



# Managing Systemic Risk in Energy and Financial Markets: Evidence from Five Portfolio Strategies Based on Connectedness

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## ABSTRACT

This study investigates the evolving interdependencies and risk transmission mechanisms between energy and financial markets, focusing on gold, West Texas Intermediate (WTI) crude oil, the S&P 500 Index (SP500), and the Shanghai Stock Exchange Composite Index (SSE) from January 2019 to August 2025. Employing daily return data, the analysis integrates multivariate linear regression, Dynamic Conditional Correlation-GARCH (DCC-GARCH) models, and dynamic conditional  $R^2$  decomposition to capture time-varying connectedness and explanatory power. Grounded in the Diebold and Yilmaz (2012; 2014) framework, augmented by  $R^2$  decomposition, the findings evince heightened systemic risk during major global disruptions, notably the COVID-19 pandemic and the Russia Ukraine conflict. WTI crude oil emerges as a principal conduit of volatility spillovers, underscoring its pivotal role in the energy-finance nexus. Five portfolio strategies Minimum Variance, Minimum Correlation, Minimum Connectedness, Minimum  $R^2$ , and Minimum Decomposed  $R^2$  are constructed and evaluated under systemic stress. While the Minimum Variance Portfolio delivers robust risk-adjusted returns, strategies based on connectedness metrics demonstrate superior resilience during crises. These insights offer valuable implications for policymakers, investors, and scholars concerned with energy market stability, financial contagion, and adaptive portfolio design.

**Keywords:** Energy Finance Linkages, Oil and Gold Markets, Dynamic Connectedness, Volatility Spillovers, DCC-GARCH Models, Systemic Risk Management

**JEL Classifications:** C58, G11, G15, Q43

## 1. INTRODUCTION

Energy and financial markets have become increasingly interconnected, facilitating the rapid transmission of economic, financial, and geopolitical shocks across commodities, financial instruments, and regions. Within this network, certain assets hold critical influence over systemic risk. West Texas Intermediate (WTI) crude oil is a key energy commodity that reflects broader macroeconomic trends and geopolitical tensions, serving as a major conduit for volatility spillovers (Nerurkar and Jickling, 2012; Toyoshima and Hamori, 2018; Wen et al., 2022; Filippidis et al., 2023; Belkhir et al., 2025). In contrast, gold is frequently viewed as a safe-haven asset amid uncertainty (Fang et al., 2018; Shahzad et al., 2020; Shankar, 2025). Equity markets such as the

S&P 500 in the United States and the Shanghai Stock Exchange Composite Index in China act as important indicators of economic health in both developed and emerging markets (Xu et al., 2023; Li et al., 2024). An integrated study of these assets is thus vital for understanding systemic risk dynamics and their implications for global financial stability and energy policy.

Previous research has extensively analyzed volatility and interconnectedness among financial assets. Markowitz's (1952) foundational work provides evidence of the benefits of diversification, but later studies evince that correlations tend to increase during crises, weakening diversification benefits when most needed (Wen et al., 2022; Ghorbel et al., 2022; Leong, 2025; Enow, 2025). In commodities, oil delves into a significant role in transmitting

macro-financial shocks (Kilian and Park, 2009; Broadstock and Filis, 2014), with sensitivity to economic and geopolitical events well documented (Hu et al., 2020; Le et al., 2023; Bouzguenda and Jarboui, 2025). Gold's role as a hedge and safe haven varies depending on crisis conditions (Baur and Lucey, 2010; Baur and McDermott, 2010; Reboredo, 2013; Chkili, 2016). Studies on equity market connections (Diebold and Yilmaz, 2012; 2014) give proof that linkages intensify under stress, increasing contagion risk.

However, two key gaps remain. Most research examines commodity and equity markets separately, overlooking their interdependence (Filippidis et al., 2023; Guidi et al., 2025). Moreover, portfolio management often relies on static variance-covariance matrices, which do not capture evolving systemic risk and volatility spillovers. Addressing these limitations requires dynamic models that incorporate systemic risk into portfolio construction.

This study fills these gaps by jointly analyzing gold, WTI crude oil, the S&P 500, and the Shanghai Stock Exchange Composite Index over 2019-2025, encompassing shocks like the COVID-19 pandemic and the Russia Ukraine war. It employs the DCC-GARCH model (Engle, 2002), along with dynamic conditional  $R^2$  measures and decomposed connectedness indices (Diebold and Yilmaz, 2012; 2014), to quantify volatility interactions and systemic risk. The study also tests five portfolio strategies incorporating connectedness metrics.

The main contribution aims to highlight the integration of systemic risk assessment with portfolio management, using conditional volatility and connectedness measures in a cohesive framework. Empirically, it provides evidence of the distinct roles of gold, oil, and equity markets in shock transmission. Practically, it proposes dynamic portfolio strategies that show greater resilience during crises. The paper proceeds as follows: Section 2 reviews literature; Section 3 details methods and data; Section 4 reports findings; and Section 5 concludes with implications for stakeholders.

## 2. LITERATURE REVIEW

Over the past decades, increasing instability in financial linkages has drawn wide scholarly interest. Multiple crises revealed the shortcomings of traditional diversification, as correlations and volatility spillovers tend to escalate during turbulent times, reducing diversification benefits when they are most crucial (Ghorbel et al., 2022; Frikha et al., 2023; Pandey et al., 2023). To better capture systemic risk and market interdependencies, researchers have shifted towards dynamic methodologies, resulting in extensive empirical contributions (Wang et al., 2022a; Yousaf et al., 2023). Within this context, commodities such as oil and gold have been central topics (Hu et al., 2020; Zhang and Wu, 2024; Bouzguenda and Jarboui, 2025). Oil remains a critical component in the global economy, acting as a transmitter of macroeconomic and financial shocks. Its volatility is influenced by supply-demand fundamentals, inventory changes (Bai, 2014; Le et al., 2023), and financial indicators including the U.S. dollar index and VIX (Hu et al., 2020; Bhagat et al., 2022). Geopolitical risks and policy uncertainty further intensify oil's volatility and its role in cross-market contagion (Zhao, 2022;

Filippidis et al., 2023). Gold is widely recognized as a safe-haven asset during market distress. Evidence supports its tendency to decouple from risky assets during crises, though effectiveness varies with shock type and intensity (Baur and Lucey, 2010; Baur and McDermott, 2010; Reboredo, 2013; Chkili, 2016; Ghorbel et al., 2022; Widjaja et al., 2023). Its volatility is sensitive to macroeconomic uncertainty, interest rate changes, and investor sentiment, reinforcing its defensive qualities (Fang et al., 2018; Hu et al., 2020; Gupta et al., 2023). Equity markets also contribute significantly to systemic risk analysis. The connectedness framework proposed by Diebold and Yilmaz (2012; 2014) offers a robust measure of contagion through forecast error variance decomposition. Their studies demonstrate that market linkages strengthen in crises, heightening systemic stress. This method has since been expanded to analyze equities, bonds, and commodities, revealing systemic risk fluctuations in response to macro-financial conditions (Wang et al., 2022b; Xu et al., 2023).

There is broad agreement that financial interconnections lack stability and depend heavily on context. Global events such as economic downturns, pandemics, and geopolitical conflicts reshuffle asset correlations and increase systemic risk transmission (Wen et al., 2022; Frikha et al., 2023). These dynamics challenge risk management and limit the effectiveness of conventional diversification during adverse periods.

Many previous studies have examined the connections between various assets using different methods, often revealing diverse and sometimes contradictory results. Anand and Paul (2021) employ a TVP-SVAR model to reveal that Brent oil demand shocks impact returns and volatility in the Bombay stock exchange. Likewise, Mensi et al. (2022), using a bivariate FIAPARCH approach, identify shifting correlations between gold, oil, and U.S. equities during the COVID-19 pandemic, highlighting the complexity of market interactions in turbulent times. During the invasion crisis, Alam et al. (2022) find that gold and silver primarily transmitted shocks, while crude oil, platinum, and natural gas mainly absorbed them. In China, Dai et al. (2022) report that WTI oil and gold mostly received shocks, with certain stock sectors acting as transmitters; this pattern is echoed by Li et al. (2024), who point to copper as a key shock transmitter. Moreover, Chen et al. (2024) show that uncertainty measures like the EPU, VIX, and GPR strengthen return connectedness among assets. Recently, Bouzguenda and Jarboui (2025) use quantile connectedness to demonstrate that relationships between BRICS assets and alternative investments were quite unstable and sensitive to crises from 2016 to 2023.

Despite these advances, critical gaps persist. Most literature analyzes commodities and equities separately, lacking a comprehensive unified framework. Additionally, portfolio management often depends on static variance-covariance models that fail to represent systemic risk dynamics. Although connectedness-based metrics have been suggested to improve portfolio robustness during crises, their practical use remains limited.

In sum, three main themes emerge from the literature:

- Asset correlations are time-varying and tend to intensify in crisis periods

- Oil and gold play distinct but complementary roles within global financial dynamics
- Advances in connectedness models and dynamic volatility frameworks have enhanced systemic risk insight, yet their integration into portfolio management is still underdeveloped.

### 3. DATA AND METHODOLOGY

#### 3.1. Data

This study examines daily closing prices for Gold, West Texas Intermediate (WTI) crude oil, the Shanghai Stock Exchange Composite Index (SSE), and the SandP 500 Index (SP500) during the period from January 03, 2019 to August 01, 2025. All data were collected from reliable financial databases, including Bloomberg, Refinitiv, and Yahoo Finance, and are expressed in U.S. dollars to ensure consistency. Daily returns were calculated using continuously compounded log-returns. The descriptive statistics reported in Table 1 show that the mean daily returns are close to zero for each asset, with gold and the SP 500 displaying weak but statistically significant positive means, while WTI and the SSE present means not significantly different from zero. Variance is lowest for gold and the SP 500 and highest for WTI, indicating greater return volatility in the oil market. All series exhibit pronounced non-normality: gold, WTI, the SP 500, and the SSE have negative skewness, which is particularly extreme for WTI, and all display high excess kurtosis, once more most striking for WTI. The Jarque–Bera test confirms significant departures from normality for every market. Unit-root testing with the Elliott–Rothenberg–Stock (ERS) statistic rejects the null of a unit root for all return series, proving stationarity. The Ljung–Box Q test on raw returns detects no autocorrelation for gold but significant serial dependence for WTI, the SP 500, and the SSE. The  $Q^2$  test on squared returns reveals strong conditional heteroskedasticity amid all markets, consistent with time-varying volatility. Kendall’s rank correlations highlight globally low but significant dependence between assets. Gold’s correlations with WTI, the SP 500, and the SSE are weak, though positive. WTI shows modest positive associations with the SP 500 and SSE, and the SP 500 and SSE share the highest cross-market dependence among the pairs. Whole, these results portray gold as the least connected asset, WTI as the most volatile, and all markets as non-normally distributed with evidence of conditional heteroskedasticity.

#### 3.2. Methodology

Following Cocco et al. (2024), this study adopts a comprehensive, multi-step methodology to examine evolving interdependencies and translate them into actionable portfolio strategies. It begins by revisiting the multivariate linear regression (MLR) model to estimate coefficients and assess model fit, then extends the analysis through the DCC-GARCH framework (Engle, 2002) to capture time-varying correlations and covariance dynamics. Subsequently, the dynamic conditional  $R^2$  and its decomposition (Genizi, 1993) are employed to evaluate the shifting explanatory power of key variables. These results are embedded within a dynamic connectedness framework to identify systemic transmission channels among assets. Finally, the insights are leveraged to develop advanced portfolio strategies, namely connectedness-driven and multivariate hedging portfolios, benchmarked against traditional models to evaluate performance and risk efficiency.

##### 3.2.1. Multivariate linear regression (MLR) framework

This section outlines the fundamental principles of the multivariate linear regression (MLR) model, establishing the basis for more advanced approaches such as the DCC-GARCH framework. It focuses on the estimation of regression coefficients and the assessment of model adequacy using the R-squared statistic. The MLR model remains a foundational tool in empirical finance and econometric analysis, typically formulated as follows:

$$y = Xb + \varepsilon \quad (1)$$

Where  $y$  is the dependent variable,  $X$  denotes the matrix of independent variables, and  $b$  is the  $K \times 1$  dimensional vector of regression coefficients, and  $\varepsilon$  is the  $T \times 1$  dimensional error vector. All series, collectively denoted as  $Z = [y, X]$ , have been adjusted for their mean values, thus excluding the intercept term. The unconditional variance–covariance matrix ( $H$ ) and correlation matrix ( $R$ ) are partitioned as follows:

$$H = \begin{bmatrix} H_{yy} & H_{yx} \\ H_{xy} & H_{xx} \end{bmatrix} \quad (2)$$

and

$$R = \begin{bmatrix} 1 & R_{yx} \\ R_{xy} & R_{xx} \end{bmatrix} \quad (3)$$

Where:

$H_{yy}$  represents the variance of the dependent variable ( $y$ ),

$H_{xx}$  represents the variance-covariance matrix of the independent variables ( $X$ ),

$H_{xy}$  ( $H_{yx} = H'_{xy}$ ) contains the covariances between the independent and dependent variables.

The vector of regression coefficients,  $b$ , can be estimated using Ordinary Least Squares (OLS), yielding the estimator  $\hat{b}$ :

$$\hat{b} = (X'X)^{-1}X'y \quad (4)$$

This coefficient can be expressed directly in terms of the variance-covariance matrix  $H$ :

$$\hat{b} = H_{xx}^{-1}H_{xy} \quad (5)$$

This relationship underscores that the regression coefficients are solely governed by the underlying variance–covariance structure of the variables. The R-squared ( $R^2$ ) statistic captures the share of the dependent variable’s ( $y$ ) variance that can be explained by the independent variables ( $X$ ):

$$R^2 = 1 - ((y - X\hat{b})'(y - X\hat{b}) / y'y) \quad (6)$$

As the regression coefficients, the  $R^2$  can also be computed directly from the partitioned variance-covariance matrix  $H$  or the correlation matrix  $R$ , as follows:

$$R^2 = H'_{xy} H_{xx}^{-1} H_{xy} H_{yy}^{-1} \quad (7)$$

$$R^2 = R'_{xy} R_{xx}^{-1} R_{xy} \quad (8)$$

Accordingly, both the regression coefficients and the  $R^2$  statistic can be derived directly from the unconditional variance–covariance matrix. Crucially, by substituting this static matrix HHH with its time-varying counterpart  $H_t$  obtained from the DCC-GARCH framework, one obtains dynamic conditional betas and dynamic conditional  $R^2$  measures (Engle, 2016). This substitution enables a time-varying assessment of model fit and explanatory power across evolving market conditions.

### 3.2.2. DCC-GARCH model specification

Ensuing Engle (2002), the Dynamic Conditional Correlation GARCH model is specified follow:

$$\varepsilon_t = D_t^{-1} z_t, z_t \sim N(0, H), \varepsilon_t \sim N(0, I), D_t = \text{diag}(h_{1t}^2 \dots h_{Kt}^2) \quad (9)$$

$$Q_t = (1 - a - b) \bar{Q} + a \varepsilon_{t-1} \varepsilon_{t-1}' + b Q_{t-1} \quad (10)$$

$$R_t = \text{diag}(Q_t^{-1/2} Q_t \text{diag}(Q_t^{-1/2})) \quad (11)$$

$$H_t = D_t R_t D_t \quad (12)$$

Here,  $z_{it}$  and  $\varepsilon_{it}$  are  $K + 1 \times 1$  dimensional return and standardized residual vectors, respectively. Error terms are independent and identically distributed (*i.d.*) random variables with zero mean and unit variance, often assumed to follow a specific distribution (e.g., Normal, Student's  $t$ , Skewed Student's  $t$ ),  $\bar{Q}$ ,  $H_t$  and  $R_t$  are  $K \times 1 + K \times 1$  dimensional unconditional variance, conditional variance-covariance, and conditional correlation matrix, respectively.

The estimation follows a two-step procedure: (i) univariate GARCH models yield conditional variances  $D_t$  and (ii) DCC parameters  $a$  (shock) and  $b$  (persistence) are estimated to derive dynamic correlations,  $R_t$  and ultimately  $H_t$ .

The full log-likelihood is decomposed into variance and correlation components:

$$L(\theta, \emptyset) = L_v(\theta) + L_c(\theta, \emptyset) \quad (13)$$

$$L_v(\theta) = -\frac{1}{2} \sum_{i=1}^K (\log(2\pi) + \log(h_{it} + \frac{z_{it}^2}{h_{it}})) \quad (14)$$

$$L_c(\theta, \emptyset) = \sum_{t=1}^T (K \log(2\pi) + \log(|D_t|) + \log(|R_t|) + \varepsilon_t' R_t^{-1} \varepsilon_t) \quad (15)$$

The log-likelihood function is divided into two parts: volatility.  $L_v(\theta)$  and correlation  $L_c(\theta, \emptyset)$ . In step one,  $\theta$  refers to univariate GARCH parameters; in step two,  $\emptyset$ , who represents DCC parameters. These steps can be estimated separately.<sup>1</sup> Following Antonakakis et al. (2021), we use their selection criterion to

estimate the univariate GARCH (1,1) model from the flexible class proposed by Hentschel (1995) and discussed by Hansen and Lunde (2005).

$$h_t^{\lambda/2} = \omega + \alpha h_{t-1}^{\frac{\lambda}{2}} (|z_{t-1} - \eta| - \gamma(z_{t-1} - \zeta)^\delta + \beta h_{t-1}^{\frac{\lambda}{2}}) \quad (16)$$

To allow greater flexibility, we follow Antonakakis et al. (2023) by estimating multiple bivariate DCC-GARCH models, permitting correlation dynamics  $a$  (shock) and  $b$  (persistence) to differ across asset pairs. This contrasts with a single multivariate DCC specification that imposes uniform correlation dynamics. Engle and Sheppard's (2001) DCC test is then employed to distinguish between constant and dynamic correlations.

### 3.3. Dynamic Conditional $R^2$ and $R^2$ Decomposed Measures

The dynamic conditional  $R^2$  is obtained by replacing the components of  $H$  in Equation (6) by the dynamic components of  $H_t$ . Thus, dynamic conditional  $R^2$  goodness-of-fit measures are obtained as follows,

$$R_t^2 = R_{xy,t}' R_{xx,t}^{-1} R_{xy,t} \quad (17)$$

This time-varying  $R_t^2$  captures the explanatory power of the model in a dynamic context, providing insights into hedging effectiveness and portfolio dependencies.

Since the sum of bivariate  $R_t^2$  values typically differ from multivariate  $R_t^2$  (due to correlations among regressors), Genizi (1993) proposed a decomposition method based on PCA of the correlation matrix. The procedure transforms correlated regressors into orthogonal latent factors  $f_t$ , that preserve total variance. Using eigenvalue decomposition of  $R_{xx,t}$ , the decomposed  $R_t^2$  is derived:

$$R_{xx,t} = V_t E_t^2 V_t' = R_{xf,t} R_{xf,t}' \quad (18)$$

$$R_{xf,t} = V_t E_t V_t' \quad (19)$$

$$R_t^2 = R_{xf,t}^2 (R_{xy,t}^{-1} R_{xy,t})^2 \quad (20)$$

$$\text{Where } R_{fy,t} = R_{xf,t}^{-1} R_{xy,t}$$

$$R_t^2 = R_{xf,t}^2 R_{fy,t}^2 \quad (21)$$

This decomposition attributes the overall explanatory power to each explanatory variable, clarifying their relative contributions over time.

### 3.4. Dynamic Connectedness Measures

To analyze systemic risk, we extend the Diebold and Yilmaz (2012; 2014) framework by replacing the standard forecast-error variance decomposition with the decomposed  $R_t^2$  matrix, yielding:

$$R_t^{2d} = [R_{1t}^2, \dots, R_{it}^2, \dots, R_{Kt}^2]$$

<sup>1</sup> In simple terms, Antonakakis et al. (2021) introduce a univariate GARCH selection criterion, which is essentially an adjusted Bayesian Information Criterion (BIC). This criterion not only penalizes based on the total number of parameters but also considers the number of statistically insignificant parameters. Moreover, it incorporates knock-out criteria derived from significant misspecification test statistics such as VaR, CVaR, VaR Duration, Sign Bias, and Q2(20) test statistics. For more details, interested readers are directed to Antonakakis et al. (2021).



From this, directional connectedness is computed:

$$FROM_{it} = \sum_{k=1, k \neq i}^K R_{ik,t}^{2d} = R_{i,t}^2 \quad (22)$$

$$TO_{it} = \sum_{k=1, k \neq i}^K R_{ki,t}^{2d} \quad (23)$$

$$NET_{i,t} = TO_{i,t} - FROM_{i,t} \quad (24)$$

Positive (negative) net values indicate whether a variable is a net transmitter (receiver) of shocks. Bilateral spillovers are captured through the net pairwise directional connectedness (NPDC):

$$NPDC_{ij,t} = R_{ij,t}^{2d} - R_{ji,t}^{2d} \quad (25)$$

The Total Connectedness Index (TCI) summarizes systemic risk as the average conditional  $R_t^2$ :

$$TCI_t = \frac{1}{K} \sum_{k=1}^K TO_{k,t} = \frac{1}{K} \sum_{k=1}^K FROM_{k,t} = \frac{1}{K} \sum_{k=1}^K R_{k,t}^2 \quad (26)$$

As  $R_t^2 \in [0,1]$ , the TCI is naturally normalized, rising sharply during crises when assets move more synchronously. Pairwise connectedness indices (PCI) are also employed:

$$PCI_{ij,t}^C = 2 \cdot \left( \frac{R_{ij,t}^{2,d} + R_{ji,t}^{2,d}}{R_{ii,t}^{2,d} + R_{jj,t}^{2,d} + R_{ij,t}^{2,d} + R_{ji,t}^{2,d}} \right) \quad (27)$$

$$PCI_{ij,t}^C = \frac{2 \cdot R_{ij,t}^2}{1 + R_{ij,t}^2} \quad (28)$$

### 3.5. Multivariate Portfolio Analysis

This section develops both bivariate and multivariate portfolio strategies, emphasizing dynamic rebalancing under shifting market conditions. Following Kroner and Sultan (1993), the bivariate hedge ratio is:

$$hr_{ij,t} = \frac{H_{ij,t}}{H_{jj,t}} \quad (29)$$

Where  $H_{ij,t}$  and  $H_{jj,t}$  stand for the conditional covariance between asset  $i$  and  $j$  and the conditional variance of asset  $j$ .

#### 3.5.1. Multivariate hedging portfolios

Using Engle's (2016) dynamic beta framework:

$$\beta_t = H_{xx,t}^{-1} H_{xy,t} \quad (30)$$

A long position in  $y_t$  can be hedged by shorting  $\beta_t$  in other assets. This strategy benefits from higher predictive accuracy during periods of strong correlations.

#### 3.5.2. Optimal bivariate portfolio weights

Kroner and Ng (1998) define the optimal portfolio weight

$$\omega_{ij,t} = \frac{H_{jj,t} - H_{ij,t}}{H_{ii,t} - 2H_{ij,t} + H_{jj,t}} \quad (31)$$

Weights are constrained between 0 and 1 to satisfy the no-short selling assumption:

$$\omega_{ij,t} = \begin{cases} 0 & \text{if } \omega_{ij,t} < 0 \\ \omega_{ij,t} & \text{if } 0 < \omega_{ij,t} < 1 \\ 1 & \text{if } \omega_{ij,t} > 1 \end{cases} \quad (32)$$

#### 3.5.3. Global minimum risk portfolios

Extending Markowitz (1952), Christoffersen et al. (2014), and Broadstock et al. (2022), the optimization problem is:

$$\argmin_{\omega} \omega' P_t \omega \text{ s.t. } \omega' \omega = 1, 0 \leq \omega'_i \leq 1$$

Here,  $P_t$  may be based on variance-covariance ( $\sigma$ ), correlations ( $R_t R_t'$ ), or PCI measures. Minimizing PCI-based risk often yields superior performance in reducing systemic exposures.

#### 3.5.4. Portfolio performance

Performance is evaluated using the Sharpe Ratio (SR) (Sharpe, 1994) and Hedging Effectiveness (HE) (Ederington, 1979).

$$SR = \frac{\bar{x}_p}{\sqrt{VAR(x_p)}} \quad (33)$$

These metrics assess both risk-adjusted returns and the effectiveness of hedge strategies.

## 4. EMPIRICAL RESULTS

This section is structured as follows. Primary, we present the estimation results of the DCC-GARCH model and the selection process for univariate GARCH models, including misspecification tests. Following, we examine the dynamic conditional  $R^2$  and its decomposed measures to assess time-varying relationships. We then discuss the outcomes of our connectedness analysis, starting with an overview of return spillovers using averaged connectedness measures. This is followed by an in-depth exploration of dynamic connectedness plots to capture temporal variations. Finally, we compare bivariate and multivariate portfolio strategies, evaluating their performance using various portfolio statistics to assess hedging effectiveness and risk diversification.

### 4.1. DCC-GARCH Estimation

Table 2 reports the diagnostic results for the best-fit univariate GARCH models among the four-return series. Overall, these diagnostics show that the chosen univariate GARCH models capture the conditional volatility dynamics effectively, with no evidence of misspecification or residual ARCH effects for any of the examined assets.

The sign-bias test statistics range from about 0.25 to 1.21 with P-values greater than 10%, indicating no significant leverage or asymmetry effects. The WARCH(20) statistics (8.45-14.93) also have non-significant P-values (0.68-0.12), indicating no remaining ARCH effects after fitting the GARCH specification. Value-at-Risk (VaR) backtests show non-significant statistics between 0.006 and 0.42, confirming adequate VaR coverage.

**Table 1: Summary statistics**

Statistic / Asset	Gold	WTI	SP500	SSE
Mean	0.000** (0.032)	0.000 (0.647)	0.000** (0.047)	0.000 (0.976)
Variance	0	0.001	0	0
Skewness	-0.296*** (0.000)	-2.961*** (0.000)	-0.868*** (0.000)	-0.515*** (0.000)
Ex.Kurtosis	3.059*** (0.000)	76.340*** (0.000)	17.287*** (0.000)	7.936*** (0.000)
JB	939.604*** (0.000)	567480.808*** (0.000)	29216.525*** (0.000)	6198.567*** (0.000)
ERS	-7.047 (0.000)	-9.799 (0.000)	-21.852 (0.000)	-21.608 (0.000)
Q (20)	9.315 (0.581)	69.370*** (0.000)	214.865*** (0.000)	43.510*** (0.000)
Q2 (20)	183.969*** (0.000)	436.982*** (0.000)	2662.423*** (0.000)	284.912*** (0.000)
Kendall	Gold	WTI	SP500	SSE
Gold	1.000***	0.063***	0.013	0.046***
WTI	0.063***	1.000***	0.122***	0.068***
SP500	0.013	0.122***	1.000***	0.073***
SSE	0.046***	0.068***	0.073***	1.000***

Significance levels are indicated by \*\*\*, \*\*, and \* for 1%, 5%, and 10%, respectively, with P-values shown in parentheses. Skewness is assessed via the D'Agostino (1970) test, kurtosis via Anscombe and Glynn (1983), and normality via the Jarque-Bera (1980) test. Unit-root properties are evaluated using Elliott et al. (1996), while the  $Q^2$  (20) statistic represents the weighted Portmanteau test of Fisher and Gallagher (2012)

**Table 2: Evaluation of univariate GARCH performance**

Measure Type	SignBias	WARCH (20)	VaR	CVaR	VaR duration
Statistics	0.2522450	8.4533406	0.0065327	-463.6548	1.0086207
P-values	0.8008741	0.6762910	0.9355809	0.4220	0.5953409
Statistics	0.7772550	13.1708005	0.1329486	-472.6075	1.0344828
P-values	0.4370877	0.2203258	0.7153943	0.1990	0.3161116
Statistics	1.2139154	14.9294435	0.4175507	-481.4844	1.0603448
P-values	0.2249037	0.1244114	0.5181617	0.7860	0.7045404
Statistics	0.9570174	14.3057750	0.2438077	-445.5148	0.9568966
P-values	0.3386583	0.1535730	0.6214694	0.1130	0.1656837

Conditional Value-at-Risk (CVaR) statistics are large and negative, but their P-values (0.42-0.11) again fail to reject model adequacy. Lastly, VaR duration tests yield statistics close to 1.0 and P-values from 0.70 to 0.17, indicating correct independence and clustering of VaR exceedances.

#### 4.1.1. Dynamic conditional variance-covariance

Figure 1 depicts the dynamic variance-covariance based on the time-varying volatility and interdependencies across SP500, SSE, Gold, and WTI.

Empirical results show that WTI exhibits the most pronounced variance shifts, with sharp spikes particularly during the COVID-19 crisis in 2020 and the subsequent energy market shocks, reflecting the commodity's vulnerability to global demand-supply disruptions and geopolitical tensions. The SSE also records substantial variance increases, especially in early 2020 and during later episodes of domestic growth uncertainty, underscoring the sensitivity of Chinese equities to both pandemic-related disruptions and structural policy risks. By contrast, the SP500 displays more moderate variance fluctuations, with values rising from near-zero levels in tranquil periods to peaks

around 0.006 during 2020, highlighting the surge in U.S. market uncertainty under systemic stress. Gold shows the lowest variance among the assets, with temporary increases in 2020 and again in 2022, consistent with its conventional role as a hedge during inflationary pressures and heightened risk aversion. The covariance structures also reveal significant time variation. For instance, the co-movement between SP500 and WTI intensified during 2020, indicating their joint exposure to systemic shocks, which reduced their diversification potential. Conversely, Gold's covariance with both equities and commodities weakened in stress periods, reflecting its safe-haven properties and its capacity to mitigate portfolio risk when traditional assets became more correlated. These dynamics, confirmed by the covariance structures, imply that investors should integrate gold as a strategic hedge and closely monitor oil spillovers, while policymakers need to strengthen macroprudential coordination to limit the amplification of systemic risks.

#### 4.1.2. Dynamic conditional betas

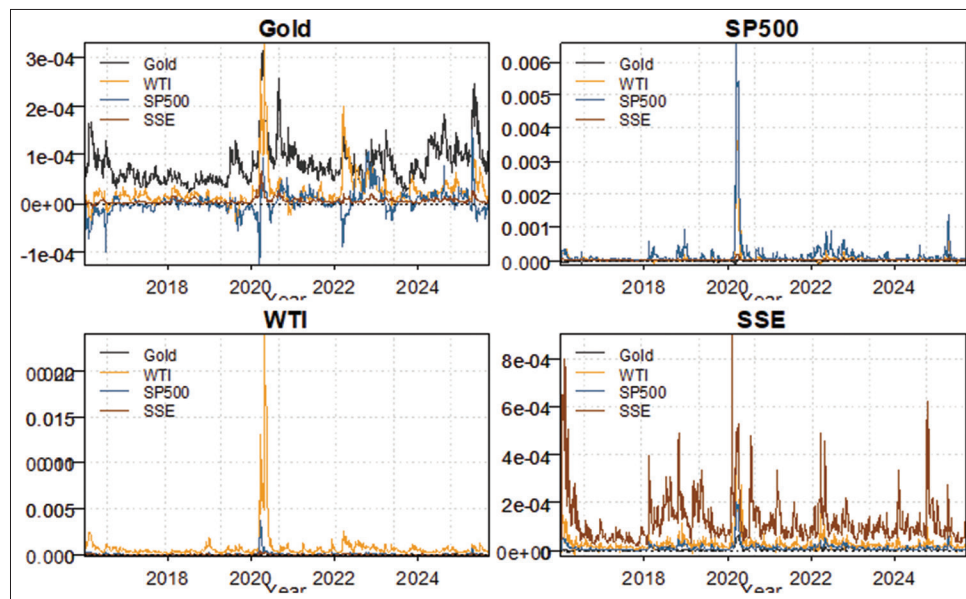
Figure 2 displays the dynamic conditional betas derived from the DCC-GARCH framework exhibit substantial time variation across Gold, WTI, the SP500, and the SSE, emphasizing the importance of capturing conditional dynamics rather than relying on static OLS estimates.

The results indicate that exposures among these assets are far from constant, with the most pronounced deviations occurring during episodes of systemic stress. For the SP500, conditional betas with Gold, WTI, and the SSE remain close to zero for most of the period but display notable spikes during crisis episodes, such as the COVID-19 outbreak in 2020 and the subsequent inflationary and monetary tightening shocks of 2022-2023. This highlights the vulnerability of U.S. equities to global disturbances, where cross-asset linkages intensify under stress. WTI demonstrates the highest volatility in conditional betas, particularly against the SP500 and Gold. These elevated and fluctuating values underscore oil's sensitivity to macro-financial turbulence, where global demand shocks, supply disruptions, and geopolitical risks sharply alter its co-movements with other assets. Gold, by contrast, shows relatively moderate conditional betas, often hovering near zero but rising intermittently during stress episodes. These dynamics reflect gold's dual role as both a safe-haven and a risk-sensitive asset, with its hedging capacity being episodic and dependent on prevailing macroeconomic and financial conditions. The SSE, meanwhile, exhibits largely muted conditional betas across the sample, pointing to a relatively weak transmission of Chinese market shocks to other assets. Nevertheless, occasional upward movements are observed during periods of heightened uncertainty, suggesting that Chinese equities can act as a marginal source of global spillovers in times of systemic stress.

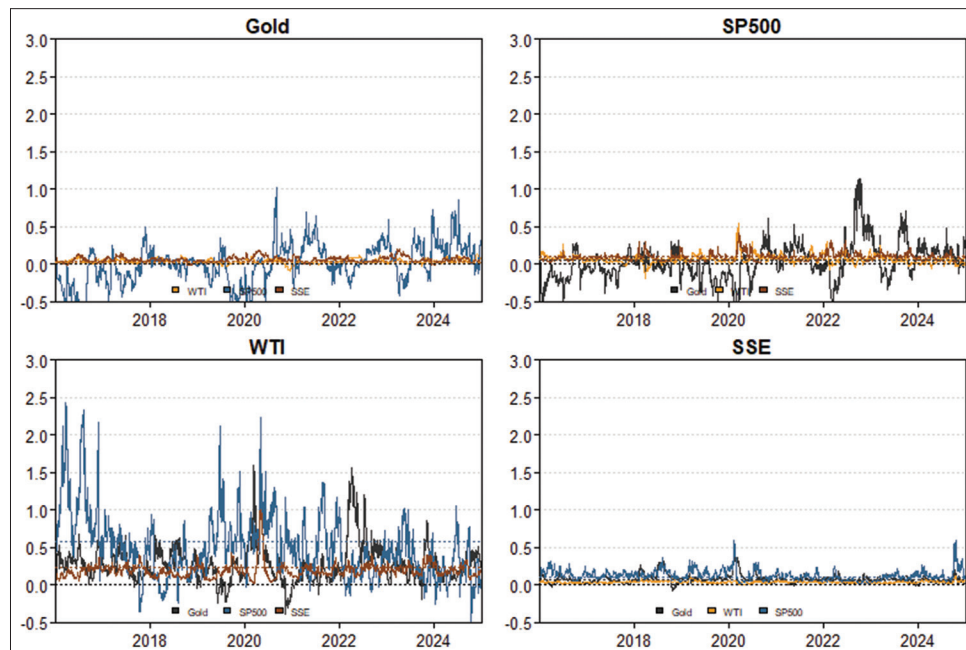
#### 4.1.3. Dynamic conditional correlations

Figure 3 displays the dynamic conditional correlations among Gold, WTI, the SP500, and the SSE, capturing how their co-movements evolve across time.

The results reveal important differences in diversification and hedging potential across assets. For the SP500, correlations with

**Figure 1:** Dynamic conditional variance-covariance

The dynamic conditional variances and covariances are estimated using the DCC-GARCH framework (Engle, 2002) combined with mixed univariate GARCH models (Antonakakis et al., 2021)

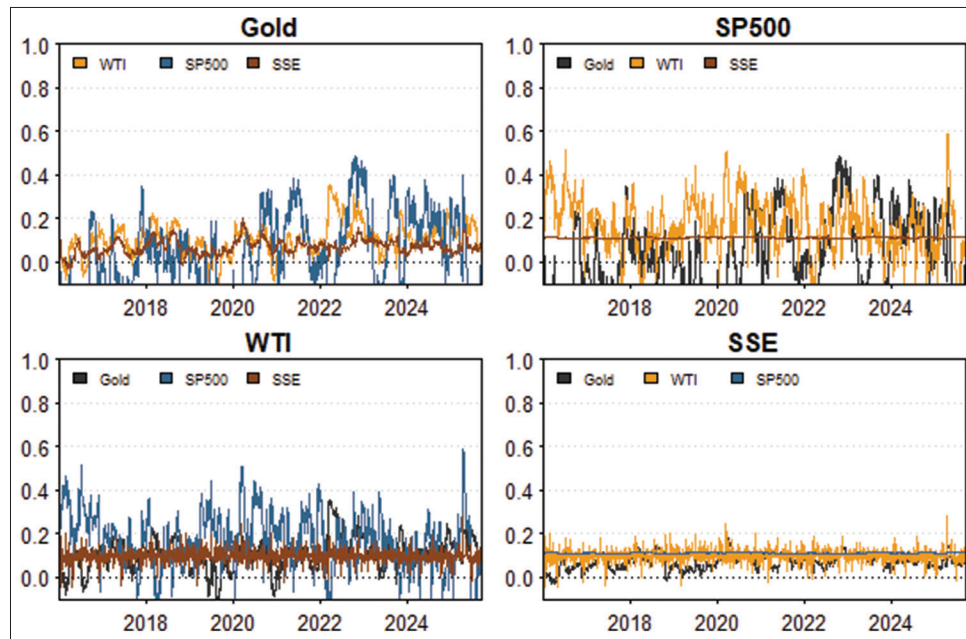
**Figure 2:** Dynamic conditional betas

The dynamic conditional betas are retrieved from the DCC-GARCH framework (Engle, 2002) with mixed univariate GARCH models (Antonakakis et al., 2021)

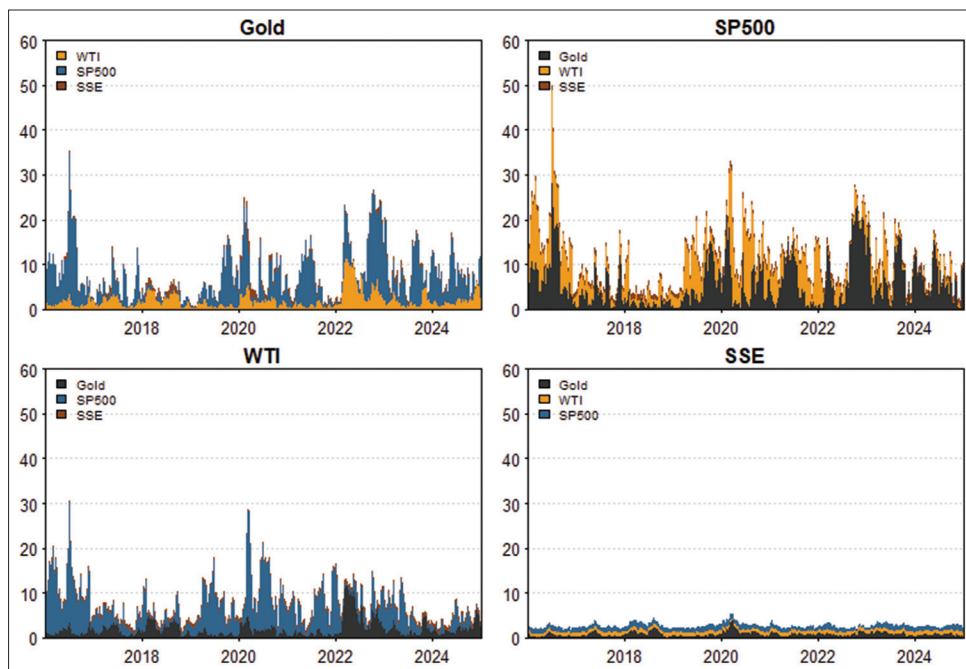
both WTI and Gold exhibit pronounced spikes during systemic stress, such as the COVID-19 shock in 2020 and the monetary tightening episodes of 2022-2023. These increases, often exceeding 0.4, indicate that U.S. equities became more synchronized with commodity and safe-haven assets during crises, thereby reducing the scope for diversification precisely when it is most needed. WTI shows the highest degree of correlation variability, with correlations against Gold and the SP500 fluctuating sharply during periods of heightened market uncertainty. These patterns highlight oil's sensitivity to macro-financial turbulence and underscore its

dual role as both a driver and recipient of global risk spillovers. Gold, by contrast, maintains correlations that are generally close to zero and occasionally negative, particularly during episodes of heightened stress. This reinforces its long-established role as a hedge and safe-haven asset, capable of decoupling from risk-on assets when systemic uncertainty intensifies. Finally, the SSE exhibits the lowest and most stable correlations across the sample, rarely exceeding 0.2. This suggests that Chinese equities provide modest but steady diversification benefits, with limited transmission of domestic shocks to global markets.



**Figure 3:** Dynamic conditional correlations

The dynamic conditional correlations are retrieved from the DCC-GARCH framework (Engle, 2002) with mixed univariate GARCH models (Antonakakis et al., 2021)

**Figure 4:** Dynamic conditional  $R^2$  decomposed measures

The dynamic conditional  $R^2$  decomposed goodness-of-fit measures are based on the DCC-GARCH framework (Engle, 2002) with mixed univariate GARCH models (Antonakakis et al., 2021) and the  $R^2$  decomposition approach of Genizi (1993)

#### 4.1.4. Dynamic conditional $R^2$ and $R^2$ decomposed measures

Figure 4 illustrates the dynamic conditional  $R^2$  decomposed measures derived from the DCC-GARCH framework, capturing the evolving explanatory power among the studied assets. The figure highlights substantial fluctuations in the  $R^2$  values across time, suggesting that the strength of the relationships between assets is highly time-dependent and sensitive to market conditions.

In summary, the SP500 and WTI exhibit the highest  $R^2$  values, indicating that their dynamics are more strongly explained by other market variables, particularly during turbulent periods such as the COVID-19 crisis (2020-2021) and the Russia–Ukraine conflict (2022). This suggests heightened co-movements and stronger interdependence when market uncertainty surges.



In contrast, Gold and SSE display lower and more stable  $R^2$  levels, reflecting their more idiosyncratic behavior and limited exposure to systemic market fluctuations. This stability underscores Gold's traditional role as a safe-haven asset, whose returns are less driven by the same factors affecting riskier markets.

Interestingly, spikes in  $R^2$  measures during crisis episodes indicate that periods of high uncertainty amplify the explanatory power of global risk factors, leading to synchronized movements across assets. Consequently, elevated  $R^2$  values can be interpreted as indicators of market stress, where diversification opportunities diminish and cross-market connectedness intensifies.

#### 4.2. $R^2$ Decomposed Connectedness Approach

We first discuss the average connectedness measures, followed by an analysis of the dynamic connectedness plots, which offer a more detailed perspective on the time-varying behavior that may be obscured by the averaged values. The connectedness table is then presented in Figure 5.

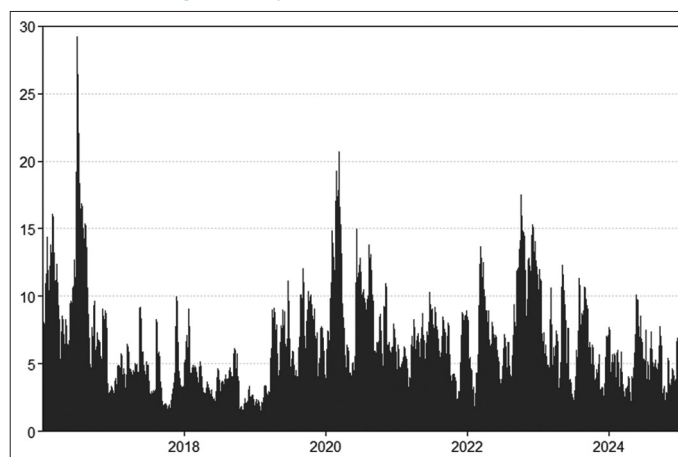
Table 3 displays the averaged connectedness measures for studied variables. Cross-market transmission remains moderate, as the total connectedness index (TCI) is about 24.41%, meaning roughly one quarter of each market's forecast error variance is explained by shocks from other markets. The SP 500 emerges as the principal transmitter, sending 9.47% of shocks to others and receiving 9.28%, which yields the highest positive net connectedness of about 0.20. Gold and WTI each transmit around 6% of shocks although receiving slightly more, about 6.4%, resulting in small negative net spillovers of (−0.07). The SSE is the least influential market, transmitting only 2.39% and receiving 2.44%, with a nearly neutral net effect of (−0.05). "Inc.Own" values show that while own variance shares are included, each market's total variance share slightly exceeds 100%, consistent with an overall connectedness of roughly 8.14% compared with the baseline TCI of 6.10%. Net pairwise transmission counts further designate that the SP 500 acts as a net transmitter to three markets, WTI to one, gold to two, and the SSE to none. Overall, the evidence highlights the SP 500 as the primary source of return spillovers, with WTI and gold playing secondary roles as mild net receivers and the SSE remaining largely insulated from external shocks.

##### 4.2.1. Total connectedness index

Figure 5 illustrates the evolution of dynamic total connectedness (TCI), which captures the overall intensity of spillovers and interdependence among the studied assets over time. The fluctuations in TCI reflect the degree of market-wide risk transmission and systemic connectedness across periods of calm and turmoil.

Several notable peaks can be observed, corresponding to periods of heightened market uncertainty. The first major spike appears around 2018, likely linked to global financial tensions and energy market adjustments. A sharp rise is again evident in early 2020, coinciding with the outbreak of the COVID-19 pandemic, when uncertainty surged across global financial and commodity markets.

Figure 5: Dynamic total connectedness



Dynamic total connectedness measures rest on the DCC-GARCH framework (Engle, 2002) with mixed univariate GARCH models (Antonakakis et al., 2021)

Table 3: Averaged connectedness measures

Asset	Gold	WTI	SP500	SSE	FROM
Gold	100.00	1.48	4.22	0.55	6.24
WTI	1.48	100.00	4.18	0.79	6.45
SP500	4.13	4.10	100.00	1.05	9.28
SSE	0.56	0.80	1.08	100.00	2.44
TO	6.17	6.38	9.47	2.39	24.41
Inc.Own	106.17	106.38	109.47	102.39	cTCI/TCI
NET	−0.07	−0.07	0.20	−0.05	8.14/6.10
NPT	2.00	1.00	3.00	0.00	

Averaged  $R^2$  decomposed connectedness measures are based on a DCC-GARCH (Engle, 2002) with mixed univariate GARCH models (Antonakakis et al., 2021). Inc. Own (Including Own), represents the sum of the contribution FROM (shocks received from other assets) plus the asset's own shocks. In other words, it reflects the share of forecast error variance explained by both the asset itself and external sources. NET is the difference between the contribution TO (shocks transmitted to others) and FROM (shocks received from others). NPT measures the net bilateral shock transmission between assets, capturing the dominant direction of pairwise connectedness

Another visible increase emerges around 2022, corresponding to the Russo-Ukrainian conflict, which significantly disrupted energy markets and heightened co-movements among assets. Although the 2022 surge is less pronounced than the 2020 peak, it follows a period of relatively low connectedness observed in late 2021, suggesting that the markets had temporarily stabilized before the geopolitical shock. This pattern indicates that while systemic risk remains sensitive to major global events, the overall connectedness trend appears to moderate over time potentially reflecting stronger market resilience, improved diversification strategies, and greater integration of sustainability-focused investments such as ESG-linked assets.

##### 4.2.2. Net total directional connectedness

Figure 6 illustrates the net total directional connectedness among Gold, WTI, SP500, and SSE, highlighting which assets act as net transmitters or receivers of shocks over time. Overall, the SP500 and WTI emerge as dominant net transmitters, indicating their leading roles in propagating market shocks to other assets. In contrast, Gold and the SSE generally behave as net receivers, absorbing volatility rather than generating it.

Gold's persistent negative net connectedness underscores its defensive and safe-haven nature, especially during market turmoil such as the COVID-19 pandemic (2020) and the Russia–Ukraine conflict (2022), when it absorbed spillovers from equity and energy markets. Conversely, the SP500's positive net connectedness during these crises reflects its strong influence on market-wide volatility, consistent with its status as a global risk barometer. WTI's similar transmitting behavior suggests that energy price shocks are key drivers of cross-market risk transmission. These findings collectively emphasize the asymmetric shock transmission structure, where risk flows predominantly from equity and energy markets toward safe-haven assets like Gold, confirming its enduring role as a stabilizing asset within diversified portfolios.

### 4.3. Portfolio Risk Management

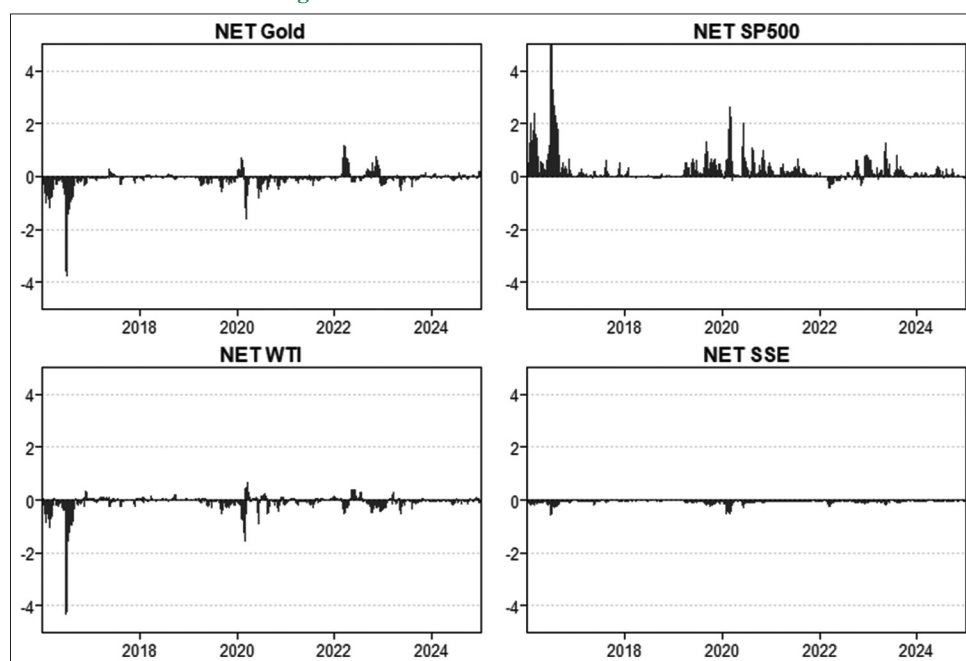
This section examines the performance of bivariate and multivariate hedging and minimum-risk portfolios. The analysis evaluates portfolio efficiency through hedging effectiveness (Ederington,

1979), Sharpe ratios (Sharpe, 1994), and the information ratio relative to the GRE market portfolio. It begins with a review of individual asset performance, annualized returns, volatility, and risk-adjusted metrics, summarized in Table 4.

#### 4.3.1. Hedging portfolios

Table 4 and Table 5 presents bivariate and multivariate hedge ratios respectively. The hedge ratio analysis, crossways all asset combinations, reveals notable differences in hedging effectiveness and risk-adjusted performance. When gold is the hedged asset, hedging with WTI provides the most favorable outcome, with a modest mean hedge ratio of 0.039, low hedging effectiveness (HE = 0.021), nevertheless the highest Sharpe ratio (SR = 0.776), indicating a meaningful improvement in risk-adjusted returns despite limited risk reduction. Hedging gold with the SP 500 or SSE yields slightly lower SR values (0.723 and 0.714, respectively) and minimal HE, suggesting that WTI offers a comparatively stronger stabilizing effect for gold portfolios. For WTI, the combination

Figure 6: Net total directional connectedness



Net total directional connectedness measures rest on the DCC-GARCH framework (Engle, 2002) with mixed univariate GARCH models (Antonakakis et al., 2021)

Table 4: Bivariate hedging portfolios

Asset Pair	Mean	Standard deviation	5%	95%	HE	P-value	Return	Standard deviation	SR
Gold/WTI	0.039	0.031	-0.014	0.094	0.021	0.659	0.001	0.001	0.776
Gold/SP500	0.023	0.252	-0.406	0.416	0.034	0.398	0.001	0.001	0.723
Gold/SSE	0.069	0.038	0.012	0.140	0.006	0.000	0.001	0.001	0.714
WTI/Gold	0.283	0.271	-0.089	0.747	0.017	0.889	0.001	0.005	0.113
WTI/SP500	0.511	0.414	-0.063	1.302	0.057	0.398	0.000	0.005	-0.019
WTI/SSE	0.259	0.139	0.121	0.408	0.018	0.000	0.001	0.005	0.159
SP500/Gold	0.049	0.252	-0.314	0.474	-0.013	0.889	0.001	0.002	0.585
SP500/WTI	0.071	0.062	-0.007	0.179	0.066	0.659	0.001	0.002	0.791
SP500/SSE	0.110	0.060	0.045	0.227	0.034	0.000	0.001	0.002	0.725
SSE/Gold	0.085	0.058	0.022	0.203	0.008	0.889	0.000	0.002	-0.042
SSE/WTI	0.047	0.019	0.025	0.084	0.017	0.659	0.000	0.002	0.020
SSE/SP500	0.142	0.074	0.053	0.289	0.013	0.398	0.000	0.002	-0.071

**Table 5: Multivariate hedging portfolios**

Asset Pair	Mean	Standard deviation	5%	95%	HE	P-value	Return	Risk	SR
Gold/WTI	0.037	0.028	-0.001	0.086	0.053	0.186	0.001	0.001	0.823
Gold/SP500	-0.001	0.259	-0.446	0.387					
Gold/SSE	0.058	0.036	0.009	0.125					
WTI/Gold	0.278	0.253	-0.007	0.705	0.077	0.054	0.000	0.005	-0.037
WTI/SP500	0.495	0.429	-0.070	1.285					
WTI/SSE	0.190	0.097	0.074	0.327					
SP500/Gold	0.024	0.251	-0.335	0.445	0.050	0.213	0.001	0.002	0.787
SP500/WTI	0.065	0.061	-0.010	0.167					
SP500/SSE	0.089	0.050	0.035	0.186					
SSE/Gold	0.076	0.060	0.013	0.179	0.029	0.477	0.000	0.002	-0.109
SSE/WTI	0.036	0.016	0.017	0.064					
SSE/SP500	0.128	0.075	0.038	0.254					

with the SP 500 shows the highest HE (0.057) amongst its pairs, though the SR is negative (-0.019), implying that while risk is somewhat mitigated, overall risk-adjusted returns deteriorate. Hedging WTI with SSE offers slightly positive SR (0.159) and low HE (0.018), offering a safer but moderate hedging benefit. Hedging WTI with gold produces low HE and poor SR, indicating limited effectiveness. Using the SP 500 as the hedged asset, hedging with WTI stands out as the most effective strategy, delivering both the highest HE (0.066) and the highest SR (0.791) across all asset pairs, highlighting its dual benefit of risk reduction and strong risk-adjusted performance. Hedging the SP 500 with SSE or gold provides moderate SR values (0.725 and 0.585, respectively) yet lower HE, suggesting limited risk mitigation.

Lastly, for the SSE, all hedging strategies offer limited benefit. Hedging with gold, WTI, or the SP 500 results in low HE values (0.008-0.017) and mostly negative or marginally positive SR, reflecting minimal improvements in portfolio performance. Generally, the results indicate that SP500/WTI is the most efficient hedging combination, offering substantial risk-adjusted benefits, gold benefits moderately from WTI hedging, and SSE remains largely insensitive to cross-asset hedging, providing minimal diversification advantages. Worth noting that Gold is not an effective hedge for SP500.

The comparison between bivariate and multivariate hedging strategies reveals that multivariate portfolios generally enhance hedging performance across most assets. For gold, the bivariate hedges show modest HE, ranging from 0.006 (Gold/SSE) to 0.034 (Gold/SP500), with Sharpe ratios between 0.714 and 0.776, the highest being for Gold/WTI (0.776). When gold is hedged using all three other assets simultaneously, the multivariate hedge achieves a higher HE of 0.053 and an improved SR of 0.823. This demonstrates that combining WTI, SP500, and SSE provides a stronger risk reduction and better risk-adjusted performance than any single bivariate combination.

For WTI, bivariate HE ranges from 0.017 (WTI/Gold) to 0.057 (WTI/SP500), while SR varies from -0.019 (WTI/SP500) to 0.159 (WTI/SSE). In the multivariate portfolio, although HE improves to 0.077, the SR becomes slightly negative at -0.037. This indicates that, despite a more effective reduction in risk, the multivariate hedge reduces expected returns relative to risk, highlighting a trade-off between risk mitigation and risk-adjusted performance for WTI.

When SP500 is hedged, the bivariate HE ranges from -0.013 (SP500/Gold) to 0.066 (SP500/WTI), and SR spans 0.585 to 0.791, with SP500/WTI offering both the highest HE and SR among bivariate options. The multivariate hedge preserves or slightly enhances HE and maintains a high SR of 0.787, showing that combining WTI, gold, and SSE provides a robust hedge while sustaining strong risk-adjusted performance. For SSE, bivariate HE is low across all pairs (0.008-0.029), and SR is negative or close to zero (-0.109-0.020), indicating weak risk reduction. In the multivariate hedge, HE improves modestly, but SR remains negative at -0.109, reflecting that while risk is partially reduced, the combination of hedging assets does not compensate enough to achieve positive risk-adjusted returns. This suggests that SSE is difficult to hedge effectively with the selected assets and may require alternative strategies for meaningful risk-adjusted gains.

In summary, Gold and SP500 benefit from both higher HE and improved SR, whereas WTI and SSE, despite higher HE, experience negative SR in the multivariate portfolio, indicating a trade-off between risk reduction and risk-adjusted returns for these markets.

#### 4.3.2. Minimum-risk portfolios

We now turn to an examination of both bivariate and multivariate portfolio strategies designed to minimize risk while indirectly enhancing the Sharpe ratio, which measures the highest return achievable for a given level of investment risk. Our analysis begins with the empirical results of Kroner and Ng (1998), who report optimal bivariate portfolio weights, as summarized in Table 6.

The results show substantial variation in optimal weights across asset pairs. For instance, the Gold/WTI portfolio shows a relatively low hedging effectiveness (HE = 0.098) but maintains a Sharpe ratio of 0.653, suggesting moderate risk reduction with decent risk-adjusted performance. In contrast, Gold/SP500 and Gold/SSE provide higher HE values of 0.454 and 0.421, respectively, with Sharpe ratios of 1.023 and 0.504, indicating that the inclusion of particularly SP500 improves risk-adjusted returns for gold. Since the best SR is for Gold/SP500, this combination appears to be the most efficient in terms of risk-adjusted performance.

Interestingly, the WTI-hedged portfolios reveal that high HE does not always translate into improved Sharpe ratios. The WTI/Gold portfolio achieves the highest HE (0.929) but a moderate

**Table 6: Optimal bivariate portfolio weights**

Asset Pair	Mean	Standard deviation	5%	95%	HE	P-value	SR
Gold/WTI	0.881	0.075	0.733	0.987	0.098	0.013	0.653
Gold/SP500	0.491	0.217	0.171	0.879	0.454	0.000	1.023
Gold/SSE	0.547	0.159	0.324	0.847	0.421	0.000	0.504
WTI/Gold	0.119	0.075	0.013	0.267	0.929	0.000	0.653
WTI/SP500	0.109	0.119	0.000	0.360	0.867	0.000	0.209
WTI/SSE	0.158	0.116	0.036	0.400	0.900	0.000	-0.154
SP500/Gold	0.509	0.217	0.121	0.829	0.666	0.000	1.023
SP500/WTI	0.891	0.119	0.640	1.000	-0.035	0.407	0.209
SP500/SSE	0.561	0.226	0.166	0.895	0.593	0.000	0.126
SSE/Gold	0.453	0.159	0.153	0.676	0.612	0.000	0.504
SSE/WTI	0.842	0.116	0.600	0.964	0.148	0.000	-0.154
SSE/SP500	0.439	0.226	0.105	0.834	0.555	0.000	0.126

SR of 0.653. In contrast, the WTI/SSE portfolio, despite an HE of 0.900, results in a negative SR of -0.154. Similarly, the WTI/SP500 portfolio has an HE of 0.867 but a SR of 0.209, reflecting that effective risk reduction may come at the cost of risk-adjusted performance. These findings underscore the importance of not solely relying on hedging effectiveness as a measure of portfolio quality. A high HE indicates effective risk reduction, but it does not guarantee improved risk-adjusted returns. Conversely, a negative Sharpe ratio suggests that the portfolio's returns do not adequately compensate for the risk taken, highlighting the need for a balanced approach in portfolio construction that considers both risk reduction and return enhancement.

Portfolios involving the SP500 generally produce moderate to high HE, ranging from -0.035 (SP500/WTI) to 0.666 (SP500/Gold), with SR values between 0.126 and 1.023. Gold again consistently contributes to higher Sharpe ratios, whereas WTI and SSE combinations, despite high HE, often fail to achieve strong risk-adjusted returns.

Finally, SSE portfolios show mixed results. SSE/Gold and SSE/SP500 deliver moderate HE (0.612 and 0.555) with positive SR (0.504 and 0.126), while SSE/WTI has relatively lower HE (0.148) and a negative SR (-0.154), indicating limited improvement in risk-adjusted performance.

Overall, these results suggest that while bivariate hedging can reduce risk, the impact on risk-adjusted returns depends heavily on the chosen combination. Gold-based portfolios generally provide the most favorable balance between risk reduction and Sharpe ratio, whereas WTI and SSE hedges may lower risk but not always enhance risk-adjusted performance.

Next, we pay particular attention to the analyze the multivariate approach for the five distinct portfolios (Table 7). Primary, for the Minimum Variance Portfolio (MVP), the asset weights are 0.347 for Gold, 0.018 for WTI, 0.379 for SP500, and 0.256 for SSE. This portfolio places the largest emphasis on SP500 and Gold, with a substantial allocation to SSE, while limiting exposure to WTI to control risk. The HE values; 0.597 for Gold, 0.968 for WTI, 0.754 for SP500, and 0.730 for SSE, indicate varying levels of connectedness, where WTI's high HE suggests it still plays an important role in diversification despite its low weight. The overall Sharpe Ratio for this portfolio is 0.624, reflecting a

**Table 7: Multivariate portfolio analysis**

Portfolio / Asset	Mean	Standard deviation	5%	95%	HE	P-value	SR
Minimum variance portfolio (MVP)							
Gold	0.347	0.148	0.131	0.628	0.597	0.000	0.624
WTI	0.018	0.024	0.000	0.070	0.968	0.000	
SP500	0.379	0.202	0.063	0.687	0.754	0.000	
SSE	0.256	0.142	0.050	0.519	0.730	0.000	
Minimum correlation portfolio (MCP)							
Gold	0.276	0.038	0.223	0.344	-0.119	0.007	0.510
WTI	0.224	0.040	0.152	0.277	0.912	0.000	
SP500	0.252	0.038	0.194	0.319	0.315	0.000	
SSE	0.248	0.035	0.184	0.299	0.251	0.000	
Minimum connectedness portfolio (MPP)							
Gold	0.250	0.014	0.226	0.272	-0.272	0.000	0.440
WTI	0.249	0.012	0.229	0.272	0.900	0.000	
SP500	0.235	0.017	0.205	0.257	0.222	0.000	
SSE	0.266	0.014	0.248	0.291	0.149	0.000	
Minimum bivariate R2 portfolio (MRP)							
Gold	0.251	0.015	0.226	0.275	-0.267	0.000	0.439
WTI	0.249	0.013	0.227	0.272	0.900	0.000	0.439
SP500	0.234	0.018	0.203	0.258	0.225	0.000	0.439
SSE	0.266	0.014	0.248	0.294	0.152	0.000	0.439
Minimum R2 decomposed connectedness portfolio (M2P)							
Gold	0.250	0.008	0.236	0.262	-0.287	0.000	0.449
WTI	0.249	0.007	0.237	0.262	0.899	0.000	0.449
SP500	0.242	0.009	0.227	0.254	0.212	0.000	0.449
SSE	0.259	0.008	0.249	0.274	0.138	0.000	0.449

significant risk-adjusted return mainly driven by the concentration in equities and gold.

The Minimum Correlation Portfolio (MCP) spreads weights more evenly across assets, with 0.276 in Gold, 0.224 in WTI, 0.252 in SP500, and 0.248 in SSE. This balanced allocation seeks to minimize correlations among assets to improve diversification. The HE values vary: Gold has a slightly negative HE of -0.119, indicating less integration, while WTI's high HE of 0.912 points to a central diversification role. SP500 and SSE have positive HE values of 0.315 and 0.251 respectively. The portfolio's Sharpe Ratio is 0.510, showing a modest but meaningful risk-adjusted return from this diversification approach.

For the Minimum Connectedness Portfolio (MPP), the asset weights are fairly balanced, with Gold at 0.250, WTI at 0.249, SP500 at 0.235, and SSE at 0.266. This portfolio focuses on minimizing systemic risk by reducing connectedness among



assets within the network. The HE values reflect this risk reduction approach: WTI's high positive HE of 0.900 indicates a strong contribution to lowering portfolio risk through connectedness reduction, while Gold's negative HE of  $-0.272$  suggests it contributes less to this risk mitigation. SP500 and SSE show moderate positive HE values of 0.222 and 0.149, respectively, indicating some role in risk reduction. The overall Sharpe Ratio of 0.440 is lower than those of the MVP and MCP, signaling more modest risk-adjusted returns but reflecting a more effective control of risk by strategically managing asset connectedness.

In the Minimum Bivariate  $R^2$  Portfolio (MRP), the asset weights closely mirror those of the Minimum Connectedness Portfolio, with Gold at 0.251, WTI at 0.249, SP500 at 0.234, and SSE at 0.266. The HE values show a consistent pattern where Gold has a negative HE of  $(-0.267)$ , indicating it contributes less to reducing overall portfolio risk, while WTI has a high positive HE of 0.900, reflecting its strong role in risk mitigation. SP500 and SSE have moderate positive HE values, supporting some degree of risk reduction. The Sharpe Ratio for this portfolio is 0.439, closely matching the MPP, which suggests similar effectiveness in balancing return and risk.

The Minimum  $R^2$  Decomposed Connectedness Portfolio (M2P) allocates weights of 0.250 to Gold, 0.249 to WTI, 0.242 to SP500, and 0.259 to SSE. The HE values continue the trend of Gold having a negative value at  $-0.287$ , WTI maintaining a high positive HE of 0.899, while SP500 and SSE have moderate HE values of 0.212 and 0.138 respectively. These HE values imply that WTI remains the primary asset contributing to risk reduction, whereas Gold's negative HE indicates a lesser role in mitigating portfolio risk. SP500 and SSE provide moderate benefits in risk control through their HE values. The Sharpe Ratio of 0.449 is slightly higher than that of MRP and MPP, indicating a modest improvement in risk-adjusted returns while maintaining a balanced portfolio.

In summary, the Minimum Variance Portfolio (MVP) concentrates on SP500, Gold, and SSE with relatively higher risk-adjusted returns, as seen in its highest Sharpe Ratio of 0.624. The other portfolios distribute weights more evenly to reduce correlations among assets and enhance diversification, resulting in better risk control but generally lower Sharpe Ratios ranging from 0.439 to 0.510. These findings illustrate the trade-off between a concentrated portfolio targeting maximum efficiency and more balanced portfolios focused on reducing risk through diversified asset allocation and interaction metrics.

#### 4.3.3. Portfolio performance

The portfolio performance presented in Table 8 examines the performance of the constructed portfolios. The MVP

has the lowest return at 0.0005519 but also the lowest standard deviation at 0.0008841, reflecting the portfolio's emphasis on risk minimization. Its Sharpe Ratio calculated by standard deviation is the highest at 0.6242, indicating the best risk-adjusted return among the portfolios when volatility is the measure of risk. However, MVP has not the highest Sharpe Ratios when assessed with Value at Risk (VaR) and have the lowest one when calculated by Conditional VaR (CVaR), scoring 5.9791 and 3.2311 respectively, which suggests it performs relatively less favorably when risk is evaluated with tail risk measures.

The MCP shows a higher return of 0.0007516 but also a substantially higher standard deviation of 0.0014739, indicating greater volatility than MVP and lower than the others. Its Sharpe Ratio based on standard deviation is 0.5099, lower than MVP but still respectable. The MCP has the highest Sharpe Ratios for both VaR and CVaR risk assessments, at 6.9254 each, highlighting its strong performance when accounting for downside risk and tail events.

MPC and MRP portfolios have similar returns (0.0006909 and 0.0006889) and standard deviations (around 0.00157), with Sharpe Ratios based on standard deviation around 0.44. Both exhibit slightly lower performance on VaR and CVaR Sharpe Ratios compared to MCP but higher than MVP, indicating moderate effectiveness in risk adjustment across different risk frameworks.

The M2P offers a return of 0.0007103 and the highest standard deviation at 0.0015807, signaling the greatest volatility among the portfolios. Its Sharpe Ratio based on standard deviation is 0.4494, slightly better than MPC and MGP but lower than MCP and MVP. The VaR and CVaR Sharpe Ratios of 5.5132 suggest moderate performance in controlling tail risk, better than MVP but not as strong as MCP.

In conclusion, the MVP emphasizes low volatility and achieves the best risk-adjusted return measured by standard deviation but underperforms with tail risk metrics. MCP provides higher returns with increased volatility but excels in managing downside risks shown by its superior VaR and CVaR Sharpe Ratios. MPC, MGP, and MRP portfolios demonstrate modest returns and risk profiles, balancing between volatility and tail risk performance. These results suggest that investors should adapt their portfolio choices according to their priorities: favoring the MVP for short-term stability, the MCP for stronger protection against extreme losses, or intermediate portfolios for a balance between return and risk management. For policymakers and regulators, these findings highlight the importance of promoting dynamic portfolio management approaches that explicitly incorporate tail

**Table 8: Portfolio performance**

Performance Measures	MVP	MCP	MPC	MRP	M2P
Return	0.0005519	0.0007516	0.0006909	0.0006889	0.0007103
Standard deviation	0.0008841	0.0014739	0.0015713	0.0015684	0.0015807
Sharpe Ratio (Standard deviation)	0.6242339	0.5098996	0.4396996	0.4392650	0.4493604
Sharpe Ratio (VaR)	5.9791028	6.9254220	5.3911580	5.3915879	5.5131796
Sharpe Ratio (CVaR)	3.2311361	6.9254220	5.3911580	5.3915879	5.5131796

risks in order to strengthen the resilience of financial markets to systemic shocks.

## 5. CONCLUSION AND POLICY RECOMMENDATIONS

This study delves into the evolving interconnections among gold, WTI crude oil, the S&P 500, and the Shanghai Stock Exchange Composite Index from 2019 to 2025, using an advanced econometric framework that combines multivariate regression, the DCC-GARCH model, dynamic conditional  $R^2$  metrics, and decomposed connectedness indices. The statistically significant findings evince that these relationships fluctuate over time and strengthen during systemic stress periods, such as the COVID-19 pandemic and the Russia–Ukraine conflict. Consistent with prior research (Wang et al., 2022a; Ghorbel et al., 2022; Pandey et al., 2023; Yousaf et al., 2023; Filippidis et al., 2023; Zhang and Wu, 2024), the results provide evidence that gold remains the least interconnected asset, reinforcing its longstanding role as a safe haven. Conversely, WTI crude oil exhibits high volatility and significant contagion effects. The S&P 500 is identified as the main channel for shock transmission, underscoring its key influence on global financial instability, while the SSE appears more insulated, offering moderate diversification benefits. By incorporating connectedness indicators into portfolio construction, the analysis demonstrates that adaptive strategies, which adjust to changing asset interdependencies, outperform traditional approaches based on variance-covariance or correlation matrices, particularly in terms of hedging effectiveness and risk-adjusted returns during crises. These findings suggest that diversification benefits are conditional and require flexible models that explicitly incorporate systemic linkages. Furthermore, empirical evidence confirms that dynamic connectedness-based methods evince greater resilience compared to static diversification frameworks (Broadstock et al., 2022; Gupta et al., 2023; Frikha et al., 2023; Belkhir et al., 2025).

These conclusions carry important implications for both policymakers and investors. For regulators, rising connectedness during crises calls for coordinated macroprudential policies and targeted interventions to stabilize energy markets. For investors, the study highlights the limitations of fixed diversification techniques and advocates strategic inclusion of gold as a hedging asset while closely monitoring contagion effects stemming from oil markets. Overall, applying connectedness analysis to regulatory frameworks and investment strategies serves as a valuable tool to enhance financial stability, effectively bridging theory and practice.

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