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Mathematical Modeling and Optimization of the Monopoly Heating Market

Andrey Penkovskii*, Valery Stennikov

Melentiev Energy System Institute of Siberia Branch of the Russian Academy of Sciences, Irkutsk, 664033, Lermontov St., 130, Russian Federation. *Email: penkoffsky@isem.irk.ru

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ABSTRACT

District heating plays a major role in many countries. Unlike markets of natural gas and electric power, district heating systems are local markets and in most cases, they organized as natural monopoly markets. This article examines the several variants for the formation of prices for household consumers on the heat energy market. For the conditions liberalized economy proposed a method of forming the price on the heat energy based on the search for market equilibrium of supply and demand on the monopoly heating market. For the conditions of the regulated heating market, the variants of forming the price of heat energy on the basis of average total and marginal costs are considered. To perform calculations, a mathematical model of the monopoly heating market is developed. It is based on the classical model of monopoly, basic models of the theory of hydraulic circuits and the theory of industrial markets and allows us to take into account the economic interests of the parties in fulfillment of physical and technical conditions and restrictions on the heat sources and heat networks. Practical calculations with help developed mathematical models were made for a real district heating system.

Keywords: District Heating, Monopoly, Regulation, Mathematical Modeling, Optimization, Analysis
JEL Classifications: L43, L51, L95

1. INTRODUCTION

District heating (DH) plays an important role on the heating market. Main consumption share of heat energy in district heating are household and industrial consumers. The total number of heating markets has been estimated to 80 000 systems (Frederiksen and Werner, 2013), thereof about 50 000 in Russia (Stennikov et al., 2016), 6000 in the Europe (Werner, 2017) and the remaining 24,000 are in China and the countries of the former Commonwealth of Independent States (such as Kazakhstan, Ukraine, Georgia, Belarus, etc.).

There are two main forms organization of heating market: competition and monopoly. Competition model on the heating market is an important element of a market economy, since it contributes to the growth of heat energy efficiency production, improving its quality and, as a result, reducing its price, which can have a favorable effect on the development of district heating. In condition of competitive market, the purchase price for heat energy is not regulated due to the possibility of a market choice of supplier (heat source). Among the main countries where currently operates a competitive model on the heating market can be identified Sweden (Werner, 2017), Finland (Paiho and Saastamoinen, 2018), Germany (Wissner, 2014).

Competition on the heating market can manifest itself in the following forms:

1. Competition between heat sources. This type of competition arises in connection with the total excess power of heat sources as compared with the total demand for heat energy for consumers
2. Competitive threat of building more cost-effective new heat sources.
3. Competition between types of heat supply.

Nomenclature	
m	Number of nodes
n	Number of branches
J	Set of nodes
J_{hs}	Set of heat sources
J_{cos}	Set of consumers
J_0	Set of branching nodes without producers and consumers
$J_{\mathrm{con}}^{\mathrm{hh}}$	Set of household consumers
$J_{\mathrm{con}}^{\mathrm{ic.hn}}$	Set of industrial consumers connected to heat networks
$J_{\mathrm{con}}^{\mathrm{ic.hs}}$	Set of industrial consumers located on collectors of heat source
I	Set of branches
Q_j^{hs}	Volume production of heat energy of j-th heat source, GJ/h
Q_j^{con}	Total consumer demand of heat energy in j-th node, GJ/h
Q_j^{hh}	Heat energy demand of j-th household consumer, GJ/h
$Q_j^{\mathrm{ic.hn}}$	Heat energy demand of j-th industrial consumer connected to heat networks, GJ/h
$Q_j^{\mathrm{ic.hs}}$	Heat energy demand of j-th industrial consumer located on collectors of heat source, GJ/h
$Q_j^{(\mathrm{H})}$	Design heating load of j-th household consumer, GJ/h
$Q_j^{(\mathrm{HW})}$	Design hot water supply load of j-th household consumer, GJ/h
r, g	Coefficients of heat load curve non-uniformity
ω	Share of hot water supply load, %
t_1	Design outdoor temperature, °C
t_2	Temperature that corresponds to the beginning of heating period, ºC
t_3	Average temperature for the heating period, °C
t_4	Design indoor temperature, °C
$\tau_{(\mathrm{HP})}$	Duration of heating period, h
$\xi_j, \upsilon_j, \mu_j, \pi_j$	Coefficients obtained from the approximation of the factual data on the heat volume purchased by an industrial consumer
P^{HSC}	Profit of Heat Supply Company, EUR
$Z_j^{\mathrm{hs}}(Q_j^{\mathrm{hs}})$	Function of costs on the production heat energy of j-th heat source, EUR
$\alpha_j, \beta_j, \gamma_j$	Coefficients of approximation of cost function of j-th heat source
$Z_i^{\mathrm{hn}}(x_i)$	Costs in the i–th branch of the heat network, EUR
$x = (x_1, \ldots, x_n), x_i$	Heat carrier flow rate in the i–th branch of the heat network, t/h
n_{year}	Number of operating hours of the pumping facility per year, h
f_{a}	Share of conditionally constant and operating costs for the heat network
a_i, b_i, c_i	Approximation coefficients of numerical values for unit cost of laying pipelines of different diameters
χ_i	Coefficient of depending on the i-th branch (pipe) inner surface roughness
s_i	Coefficient of hydraulic resistance of the i–th branch, mh²/t²
l_i	Length of i–th branch, m

Nomenclature	
C_{el}	Unit cost of electricity, EUR/kWh
η	Pumping station efficiency, %
$\bar{\mathrm{A}}$	Complete incidence matrix
A	$(m-1) \times$ n incidence matrix for linearly independent nodes, which is obtained on the basis of complete matrix $\bar{\mathrm{A}}$ by deleting any of its rows
$\bar{\mathrm{A}}^{\mathrm{T}}$	Transposed complete matrix $\bar{\mathrm{A}}$
$\mathrm{P} = (\mathrm{P}_1, \ldots, \mathrm{P}_m), \mathrm{P}_j$	Pressure in the j-th node, Pa
h_i	Pressures loss at the i-th of branch, Pa
H_i	Effective head at the i-th branch, Pa
w^{hh}	Final heat energy price for household consumers, EUR/GJ
$w^{\mathrm{gen.hh}}$	Price for generation of heat energy for household consumers, EUR/GJ
$w_j^{\mathrm{ic.hn}}$	Purchase price of heat energy of the j-th industrial consumer connected to heat networks, which includes production and transportation of heat energy, EUR/GJ
$w_j^{\mathrm{gen.ic.hn}}$	Purchase price of heat energy of the j-th industrial consumer connected to heat networks, which includes only production of heat energy, EUR/GJ
$w_j^{\mathrm{gen.ic.hs}}$	Purchase price of heat energy of the j-th industrial consumer located on collectors of heat source, which includes only production of heat energy, EUR/GJ
w^{hn}	Price for transportation of heat energy, EUR/GJ
w^{HSC}	Equilibrium heat energy price of production and transportation of Heat Supply Company, EUR/GJ
$w^{\mathrm{gen.HSC}}$	Equilibrium heat energy production price of Heat Supply Company, EUR/GJ
θ	Number of considered categories of heat energy consumers
Θ^{hh}	Share of heat energy consumption of household consumers, %
$\Theta_j^{\mathrm{ic.hn}}$	Share of heat energy consumption of the j-th industrial consumer connected to heat networks, %
$\Theta_j^{\mathrm{ic.hs}}$	Share of heat energy consumption of the j-th industrial consumer located on collectors of heat source, %
$Q_{j\_\min}^{\mathrm{hs}}, Q_{j\_\max}^{\mathrm{hs}}$	Minimum and maximum levels of the j-th heat source productive capacity, respectively, GJ/h
$Q_{j\_\max}^{\mathrm{ic.hn}}$	Maximum levels consumption of the j-th industrial consumer connected to heat networks, GJ/h
$Q_{j\_\max}^{\mathrm{ic.hs}}$	Maximum levels consumption of the j-th industrial consumer located on collectors of heat source, GJ/h

Another type of heating market that is most prevalent in countries with developed of DH is a natural monopoly with tariff regulation on the heat energy for consumers. These are the large heating markets of some countries European Union (Netherlands (Barriers to district heating development in the Netherlands, 2017), Poland (Wojdyga and Chorzelski, 2017), Lithuania (District Heating and Cooling, Combined Heat and Power and Renewable Energy Sources, 2014), Latvia (Ziemele et al., 2014; Sarma and Bazbauers, 2016), Norway (District Heating in Norway, 2017), Estonia

(Šommet, 2013), et al.), Russia (Dyomina, 2017), China (Zhang, et al., 2015) and other. In these countries the regulator, whose duties include the management of the tariff for heat energy, are the different bodies of government management (Table 1).

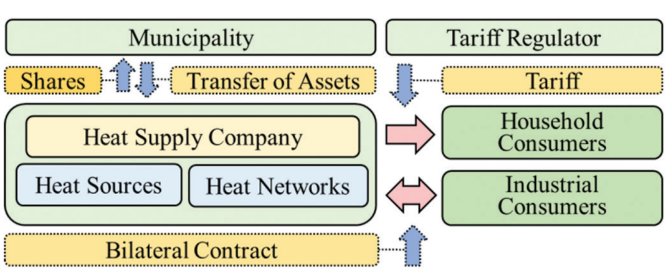
The monopoly model of DH provides for the integration of all heat supply aspects, including production, transportation and sales of heat energy in the unified Heat Supply Company (HSC). The organizational model of HSC can be represented by the scheme shown in Figure 1.

In the organizational model of heat supply management (see Figure 1), simultaneous control over the production, transportation and sales of heat energy is justified in the context of maintaining system reliability (Postnikov et al. 2018), reducing technical and economic risks, and sustainable functioning of DH. Within the framework of this model, there is no competition among heat sources, and heat networks are a monopoly structure.

With this monopoly model, the HSC should become the owner of all the municipal assets including heat sources and distribution heat networks. Such a merging of the main assets and heat supply control processes shapes the HSC as a single seller on the heating market, i.e. a monopolist. Thus, the HSC will have a total control of heat supply on the market and market heat price.

The relationships among the market participants in the form of the monopoly model of the HSC develop according to a certain pattern and imply the following. Based on the forecasts of demand and optimal variants of heating system expansion, the HSC delivers heat (under medium-term and long-term contracts) to consumers at a price calculated as a sum of a price of heat source heat production and a price of heat transportation from the heat source to the consumer. In this case, the HSC produces the volumes of heat that on the one hand would maximize its profit, given physical-technical constraints on heat sources and heat networks, and meet the demand for heat specified by the consumer, and on the other hand would correspond to the consumer wish to pay for this demand.

Figure 1: The organizational model of monopoly heating market



Currently, among the most common approaches for modeling medium-term (or long-term) forecasting of possible situations in a monopoly heating market, we can distinguish the classical microeconomic model of monopoly (Tirole, 1988). It is one of the universal models for analyzing the functioning and development of various markets, including those that are adequate to the heat business.

There are many possible variants for the formation of prices for heat energy for household consumers of in the context of the HSC model, the most common of which are:

- free pricing based on market equilibrium of demand and supply for heat energy;
- regulation tariff for household consumers at the level of average total costs;
- regulation tariff for household consumers at the level of marginal costs.

To perform the analysis of models of prices heat energy formation for in the conditions of the market the corresponding mathematical model of district heating system which will allow to carry out multivariate calculations on optimization of the heating market according to the established economic criteria and taking into account the available technological restrictions of the district heating system.

2. THE MATHEMATICAL MODELING OF MONOPOLY HEATING MARKET

District heating system is modeled by a network with m nodes and n branches (Merenkov and Khasilev, 1985). Let us denote the set of nodes by $J = \{j\colon j=1, \ldots m\}$ and the set of branches by $I=\{i\colon i=1,\ldots n\}$. The nodes set of network consist of the sets of heat sources J_{hs}, consumers J_{con} and set of branching nodes without heat sources and consumers J_0:

$$J = J_{\text{hs}} \cup J_{\text{con}} \cup J_0.$$

Modeling of such a system takes into account time intervals: τ_0 is starting time interval, T is final time interval (for example, number of hours in the year, 8760).

Let Q_j^{con} – total consumer demand of heat energy in j-th node. Then, taking into account the accepted notation, the following relations are true:

Table 1: The government management bodies on regulation of heat energy tariff

Country	Netherlands	Poland	Lithuania	Latvia
Regulator	Authority for Consumer and Market	Energy Regulatory Office	National Control Commission for Prices and Energy	Public Utilities Commission
Country	Norway	Estonia	Russia	China
Regulator	Norwegian Water Resources and Energy Directorate	Estonian Competition Authority	Regional and Municipal Energy Commissions	Regional and Municipal Energy Commissions

$$Q_j^{\text{con}} = Q_j^{\text{hh}} + Q_j^{\text{ic.hn}} + Q_j^{\text{ic.hs}},\ j \in J_{\text{con}}^{\text{hh}} \cap J_{\text{con}}^{\text{ic.hn}} \cap J_{\text{con}}^{\text{ic.hs}},$$

$$Q_j^{\text{con}} = Q_j^{\text{hh}},\ j \in J_{\text{con}}^{\text{hh}} \setminus (J_{\text{con}}^{\text{ic.hn}} \cup J_{\text{con}}^{\text{ic.hs}}),$$

$$Q_j^{\text{con}} = Q_j^{\text{ic.hn}},\ j \in J_{\text{con}}^{\text{ic.hn}} \setminus (J_{\text{con}}^{\text{hh}} \cup J_{\text{con}}^{\text{ic.hs}}),$$

$$Q_j^{\text{con}} = Q_j^{\text{ic.hs}},\ j \in J_{\text{con}}^{\text{ic.hs}} \setminus (J_{\text{con}}^{\text{hh}} \cup J_{\text{con}}^{\text{ic.hn}}).$$

The demand of household consumers (Q_j^{hh}) is determined by the heat load duration curve. The configuration of this curve is well described by the Rossander equation, according to which the heat load at every time τ can be found by the following expression (Rossander, 1913):

$$Q_{j\tau}^{\text{hh}} = \left[1-(1-r)\cdot\left(\frac{\tau}{\tau_{(\text{HP})}}\right)^{\frac{g-r}{1-g}}\right]\cdot Q_j^{(\text{H})} + Q_j^{(\text{HW})},\ j \in J_{\text{con}}^{\text{hh}}, \quad (1)$$

$$r = (1-\omega)\cdot\frac{t_4 - t_2}{t_4 - t_1}, \quad (2)$$

$$g = (1-\omega)\cdot\frac{t_4 - t_3}{t_4 - t_1}. \quad (3)$$

The heat energy demand of industrial consumers connected to heat networks is modeled by the demand characteristic which is obtained from real calculations and can be represented by the linear dependence.

The demand of industrial consumers for heat connected to heat networks is modeled by the demand characteristic, which is obtained from real calculations and can be represented by the linear dependence (Stennikov et al., 2013):

$$Q_{j\tau}^{\text{ic.hn}} = \xi_j - {}_j \cdot w_{j\tau}^{\text{ic.hn}},\ \xi_j > 0,\ {}_j > 0,\ j \in J_{\text{con}}^{\text{ic.hn}}. \quad (4)$$

For industrial consumers located on collectors of heat source, the heat energy demand function has the following form:

$$Q_{j\tau}^{\text{ic.hs}} = \mu_j - \pi_j \cdot w_{j\tau}^{\text{gen.ic.hs}},\ \mu_j > 0,\ \pi_j > 0,\ j \in J_{\text{con}}^{\text{ic.hs}}. \quad (5)$$

For each industrial consumer connected to the heat networks and located at heat source collectors, there are respective restrictions on the heat energy consumption volume:

$$Q_{j\tau}^{\text{ic.hn}} \le Q_{j\_\max}^{\text{ic.hn}},\ j \in J_{\text{con}}^{\text{ic.hn}}, \quad (6)$$

$$Q_{j\tau}^{\text{ic.hs}} \le Q_{j-\max}^{\text{ic.hs}},\ j \in J_{\text{con}}^{\text{ic.hs}}. \quad (7)$$

Heat demand volatility is the main heating market problem. Therefore, we suggest considering the interconnection between producers and consumers for every hour of the given time period. Such discrete modeling is of high practical interest since it allows us to take into account daily and seasonal demand for heat, which considerably affects profit of every heat producer.

In the market conditions, the HSC behavior is described by the classic model of natural monopoly (Carlton and Perloff, 2000; Belleflamme and Peitz, 2010), which take into account the heat energy production and transportation costs:

$$P_\tau^{\text{HSC}} = \sum_{j\in J_{\text{hs}}} (w_\tau^{\text{HSC}} \cdot Q_{j\tau}^{\text{hs}}) - \sum_{j\in J_{\text{hs}}} Z_{j\tau}^{\text{hs}}(Q_{j\tau}^{\text{hs}}) - \sum_{i\in I} Z_{i\tau}^{\text{hn}}(x_{i\tau}). \quad (8)$$

Our experience shows that the best approximation of the correspondence between costs and volumes of produced heat is given by the function (Penkovskii et al., 2018):

$$Z_{j\tau}^{\text{hs}}(Q_{j\tau}^{\text{hs}}) = \alpha_j \cdot (Q_{j\tau}^{\text{hs}})^2 + \beta_j \cdot Q_{j\tau}^{\text{hs}} + \gamma_j, \alpha_j > 0,\ \beta_j > 0,\ \gamma_j > 0\ j \in J_{\text{hs}}. \quad (9)$$

Total costs for heat networks include operational costs and costs for heat carrier pumping through heat networks, which are determined by the following analytical dependence (Sennova and Sidler, 1987):

$$\sum_{i\in I} Z_{i\tau}^{\text{hn}}(x_{i\tau}) = \frac{1}{n_{\text{year}}}\cdot f_{\text{a}} \cdot \sum_{i\in I}[a_i + b_i \cdot \chi_i^{0.19 u_i} \cdot s_i^{-0.19\cdot u_i} \cdot l_i^{0.19 u_i}]\cdot l_i + \frac{C_{\text{el}}}{367.2\cdot\cdot}\sum_{i\in I}\chi_{i\tau}^2 \cdot |\chi_{i\tau}| \cdot s_i. \quad (10)$$

The heat energy transportation costs is determined on the basis of the optimal flow distribution in the heat network. The mathematical model of optimal flow distribution in the heat network in the nodal form for district heating system conditions with a multitude of diverse consumers and heat energy sources can be represented as follows (Merenkov and Khasilev, 1985):

$$\text{A}_j \text{x}_\tau = \text{Q}_{j\tau}^{\text{hs}} - Q_{j\tau}^{\text{ic.hn}} - Q_{j\tau}^{\text{ic.hs}} - Q_{j\tau}^{\text{hs}},\ j \in J_{\text{hs}} \cup J_{\text{con}}^{\text{hh}} \cup J_{\text{con}}^{\text{ic.hs}} \cup J_{\text{con}}^{\text{ic.hs}}, \quad (11)$$

$$\text{A}_j \text{x}_\tau = \text{Q}_{j\tau}^{\text{hs}},\ j \in J_{\text{hs}} \setminus J_{\text{con}}^{\text{hh}} \cup J_{\text{con}}^{\text{ic.hs}} \cup J_{\text{con}}^{\text{ic.hs}}, \quad (12)$$

$$\text{A}_j \text{x}_\tau = -Q_{j\tau}^{\text{ic.hs}},\ j \in J_{\text{con}}^{\text{ic.hs}} \setminus J_{\text{con}}^{\text{hh}} \cup J_{\text{hs}} \cup J_{\text{con}}^{\text{ic.hn}}, \quad (13)$$

$$\text{A}_j \text{x}_\tau = -Q_{j\tau}^{\text{ic.hn}},\ j \in J_{\text{con}}^{\text{ic.hn}} \setminus J_{\text{con}}^{\text{hh}} \cup J_{\text{hs}} \cup J_{\text{con}}^{\text{ic.hs}}, \quad (14)$$

$$\text{A}_j \text{x}_\tau = -Q_{j_\tau}^{\text{hh}},\ j \in J_{\text{con}}^{\text{hh}} \setminus J_{\text{con}}^{\text{ic.hn}} \cup J_{\text{hs}} \cup J_{\text{con}}^{\text{ic.hs}} \quad (15)$$

$$\text{A}_j \text{x}_\tau = 0,\ j \in J_0, \quad (16)$$

$$\overline{\text{A}}_i^{\text{T}} \text{P}_\tau = h_{i\tau} - H_{i\tau},\ i \in I, \quad (17)$$

$$h_{i\tau} = s_i |x_{i\tau}| x_{i\tau},\ i \in I. \quad (18)$$

Equations (11)-(16) reflect Kirchhoffs first law, equation (17) reflects Kirchhoffs second law, and equation (18) is the closing relation describing the relationship between the quantities with respect to which Kirchhoffs first and second laws are formulated.

The solution of the problem of optimal flow distribution in heat networks for the conditions of a market economy is complicated since part of the heat loads (industrial consumers) are variable and depend on the heat energy price (equations (4) and (5)). To solve this problem used the approach based on the construction of redundant design scheme of district heating system (Merenkov and Khasilev, 1985). The redundant scheme is formed on the basis of design scheme of district heating system by introducing fictitious node and also fictitious branches connecting fictitious node with all consumer nodes as shown in Figure 2.

Nodes 2, 3 and 4 connect with node 6 form redundant design scheme of district heating system. Heat carrier flow rates in the branches 2-6 and 4-6 correspond to the specified loads of household consumers in nodes 2 and 4, respectively, and heat carrier flow rate in the branches 3-6 is an optimized parameter. As an additional condition to the problem (11)-(18), it is necessary to enter the material balance equation for the total production and consumption of heat energy:

$$\sum_{j\in J_{\mathrm{hs}}} Q_{j\hat{o}}^{\mathrm{hs}} - \sum_{j\in J_{\mathrm{con}}} Q_{j\hat{o}}^{\mathrm{con}} = 0.$$

For the mathematical description of the redundant design scheme, the set of nodes is expanded by fictitious node $j=m+1$. As a result, the set of nodes as follows:

$$J = J_{\mathrm{hs}} \cup J_{\mathrm{con}}^{\mathrm{in.hs}} \cup J_{\mathrm{con}}^{\mathrm{in.hn}} \cup J_{\mathrm{con}}^{\mathrm{hh}} \mathrm{Y} j_{m+1}.$$

The set of branches I of the redundant design scheme is complemented by a subset of fictitious branches I_{m+1} that connect the consumer nodes with the fictitious node. Thus, branches set of the heating network will be written as follows:

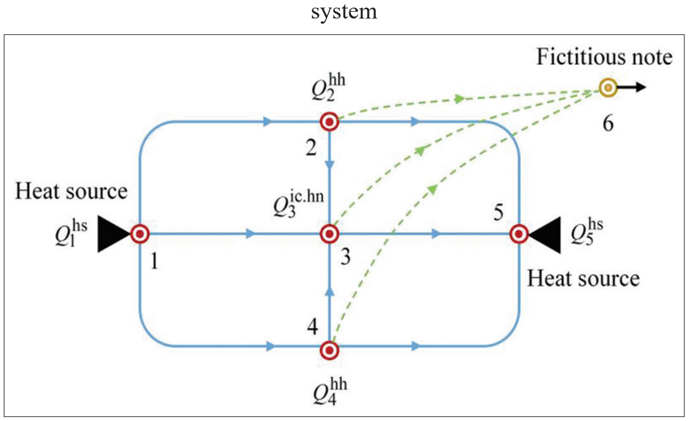
$$I \mathrm{Y} I_{m+1}.$$

The new parameters of the redundant design scheme will be as follows: the number of nodes M = m + 1; the number of branches N = n + n_f (where n_f is the fictitious branches in the design scheme); the number of contours C = c + n_f − 1. Here m, n and c are the number of nodes, branches and contours in the design scheme to its expansion.

One of the main indicators determining the compromise of interests between heat supply participants (heat sources and consumers) is the equilibrium price of produced and consumed heat energy in considered district heating system, which can be derived from the HSC economic balance:

$$w^{\mathrm{HSC}} \cdot \sum_{j\in J_{\mathrm{hs}}} Q_{j\tau}^{\mathrm{hs}} = \sum_{j\in J_{\mathrm{con}}^{\mathrm{hh}}} (w_{\tau}^{\mathrm{hh}} \cdot Q_{j\tau}^{\mathrm{hh}}) + \sum_{j\in J_{\mathrm{con}}^{\mathrm{in.hn}}} (w_{j\tau}^{\mathrm{ic.hn}} \cdot Q_{j\tau}^{\mathrm{ic.hn}}) + \sum_{j\in J_{\mathrm{con}}^{\mathrm{ic.hs}}} (w_{j\tau}^{\mathrm{gen.ic.hs}} \cdot Q_{j\tau}^{\mathrm{ic.hs}}). \tag{19}$$

Figure 2: Formation redundant design scheme of district heating system



Economic balance equation (19) without take into account the heat networks costs is as follows:

$$w^{\mathrm{HSC}} \cdot \sum_{j\in J_{\mathrm{hs}}} Q_{j\tau}^{\mathrm{hs}} - \sum_{i\in I} Z_{i\tau}^{\mathrm{hn}}(x_{i\tau}) = \sum_{j\in J_{\mathrm{con}}^{\mathrm{hh}}} (w_{\tau}^{\mathrm{gen.hh}} \cdot Q_{j\tau}^{\mathrm{hh}}) + \sum_{j\in J_{\mathrm{con}}^{\mathrm{in.hn}}} (w_{j\tau}^{\mathrm{gen.ic.hn}} \cdot Q_{j\tau}^{\mathrm{ic.hn}}) + \sum_{j\in J_{\mathrm{con}}^{\mathrm{ic.hs}}} (w_{j\tau}^{\mathrm{gen.ic.hs}} \cdot Q_{j\tau}^{\mathrm{ic.hs}}), \tag{20}$$

where

$$w_{\tau}^{\mathrm{gen.hh}} = w_{\tau}^{\mathrm{hh}} - w_{\tau}^{\mathrm{hn}}, \tag{21}$$

$$w_{j\tau}^{\mathrm{gen.ic.hn}} = w_{j\tau}^{\mathrm{ic.hn}} - w_{j\tau}^{\mathrm{hn}}, \tag{22}$$

$$w_{\tau}^{\mathrm{hn}} = \frac{\sum_{i\in I} Z_{i\tau}^{\mathrm{hn}}(x_{i\tau})}{\sum_{j\in J_{\mathrm{hs}}} Q_{j\tau}^{\mathrm{hs}} - \sum_{j\in J_{\mathrm{con}}^{\mathrm{ic.hs}}} Q_{j\tau}^{\mathrm{ic.hs}}}. \tag{23}$$

If divide equation (20) by $\sum_{j\in J_{\mathrm{hs}}} Q_{j\tau}^{\mathrm{hs}}$, then we obtain the equilibrium price of the produced heat energy of the HSC, relative to all consumer prices and their share of heat consumption:

$$w_{\tau}^{\mathrm{gen.HSC}} = w_{\tau}^{\mathrm{gen.hh}} \cdot \Theta_{\tau}^{\mathrm{hh}} + \sum_{j\in J_{\mathrm{con}}^{\mathrm{ic.hn}}} w_{j\tau}^{\mathrm{gen.ic.hs}} \cdot \Theta_{j\tau}^{\mathrm{ic.hs}} + \sum_{j\in J_{\mathrm{con}}^{\mathrm{in.hs}}} w_{j\tau}^{\mathrm{gen.ic.hs}} \cdot \Theta_{j\tau}^{\mathrm{ic.hs}}, \tag{24}$$

where

$$w_{\tau}^{\mathrm{gen.HSC}} = \frac{\sum_{j\in J_{\mathrm{hs}}} (w_{\tau}^{\mathrm{HSC}} \cdot Q_{j\tau}^{\mathrm{hs}}) - \sum_{i\in I} Z_{i\tau}^{\mathrm{hn}}(x_{i\tau})}{\sum_{j\in J_{\mathrm{hs}}} Q_{j\tau}^{\mathrm{hs}}}, \tag{25}$$

$$\Theta_{\tau}^{\mathrm{hh}} = \frac{\sum_{j\in J_{\mathrm{con}}^{\mathrm{hh}}} Q_{j\tau}^{\mathrm{hh}}}{\sum_{j\in J_{\mathrm{hs}}} Q_{j\tau}^{\mathrm{hs}}}, \tag{26}$$

$$\Theta_{j\tau}^{\mathrm{ic.hn}} = \frac{\sum_{j\in J_{\mathrm{con}}^{\mathrm{ic.hn}}} Q_{j\tau}^{\mathrm{ic.hn}}}{\sum_{j\in J_{\mathrm{hs}}} Q_{j\tau}^{\mathrm{hs}}}, \tag{27}$$

$$\Theta_{j\tau}^{\mathrm{ic.hs}} = \frac{\sum_{j\in J_{\mathrm{con}}^{\mathrm{ic.hs}}} Q_{j\tau}^{\mathrm{ic.hs}}}{\sum_{j\in J_{\mathrm{hs}}} Q_{j\tau}^{\mathrm{hs}}}. \tag{28}$$

To analyze the economic contribution of each heat source to the total heat revenue for each category of consumers, it is necessary to know how the purchase price of heat for a particular consumer relates to the equilibrium price produced by the HSC. To do this, it is suggested to use market principles of a supply-and-demand equilibrium. In market conditions the price increases with a decrease in the purchase volume and vice versa. In the district heating system with many consumer categories having different demand parameters, a relationship between the equilibrium price for the produced heat energy with its prices consumed heat energy can be represented through the average heating market share $\frac{1}{\theta}$, where θ is the number of categories of heat energy consumers. Proceeding from the above, we can write equations of constraints between the prices for generated heat energy and consumed heat energy as follows.

For household consumers:

$$w_\tau^{\text{gen.hh}} = \begin{cases} w_\tau^{\text{gen.HSC}} - w_\tau^{\text{gen.HSC}}(1-\Theta_\tau^{\text{hh}}), \text{ if } \Theta_\tau^{\text{hh}} > 1/\theta_\tau \\ w_\tau^{\text{gen.HSC}} + w_\tau^{\text{gen.HSC}}(1-\Theta_\tau^{\text{hh}}), \text{ if } \Theta_\tau^{\text{hh}} < 1/\theta_\tau \\ w_\tau^{\text{gen.HSC}}, \text{ if } \Theta_\tau^{\text{hh}} = 1/\theta_\tau \end{cases}. \quad (29)$$

For j-th industrial consumers connected to heat network:

$$w_{j\tau}^{\text{gen.ic.hn}} = \begin{cases} w_\tau^{\text{gen.HSC}} - w_\tau^{\text{gen.HSC}}(1-\Theta_{j\tau}^{\text{ic.hn}}), \text{ if } \Theta_{j\tau}^{\text{ic.hn}} > 1/\theta_\tau \\ w_\tau^{\text{gen.HSC}} + w_\tau^{\text{gen.HSC}}(1-\Theta_{j\tau}^{\text{ic.hn}}), \text{ if } \Theta_{j\tau}^{\text{ic.hn}} < 1/\theta_\tau \\ w_\tau^{\text{gen.HSC}}, \text{ if } \Theta_{j\tau}^{\text{ic.hn}} = 1/\theta_\tau \end{cases}. \quad (30)$$

For j-th industrial consumers located on collectors of heat source:

$$w_{j\tau}^{\text{gen.ic.hn}} = \begin{cases} w_\tau^{\text{gen.HSC}} - w_\tau^{\text{gen.HSC}}(1-\Theta_{j\tau}^{\text{ic.hn}}), \text{ if } \Theta_{j\tau}^{\text{ic.hn}} > 1/\theta_\tau \\ w_\tau^{\text{gen.HSC}} + w_\tau^{\text{gen.HSC}}(1-\Theta_{j\text{Ä}}^{\text{ic.hn}}), \text{ if } \Theta_{j\tau}^{\text{ic.hn}} < 1/\theta_\tau \\ w_\tau^{\text{gen.HSC}}, \text{ if } \Theta_{j\tau}^{\text{ic.hn}} = 1/\theta_\tau \end{cases}. \quad (31)$$

Based on the above mentioned, an equilibrium between demand and supply on the heat energy monopoly market is determined by solving the problem of profit maximization (Gravelle and Rees, 2004) of the HSC (for conditions liberalized economy), considering the constraints on minimum and maximum levels of the heat sources productive capacity. Find:

$$\sum_{\tau=\tau_0}^{T} P_\tau^{\text{HSC}} = \sum_{\tau=\tau_0}^{T} \sum_{j\in J_{\text{hs}}} (w_\tau^{\text{HSC}} \cdot Q_{j\tau}^{\text{hs}}) - \sum_{\tau=\tau_0}^{T} \sum_{j\in J_{\text{hs}}} Z_{j\tau}^{\text{hs}}(Q_{j\tau}^{\text{hs}})$$
$$-\sum_{\tau=\tau_0}^{T} \sum_{i\in I} Z_{i\tau}^{\text{hn}}(x_{i\tau}) \to \max, \quad (32)$$

subject to (1)-(7), (9)-(18), (21)-(31), and

$$Q_{j\_\min}^{\text{hs}} \le Q_{j\tau}^{\text{hs}} \le Q_{j\_\max}^{\text{hs}}, \; j \in J_{\text{hs}}. \quad (33)$$

The search for an optimal solution to the developed mathematical model of the heat energy monopoly market is based on the univariate relaxation method (Shoup, 1979) with the of methods of redundant design schemes and simple iteration. The method suggests the reduction of the multidimensional optimization problem to one-dimensional one and the use of a stepwise procedure for the improvement of solutions concerning the volumes of production by all heat sources. The calculation algorithm is presented in the Figure 3 as block diagram.

3. REGULATION TARIFF ON THE MONOPOLY HEATING MARKET

There are two classic methods regulation of tariff on the monopoly market (Gravelle and Rees, 2004): marginal cost (*MC*) method or average total cost (*ATC*) method.

In the first case, the regulator controls the situation so that the price set by the natural monopolist on the market does not exceed its marginal costs. The method of average total costs is that the monopolist operates on the principle of break-even. Both of these methods have certain disadvantages. The marginal cost method in most cases leads to the loss of the monopolist and the need to subsidize its expenses with public funds. The method of setting the price at the level of average total costs deprives the monopolist of the incentive to reduce his costs, since he knows that any costs will be compensated by the corresponding fixed price. These methods allow, on the one hand, to reduce the tariff for products as compared to the unregulated monopoly, and on the other hand, they allow to stimulate the monopolist's productivity increase.

3.1. The Average Total Costs Method

Consider formation of a mathematical model of a regulated monopoly heating market, in which the tariff on the heat energy for household consumers is set at the level of the corresponding share of average total costs, which were spent on the production and transportation of heat energy for household consumers. The scheme of construction of such a mathematical model (32), (33), (1)-(7), (9)-(18), (21)-(31) is fully consistent with the model with the addition of restrictions on the heat energy tariff for household consumers:

$$w_\tau^{\text{hh}} = ATC = \frac{\Theta_\tau^{\text{hh}} \cdot (\sum_{j\in J_{\text{hs}}} Z_{j\tau}^{\text{hs}}(Q_{j\tau}^{\text{hs}}) + \sum_{i\in I} Z_{i\tau}^{\text{hn}}(x_{i\tau}))}{\sum_{j\in J_{\text{con}}^{\text{hh}}} Q_{j\tau}^{\text{hh}}}$$
$$= \frac{\sum_{j\in J_{\text{hs}}} Z_{j\tau}^{\text{hs}}(Q_{j\tau}^{\text{hs}}) + \sum_{i\in I} Z_{i\tau}^{\text{hn}}(x_{i\tau})}{\sum_{j\in J_{\text{hs}}} Q_{j\tau}^{\text{hs}}}. \quad (34)$$

3.2. The Marginal Costs Method

In the case of regulation of the tariff on the heat energy for household consumers at the level of marginal costs, the type of mathematical model will be preserved in the same way as in the regulation variant at the average total costs, but with the replacement of heat energy tariff for household consumers at marginal cost.

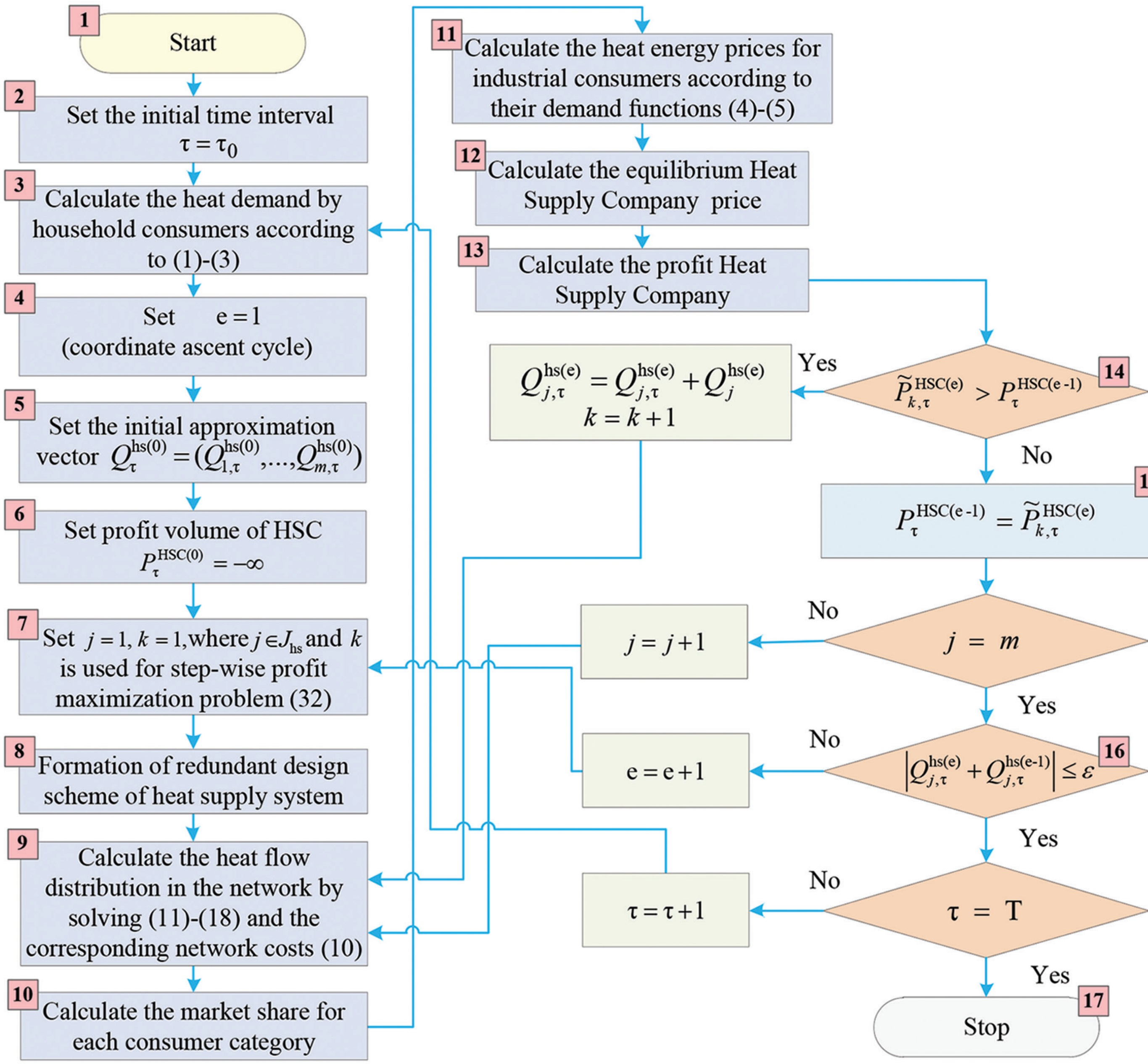
Marginal costs or costs associated with the additional production and transportation of a heat energy unit are the ratio of the total marginal costs for the production and transportation of heat energy to the volume of produced and transportation of heat energy by the HSC. For household consumers, these costs can be represented as follows:

$$w_\tau^{\text{hh}} = MC = \frac{\partial(\sum_{j\in J_{\text{hs}}} Z_{j\tau}^{\text{hs}}(Q_{j\tau}^{\text{hs}}) + \sum_{i\in I} Z_{i\tau}^{\text{hn}}(x_{i\tau}))}{\partial Q_{j\tau}^{\text{hs}} \partial x_{i\tau}}$$
$$= \sum_{j\in J_{\text{hs}}} (2\pm_j \cdot Q_{j\text{Ä}}^{\text{hs}} + ^2{}_j) + \frac{3 \cdot C_{\text{el}} \cdot \sum_{i\in I} x_{i\tau}^2 \cdot s_i}{367.2 \cdot \eta}. \quad (35)$$

4. SIMULATION AND RESULTS

The initial data for the mathematical modeling of a competitive heating market is the heat supply scheme with specified lengths and pipeline diameters, locations of sources of heat energy, cost

Figure 3: The block diagram of algorithm for search of equilibrium of supply-and-demand on monopoly heating market



functions of heat sources, climatic characteristics of the region and the demand of household and industrial consumers. Various objects represent consumers of household: apartment blocks, schools, restaurants, hospitals etc. The type of buildings, taking into account their classification according to thermophysical properties, calculates demand of household. According to the known diameters and lengths of pipelines, the resistance of the heat network sections is determined using the D'Arcy-Weisbach formula. The site resistance will then be used to simulate the heat network. The developed mathematical model was tested on the real district heating system. The design scheme is shown in Figure 4.

Considered three variants formed pricing on the heat energy:

- Variant 1: free pricing based on the market equilibrium of demand and supply for heat energy (Var.1);
- Variant 2: regulation tariff on the heat energy by average total costs method (Var.2);
- Variant 3: regulation tariff on the heat energy by marginal costs method (Var.3).

The calculations were performed using GAMS/CONOPT solver.

Table 2 presents annual technical and economic indices obtained from the calculations for HSs, heat network and consumers.

In Figure 5 are shown the heat energy production volumes and corresponding prices of the HSC for three different variants pricing on monopoly heating market.

The calculations showed that in variant 3 are achieved the maximum heat energy production volumes (38.0 million GJ) with

Table 2: Technical and economic indices of heat energy market

Calculated indices	Var.1	Var.2	Var.3
Heat production volume, million GJ, including:	30.9	32.0	38.0
HS-1	4.6	6.3	6.5
HS-2	17.7	17.5	21.8
HS-3	8.6	8.2	9.7
Heat production costs, million EUR	44.8	46.8	54.9
Heat network costs, million EUR	16.9	9.7	11.7
Total costs of the Heat Supply Company, million EUR	61.7	56.5	66.6
Heat consumption by household consumers, million GJ	27.0	27.0	27.0
Heat consumption by industrial consumer, million GJ	3.8	5.1	11.0
Equilibrium heat energy price of the Heat Supply Company, EUR/GJ	2.41	1.84	1.35
Heat energy price for industrial consumers, EUR/GJ	3.07	3.01	2.65
Heat energy price for household consumer, EUR/GJ	2.33	1.63	0.83
Profit of the Heat Supply Company, million EUR	12.80	2.80	-15.04

Figure 4: The design heat supply scheme

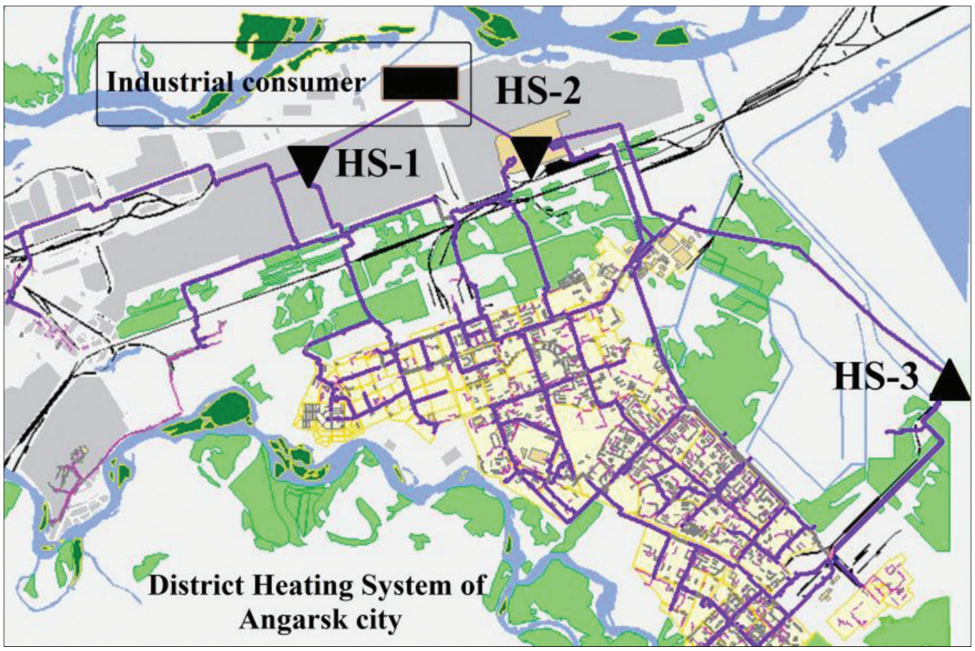
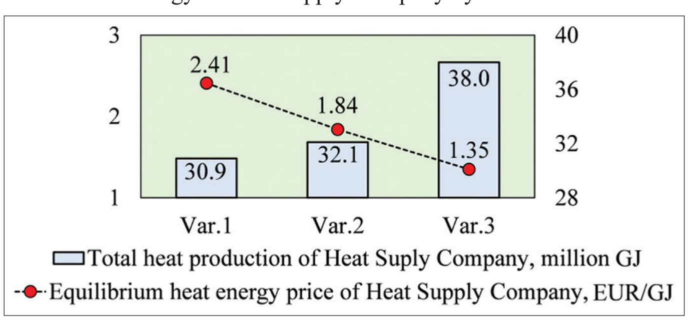


Figure 5: Optimal values for production volumes and prices for heat energy of Heat Supply Company by variants



the minimum price (1.35 EUR/GJ) on the heat energy of HSC (Figure 5). Minimum level production heat energy (30.9 million GJ) with maximum heat energy price (2.41 EUR/GJ) of the HSC correspond variant 1 (Figure 3). The results obtained for the prices and volumes of heat energy produced by HSC satisfy the market conditions of demand and supply (i.e. the price increases with a decrease in the heat energy production volumes and vice versa (Figure 5).

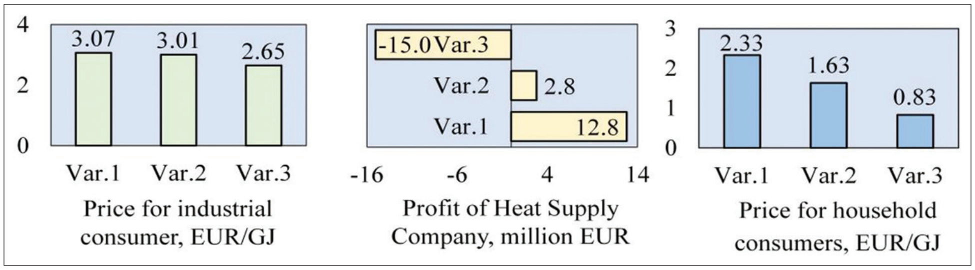
Quantitative assessment of indicators (prices and profit) reflecting the interests of participants (household and industrial consumers and HSC) on the heat energy market are show in Figure 6.

From the standpoint of the HSC (maximizing profits), variant 1 is beneficial in which it receive the greatest profits. The model with tariff regulation for household consumers at the level of marginal costs (variant 3) HSC incur losses. Since the tariff set for household consumers does not allow to cover relatively fixed costs of HSC.

In variant 2 (regulation of household tariff at the level of average total costs), HSC makes the profit (4.7% of total revenue) at the account of industrial consumer, and tariff for the household consumers and the price for the industrial consumer will be 1.63 EUR/GJ and 3.01 EUR/GJ, respectively.

Consumer preferences (price for heat energy) by variants are shown in the Figure 6. For household consumers, the best is variant 3, in which the lowest prices for heat energy are obtained - 0.83 EUR/GJ. The most expensive tariff for household and industrial consumers is variant.

Figure 6: The indicators reflecting the interests of participants on the monopoly heating market



5. CONCLUSION

The most common is the monopoly model of the heat energy market operating under two pricing conditions: free (liberalized) pricing and tariff regulation for consumers. The equilibrium mathematical model was developed for the free pricing on the heat energy market based on the microeconomic model of the monopoly market. This mathematical model makes it possible to take into account heat energy production and transportation costs as part of a single economic criterion. For the equilibrium pricing for consumers, the method based on market pricing principles was proposed. For the regulated heating market model, the methods of heat energy pricing for household consumers with allowance for average total costs and marginal costs was proposed.

The univariate relaxation algorithm (the method of coordinate ascent) with the use of redundant design schemes and simple iteration inside the cycle was developed as a computational tool for searching for the heat energy supply-and-demand equilibrium. It allows determining the optimal parameters of district heating systems (heat energy production volumes, their distribution among heat sources, optimal heat carrier distribution in the network, etc.) for both free pricing and tariff regulation in the context of the HSC. The practical studies carried out on the example of the district heating system showed that the transition from regulated tariffs to the free pricing model on the heating market would lead to a sharp increase in the heat energy tariffs for household consumers (by 30%) and gaining of excess profit by HSC (more 17%).

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