302	0.021	0.838	0.045	0.557
b_{oo}	0.965	0.000	0.913	0.000	0.918	0.000
d_{ss}	0.515	0.000	0.421	0.419	0.342	0.002
d_{so}	0.004	0.852	-----	-----	-----	-----
d_{os}	0.087	0.435	-----	-----	-----	-----
d_{oo}	0.300	0.000	0.076	0.003	0.079	0.006
ρ	-----	-----	0.200	0.000	-----	-----
θ_1	-----	-----	-----	-----	0.036	0.072
θ_2	-----	-----	-----	-----	0.952	0.000
Residual diagnostics for independent series						
	FTSE100	BRENT	FTSE100	BRENT	FTSE100	BRENT
LogLik.	−5293.86		−5315.00		−5294.07	
Q (24)	20.217	17.309	20.808	20.319	20.589	21.677
Q^2 (24)	26.659	18.004	24.457	19.809	24.182	13.662
ARCH(10)	1.050	0.298	0.905	0.342	0.874	0.202

Same as Table A1

Table A5: Multivariate GARCH results for Italy MIB

Parameters	ABEKK		AVARMA-CCC		AVARMA-DCC	
	Coeff.	Sig.	Coeff.	Sig.	Coeff.	Sig.
Mean equation						
con_s	0.048	0.558	0.057	0.497	0.081	0.319
con_o	0.045	0.738	0.107	0.412	0.155	0.260
Variance equation						
c_{ss}	0.413	0.001	0.257	0.092	0.243	0.099
c_{so}	−0.216	0.089	-----	-----	-----	-----
c_{oo}	0.000	1.000	0.469	0.122	0.488	0.187
a_{ss}	0.193	0.002	0.073	0.144	0.077	0.058
a_{so}	0.054	0.002	0.013	0.134	0.013	0.073
a_{os}	0.194	0.000	0.094	0.054	0.101	0.027
a_{oo}	−0.097	0.034	0.043	0.014	0.040	0.041
b_{ss}	0.933	0.000	0.877	0.000	0.882	0.000
b_{so}	0.005	0.319	−0.018	0.143	−0.018	0.082
b_{os}	−0.016	0.174	−0.095	0.011	−0.093	0.005
b_{oo}	0.965	0.000	0.906	0.000	0.905	0.000
d_{ss}	0.341	0.000	0.075	0.057	0.070	0.032
d_{so}	−0.005	0.871	-----	-----	-----	-----
d_{os}	−0.074	0.348	-----	-----	-----	-----
d_{oo}	0.349	0.000	0.071	0.004	0.071	0.013

(*Contd...*)

Table A5: (*Continued*)

Parameters	ABEKK		AVARMA-CCC		AVARMA-DCC	
	Coeff.	Sig.	Coeff.	Sig.	Coeff.	Sig.
ρ	-----	-----	0.194	0.000	-----	-----
θ_1	-----	-----	-----	-----	0.026	0.033
θ_2	-----	-----	-----	-----	0.968	0.000
Residual diagnostics for independent series						
	MIB	BRENT	MIB	BRENT	MIB	BRENT
LogLik.	−5658.23		−5678.93		−5662.72	
Q (24)	31.408	19.609	33.197	18.170	32.796	22.433
Q^2 (24)	12.245	25.056	13.992	31.868	15.049	25.207
ARCH(10)	0.331	0.578	0.453	0.346	0.480	0.356

Same as Table A1

Table A6: Multivariate GARCH results for Japan Nikkei225

Parameters	ABEKK		AVARMA-CCC		AVARMA-DCC	
	Coeff.	Sig.	Coeff.	Sig.	Coeff.	Sig.
Mean equation						
con_s	0.045	0.602	0.080	0.353	0.086	0.255
con_o	0.060	0.651	0.056	0.636	0.074	0.531
Variance equation						
c_{ss}	1.238	0.000	1.666	0.003	1.618	0.000
c_{so}	0.126	0.673	-----	-----	-----	-----
c_{oo}	0.197	0.736	0.624	0.450	0.562	0.391
a_{ss}	0.062	0.421	−0.004	0.928	−0.008	0.850
a_{so}	−0.119	0.033	0.022	0.089	0.023	0.027
a_{os}	−0.078	0.206	0.033	0.692	0.031	0.642
a_{oo}	0.158	0.049	0.038	0.116	0.036	0.120
b_{ss}	0.823	0.000	0.645	0.000	0.659	0.000
b_{so}	0.019	0.056	0.003	0.897	0.001	0.913
b_{os}	−0.058	0.547	−0.082	0.615	−0.070	0.591
b_{oo}	0.971	0.000	0.916	0.000	0.917	0.000
d_{ss}	0.388	0.000	0.209	0.019	0.206	0.000
d_{so}	0.080	0.154	-----	-----	-----	-----
d_{os}	0.102	0.454	-----	-----	-----	-----
d_{oo}	0.258	0.000	0.081	0.005	0.082	0.001
ρ	-----	-----	0.151	0.000	-----	-----
θ_1	-----	-----	-----	-----	0.024	0.009
θ_2	-----	-----	-----	-----	0.964	0.000
Residual diagnostics for independent series						
	Nikkei225	BRENT	Nikkei225	BRENT	Nikkei225	BRENT
LogLik.	−5676.83		−5682.39		−5673.51	
Q (24)	37.814**	21.480	37.263**	20.541	37.228**	20.875
Q^2 (24)	15.071	29.824	15.964	29.662	15.888	26.600
ARCH(10)	0.415	0.670	0.540	0.496	0.532	0.475

Same as Table A1

Table A7: Multivariate GARCH results for Canada TSX

Parameters	ABEKK		AVARMA-CCC		AVARMA-DCC	
	Coeff.	Sig.	Coeff.	Sig.	Coeff.	Sig.
Mean equation						
con_s	0.108	0.020	0.117	0.008	0.125	0.025
con_o	0.016	0.893	0.085	0.447	0.101	0.524
Variance equation						
c_{ss}	0.450	0.000	0.154	0.077	0.143	0.606
c_{so}	0.371	0.021	-----	-----	-----	-----
c_{oo}	0.000	1.000	0.123	0.214	0.108	0.317
a_{ss}	0.274	0.000	0.017	0.579	0.025	0.822
a_{so}	−0.091	0.010	0.004	0.457	0.001	0.913
a_{os}	−0.052	0.691	0.228	0.031	0.198	0.471
a_{oo}	−0.007	0.921	0.015	0.265	0.015	0.333
b_{ss}	0.868	0.000	0.656	0.000	0.724	0.451
b_{so}	0.007	0.265	0.025	0.196	0.018	0.877
b_{os}	−0.106	0.016	−0.204	0.194	−0.147	0.677
b_{oo}	0.981	0.000	0.949	0.000	0.945	0.000
d_{ss}	0.470	0.000	0.280	0.005	0.229	0.738
d_{so}	0.011	0.602	-----	-----	-----	-----
d_{os}	0.230	0.004	-----	-----	-----	-----
d_{oo}	0.284	0.000	0.057	0.005	0.057	0.134
ρ	-----	-----	0.322	0.000	-----	-----
θ_1	-----	-----	-----	-----	0.020	0.263
θ_2	-----	-----	-----	-----	0.973	0.000
Residual diagnostics for independent series						
	TSX	**BRENT**	**TSX**	**BRENT**	**TSX**	**BRENT**
LogLik.	−5213.55		−5233.95		−5221.00	
Q (24)	29.321	24.687	29.316	28.649	29.737	28.765
Q^2 (24)	15.953	21.231	16.044	24.521	14.674	17.912
ARCH(10)	0.639	0.480	0.530	0.309	0.478	0.238

Same as Table A1