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Forecasting Gasoline Market Volatility using Non-Linear Time Series Models

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ABSTRACT

This study forecasts the dynamics of gasoline price returns using daily data from January 2, 1992, to June 6, 2022, and crude oil price returns as a regressor. The non-linear dependence in the volatility of the gasoline return is confirmed and the Markov Switching (MS), the autoregressive conditional heteroskedasticity (ARCH) and the generalized autoregressive conditional heteroskedasticity (GARCH) models are estimated. To account for the linear dependence found in the initial estimates, a GARCH (1,1) model with lagged gasoline returns is used, while a GARCH (1,1) is fitted on the Markov switching residual to capture both the volatility in the conditional mean and variance. The forecasting performance of the estimated models is evaluated, and the GARCH (1,1) on the Markov Switching residual is found to be the best model to forecast the average gasoline returns, while the GARCH (1,1) with linear dependence is preferable for forecasting the volatility of gasoline returns. Identifying the best time series model is crucial for the market participants, and especially for oil companies to evaluate the market situation.

Keywords: Energy Markets, Volatility, Generalized Autoregressive Conditional Heteroskedasticity, Markov Switching
JEL Classifications: C22, C52, C53

1. INTRODUCTION

Linear time series models, such as autoregressive models (AR), moving average models (MA), autoregressive integrated moving average models (ARIMA) and vector autoregressive models (VAR), are widely used to capture the volatility dynamics of economic variables. However, these models do not fit well to data exhibiting non-linear, complex dynamic patterns, such as energy prices. One of the most popular non-linear time series models used to investigate the volatility of energy prices is the Markov switching model, proposed by Hamilton (1989). This model involves multiple equations that describe the time series behaviour in different regimes, while also allowing switching between these equations. In addition, the Markov switching (MS) model admits frequent changes at different time points, rather than exogenous or occasional changes. However, the Markov Switching model captures only the volatility in the conditional mean and not the volatility in the conditional variance. For this reason, several studies for energy commodities (e.g. gasoline, crude oil, natural gas, heating oil and gasoil) use the autoregressive conditional heteroskedasticity (ARCH) and the generalized autoregressive conditional heteroskedasticity (GARCH) models, introduced by Engle (1982) and Bollerslev (1986) respectively, to capture the volatility in conditional variance; see, for instance, the studies by Sadorsky (2006), Cheong (2009), Nomikos and Pouliasis (2011), Wang and Wu (2012), Lv and Shan (2013), Chkili et al. (2014), Lin et al. (2020), Ambya et al. (2020), Hendrawaty et al. (2021), He (2023), Osho and Oloyede (2024), Chung (2024), Roshanpour et al. (2025).

However, it should be noted that the complexity of the model specification does not guarantee high out-of-sample forecasting performance. For instance, the study by Sadorsky (2006) forecasts daily volatility in petroleum futures price returns, using univariate and multivariate models, and daily data for crude oil, heating oil, and unleaded gasoline from February 5, 1988, to January 31, 2003, and for natural gas from April 3, 1990, to January 31, 2003. The study finds that the single-equation GARCH model performs better than the state space, vector autoregressive, and bivariate GARCH models, especially in the case of unleaded gasoline and crude oil, for which the GARCH (1,1) model fits better. Similarly, Cheong (2009) investigates the time-varying volatility of the West Texas Intermediate (WTI) and Europe Brent crude oil price, using daily data from January 4, 1993, to December 31, 2008, and ARCH/GARCH-class models. The study finds that although the fractionally integrated asymmetric power ARCH model has better in-sample performance for both WTI and Brent, the simplest parsimonious GARCH model appears to have better forecasting performance for Brent crude oil. These results show that models with high complexity do not always outperform single equation GARCH models in forecasting performance.

This has led recent studies, such as Ambya et al. (2020), Hendrawaty et al. (2021) and Osho and Oloyede (2024), to use low-complexity GARCH models. Specifically, Ambya et al. (2020) use daily data from 2009 to 2019 and AR(1)-GARCH (1,1) models to forecast future natural gas prices. Hendrawaty et al. (2021) forecast crude oil prices using daily data from December 2019 to May 2020 with an AR(1)-GARCH (1,1) model. Moreover, Osho and Oloyede (2024) assess the effectiveness of the GARCH (1,1) and GARCH (1,2) models in forecasting crude oil price volatility, finding that the GARCH (1,1) model is preferred due to its forecasting accuracy and low complexity, confirming the results of earlier studies.

As the out-of-sample performance is crucial for energy market participants, this paper is intended to answer the research question: What is the best model for forecasting the dynamics of gasoline price returns?

To answer the research question, this study uses Markov switching, ARCH and GARCH models. However, as the volatility in the conditional mean and variance is not captured by separately applying the MS, ARCH or GARCH models, this study also fits a GARCH on the Markov switching residual to combine both conditional mean and variance. Moreover, except from the fact that regimes and non-linear dependence are present in the energy price returns, crude oil price volatility also plays an important role in forecasting refined product price volatility (Wang and Wu, 2012; Nomikos and Pouliasis, 2011). For this reason, this paper includes the crude oil returns as a regressor in the MS and ARCH/GARCH estimates to model the gasoline returns. In addition, to account for the linear dependence found in the initial estimates, a GARCH (1,1) model with lagged gasoline returns is used. Also, daily data are used in the analysis, focusing on the short-term dynamics, as they are important for oil companies to evaluate the market situation (Wang and Wu, 2012). Finally, to evaluate the forecasting performance of the estimated models, the root mean squared error (RMSE), mean absolute error (MAE), symmetric mean absolute percentage error (SMAPE) and Theil U1 criteria are used.

This study compares the forecasting performance of the selected models and the GARCH (1,1) on the Markov switching residual is found to be the best model to forecast the average gasoline returns (volatility of the conditional mean), while the GARCH (1,1) with linear dependence is preferable for forecasting the volatility of gasoline returns (volatility in the conditional variance). Section 2 presents the data and methods, Section 3 presents the empirical analysis, while Sections 4 and 5 present the forecasting performance evaluation and conclusion respectively.

2. DATA AND METHODS

2.1. Data

The data consists of daily (five days a week) spot price data of conventional gasoline (New York Harbor, US$ per gallon) and crude oil (West Texas Intermediate-Cushing Oklahoma, US$ per barrel) obtained from the Energy Information Administration (EIA). The data covers the time period from January 2, 1992, to June 6, 2022. To forecast the volatility of gasoline price, the gasoline and crude oil simple returns are calculated:

$$GAS_t = (P_{GAS\,t} - P_{GAS\,t-1})/P_{GAS\,t-1} \quad (1)$$

$$WTI_t = (P_{WTI\,t} - P_{WTI\,t-1})/P_{WTI\,t-1}$$

Where GAS_t and WTI_t are the gasoline and crude oil returns, $P_{GAS\,t}$ and $P_{WTI\,t}$ are the prices of the assets at period t, while $P_{GAS\,t-1}$ and $P_{WTI\,t-1}$ are the prices of the assets in t-1.

Figures 1 and 2 present the plots of the gasoline and crude oil daily prices and returns. As it can be seen from Figure 1, on July 2, 2008, the crude oil price peaked at 143.71 US$/barrel, followed by a sharp decrease to 30.28 US$/barrel on December 23, 2008. The crude oil price gradually increased to 106.07 US$/barrel on June 30, 2014, and dropped significantly on February 11, 2016, and on April 20, 2020, after increasing again to the May 2008 level. Similarly, the crude oil return is influenced by the same shocks (Figure 2). The gasoline price and gasoline return series presented in Figures 1 and 2 respectively, follow the crude oil series fluctuations, showing that the variables are related to each other. Therefore, as gasoline is a refined product, its return depends on crude oil return. The gasoline and crude oil volatility is presented in the Appendix, Figure A1.

Also, descriptive statistics for the gasoline return (GAS_t) and crude oil return (WTI_t) are presented in Table 1. The mean values are close to zero and the standard deviation is low. The Jarque-Bera statistics (Jarque and Bera, 1980) shows that the series do not follow a normal distribution ($P < 0.05$), while there is non-zero skewness and positive excess kurtosis. Also, the Q statistics for serial correlation show that the null hypothesis of no serial correlation up to 10th and 20th order is rejected at 1% significance level for both gasoline and crude oil returns and squared returns.

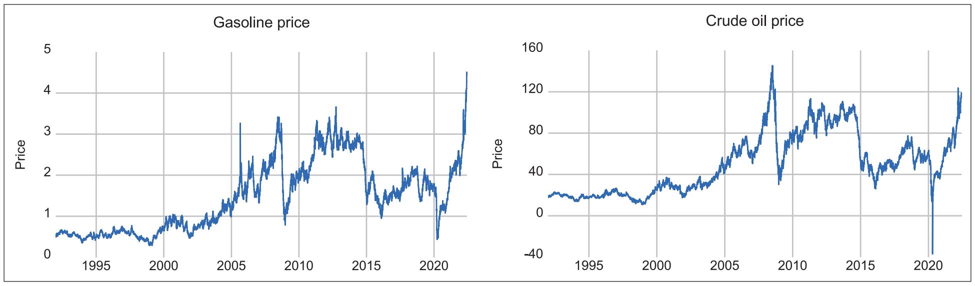
Appendix Figure B1-B4 for Q(ith) and Q^2(ith), and correlograms.

2.2. Methods

2.2.1. Unit root test

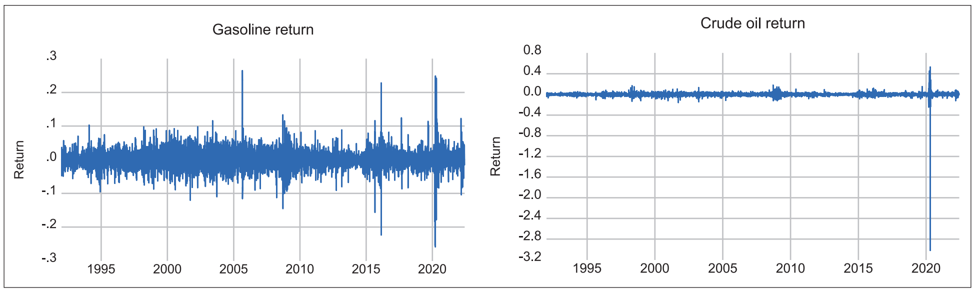
The study starts by assessing the order of integration of the gasoline and crude oil returns series, GAS_t and WTI_t respectively.

Figure 1: Plots of gasoline and crude oil prices



Source: Energy Information Administration (EIA)

Figure 2: Plots of gasoline and crude oil returns



Source: Energy Information Administration (EIA)

Table 1: Descriptive statistics for gasoline return (GAS_t) and crude oil return (WTI_t)

Statistics	GAS_t	WTI_t
Mean (%)	0.06	0.01
Maximum	0.26	0.53
Minimum	−0.26	−3.02
Standard deviation (%)	2.62	4.48
Skewness	0.11	−40.79
Kurtosis	13.36	2671.54
Jarque-Bera (Prob)	0.000***	0.000***
Q (10)	23.00***	486.49***
Q (20)	45.65***	613.65***
Q^2 (10)	2408.00***	236.86***
Q^2 (20)	3160.8***	241.92***

The variable GAS_t represents gasoline return, and WTI_t represents the crude oil return Q (ith) and Q^2(ith) are the Ljung and Box (1978) statistics of GAS_t and WTI_t series, and squared GAS_t and squared WTI_t series, respectively, for up to ith order serial correlation

*** denotes rejection of the null hypothesis of no serial correlation up to 10th and 20th order

In particular, the Augmented Dickey and Fuller (1979) (ADF), the Phillips and Perron (1988) (PP) and the Kwiatkowski et al. (1992) (KPSS) tests are applied. The ADF test involves the following equations:

$$\Delta Y_t = \alpha + \gamma_1 Y_{t-1} + \gamma_2 t + \sum_{i=1}^{p} \beta i \Delta Y_{t-i} + \varepsilon_t \quad (2)$$

$$\Delta Y_t = \alpha + \gamma_1 Y_{t-1} + \sum_{i=1}^{p} \beta i \Delta Y_{t-i} + \varepsilon_t \quad (3)$$

$$\Delta Y_t = \gamma_1 Y_{t-1} + \sum_{i=1}^{p} \beta i \Delta Y_{t-i} + \varepsilon_t \quad (4)$$

Where α and γ_2 represent the deterministic elements.

Equation (2) is a random walk with constant and trend, equation (3) is a random walk with constant, while equation (4) is a random walk (Gujarati, 2003). The error terms (ε_t) are independent with each other and identically distributed $\{\varepsilon_t \sim (0, \sigma^2)$ for $t = 1, 2, \ldots\}$. The null hypothesis, H_0: The series is non-stationary ($\gamma_1 = 0$) is tested against the alternative, H_a: the series is stationary ($\gamma_1 < 0$).

In addition, the PP unit test involves the following equations:

$$Y_t = \alpha + \gamma_1 y_{t-1} + \varepsilon_t \quad (5)$$

$$Y_t = \alpha + \gamma_1 y_{t-1} + \gamma_2 (t - T/2) + \varepsilon_t \quad (6)$$

Where α and γ_2 are the deterministic elements, T is the number of observations, while the error terms (ε_t) are allowed to be heteroscedastic and correlated with each other (Enders, 1995). The null hypothesis, H_0: the series is non-stationary ($\gamma_1 = 0$) is tested against the alternative, H_a: The series is stationary ($\gamma_1 < 0$).

It should be noted that the Kwiatkowski, Phillips, Schmidt and Shin test (KPSS) is applied, as according to Verbeek (2012:294) "Not all series for which we cannot reject the unit root hypothesis are necessarily integrated of order one."

The KPSS statistic is defined as: $KPSS = \sum_t S(t)^2/(T^2 f_0)$ (7)

Where f_0 is an estimator of the residual spectrum at frequency zero and S(t) is a cumulative residual function: $S(t) = \sum_{r=1}^{t} \hat{u}_r$, based on the residuals $\hat{u}_t$ from the equation $Y_t = \delta x_t' + u_t$.

The above tests are applied by following the Dolado et al. (1990) process regarding the inclusion of the constant and trend. Also, graphical inspection of the plots and correlograms of the GAS_t and WTI_t series is conducted to confirm the unit root test results.

2.2.2. Markov switching model

To identify the model that best fits the data, the non-linear dependence in the volatility of the gasoline returns will be examined, initially by using the squared gasoline returns. Afterwards, a regression model will be estimated for the GAS_t, assuming that the constant is the conditional mean, and the ARCH test and the residual plots will be used. If the non-linearity is completely endogenous, driven by the data itself and identified from the estimation process, the Markov Switching model[1] will be estimated. It will also be assumed that the gasoline return, GAS_t, depends on the crude oil return, WTI_t, following Wang and Wu (2012), and on the lagged gasoline return[2], accounting for linear dependence. Also, two regimes are included, as energy markets exhibit regime-switching dynamics (Figures 1 and 2). Therefore, the behaviour of the GAS_t series can be described by the following process:

$$GAS_{i,t} = \mu_{t,i,st} + \gamma_{t,i,st} WTI_{i,t} + \delta_{t,i,st}\, GAS_{i,t-1} + u_{i,t},\ u_{i,t} \sim iid(0,\sigma 2^{st}) \quad (8)$$

Where $GAS_{i,t}$ and $WTI_{i,t}$ represent the gasoline and crude oil returns, respectively, while $\mu_{t,i,st}$, $\gamma_{t,i,st}$, and $\delta_{t,i,st}$ are the state-dependent regression coefficients. In addition, the model is estimated with two regimes (states) and state dependent volatility for $u_{i,t}$. Also, the probability of transition from state 1 or 2 at time t +1 depends upon being in state 1 or 2 at the time t:

$$P = \begin{matrix} P11 & P21 \\ P12 & P22 \end{matrix} \quad (9)$$

Where:
$P(st=1 \mid st_{,t-1}=1)=P11$
$P(st=2 \mid st_{,t-1}=1)=1-P11$
$P(st=2 \mid st_{,t-1}=2)=P22$
$P(st=1 \mid st_{,t-1}=2)=1-P22$

2.2.3. ARCH and GARCH models

ARCH(p) models are estimated for gasoline returns, assuming again that GAS_t depends on the WTI_t. Therefore, the ARCH(p) with intercept, μ_{t1}, is defined as follows:

$$GAS_t = \mu_{t1} + \gamma_1 WTI_t + \varepsilon_t \quad (10)$$

$$= \mu_{t1} + \gamma_1 WTI_t + \sqrt{\sigma_t^2} z_t,\ z_t \sim N(0,1)$$

Where the conditional variance equation is defined as:

$$\sigma^2_t = \omega + \alpha_1\varepsilon^2_{t-1} + \alpha_2\varepsilon^2_{t-2} + \ldots + \alpha_p\varepsilon^2_{t-p}$$

However, ARCH (p) models cannot capture the dynamics of volatility, and for this reason, the standard GARCH(1,1) is also estimated. The GARCH (1,1) model is described as follows:

$$GAS_t = \mu_{t1} + \gamma_1 WTI_t + \varepsilon_t \quad (11)$$

$$= \mu_{t1} + \gamma_1 WTI_t + \sqrt{\sigma_t^2} z_t,\ \varepsilon_t = \sqrt{\sigma^2_t} z_t,\ z_t \sim N(0,1)$$

Where $\sigma^2_t = \omega + \alpha_1\varepsilon^2_{t-1} + \beta_1\sigma^2_{t-1}$

Where μ_{t1} denotes the conditional mean and σ^2_t is the conditional variance, with $\omega>0$, $\alpha1 \geq 0$ and $\beta1 > 0$ to ensure $\sigma^2_t > 0$.

The ARCH (p) and GARCH (1,1) are estimated, and the Schwarz Information Criterion (SIC) is used to select the best model. In addition, diagnostic tests are also performed to check for linear or non-linear dependence remaining in the selected GARCH (1,1) model. In particular, the Jarque and Bera (1980) test is conducted to check for non-normal dynamics in the residuals, the Q-correlogram is used to detect linear dependence, while the correlogram of the squared residuals and the ARCH test are used to check for non-linear dependence. In case of remaining linear-dependence in the GARCH (1,1), a one-period lag of the gasoline returns, GAS_{t-1}, is included as an explanatory variable in the model:

$$GAS_t = \mu_{t1} + \gamma_1 WTI_t + \delta_1 GAS_{t-1} + \varepsilon_t \quad (12)$$

$$= \mu_{t1} + \gamma_1 WTI_t + \delta_1 GAS_{t-1} + \sqrt{\sigma_t^2} z_t,\ \varepsilon_t = \sqrt{\sigma^2_t} z_t,\ z_t \sim N(0,1)$$

Where $\sigma^2_t = \omega + \alpha_1\varepsilon^2_{t-1} + \beta_1\sigma^2_{t-1}$

Where μ_{t1} denotes the conditional mean and σ^2_t is the conditional variance, with $\omega>0$, $\alpha1 \geq 0$ and $\beta1 > 0$ to ensure $\sigma^2_t > 0$.

However, the volatility in the conditional mean and variance is not captured by separately applying the MS, ARCH or GARCH models. For this reason, this study also fits a GARCH (1,1) on the Markov switching residual, $GARCH(1,1)_{MSut}$, to capture both the volatility in the conditional mean and variance:

$$GAS_t = \mu_{t1} + \gamma_1 WTI_t + \zeta_1 MSu_t + \sqrt{\sigma_t^2} z_t,\ \varepsilon_t = \sqrt{\sigma^2_t} z_t,\ z_t \sim N(0,1) \quad (13)$$

Where $\sigma^2_t = \omega + \alpha_1\varepsilon^2_{t-1} + \beta_1\sigma^2_{t-1}$

where μ_{t1} denotes the conditional mean, γ_1 and ζ_1 are the coefficients, MSu_t is the Markov switching residual and σ^2_t is the conditional variance, with $\omega>0$, $\alpha1 \geq 0$ and $\beta1 > 0$ to ensure $\sigma^2_t > 0$.

3. EMPIRICAL ANALYSIS

3.1. Unit Root Tests and Preliminary Analysis

The stationarity properties of the gasoline (GAS_t) and crude oil (WTI_t) returns are examined using graphical inspection of the plots and correlograms, and by performing the ADF, PP and KPSS tests. As it can be seen from Figure 2 (section 2.1),

Table 2: Unit root test results

Variable	ADF	PP	KPSS
GAS_t	−86.94 [0]***	−86.95 (2)***	0.06 (0)
WTI_t	−20.22 [22]***	−70.88 (25)***	0.15 (22)

The null hypothesis for the ADF and PP tests is that the series in non-stationary, while for the KPSS the null hypothesis is that the series is stationary. *** indicates rejection of the null hypothesis. The optimal lag for the ADF test in [ ] is selected based on the Schwarz information criterion, while the optimal bandwidths of the PP and KPSS tests in ( ) are determined based on Newey-West criterion. The Dolado *et al*. (1990) process is used for the inclusion of intercept or trend in all the equations. Based on this process, for the GAS_t series, constant is included for all the tests. For the WTI_t series, no constant or trend is included for the ADF and PP test, while constant is included for the KPSS test

1 A Threshold Autoregressive (TAR) model would also be appropriate, however, the Markov Switching model can be estimated without choosing a regime variable.

2 It should be noted that this specification provides better regression fit and residual diagnostics compared to the model without the lagged dependent variable.

Table 3: Markov switching model for gasoline return

MS	μt_1	μ_{t2}	γ_1	γ_2	δ_1	δ_2	σ_1	σ_2	P_{11}	P_{21}
Coef	−0.00	0.00	0.82***	0.09***	0.03***	−0.00	−4.35***	−3.11***	3.23***	−1.17***
z-stat	−0.75	1.63	89.81	6.99	3.78	−0.19	−323.93	−113.64	31.28	−9.55
SIC						−5.311				

*** denotes significance at 1% level. The coefficient of GAS_{t-1} is significant at 1% in regime 1, while the coefficient of WTI_t is significant at 1% in both regimes, confirming the inclusion of crude oil return in the estimates

the GAS_t and WTI_t series are stationary[3], as they do not exhibit any trend. Also, the correlograms confirm that the series are stationary, as the autocorrelations at various lags hover around zero (Appendix, Figures B1 and B2). In addition, the ADF, PP and KPSS test results show that the GAS_t and WTI_t series are stationary (Table 2). In particular, the ADF and PP tests indicate that the null hypothesis of non-stationarity is rejected for both series at the 1% significance level. The KPSS test results confirm the stationarity of the series, as the null hypothesis of stationarity cannot be rejected at any conventional level of significance.

In addition, the non-linear dependence in the volatility of the gasoline return is examined using the squared GAS_t and the ARCH test. The correlogram of the squared GAS_t shows evidence of non-linear dependence (Appendix, Figure B3), and the ARCH test confirms the existence of non-linear dependence in the conditional variance due to time series heteroskedasticity[4]. In addition, the residual plot (Appendix, Figure B5) suggests that the residual is conditionally heteroskedastic and can be represented by ARCH and GARCH. In particular, periods of high volatility are followed by periods of high volatility, while periods of low volatility are followed by periods of low volatility.

Given the evidence of non-linear dependence in the volatility of the gasoline returns, the Markov Switching model is used to capture the non-linearity in the conditional mean, while the ARCH/GARCH models to capture the non-linearity in the conditional variance.

3.2. Markov Switching model

Table 3 reports the estimation results for the Markov switching model. As it can be seen, the volatility in the regimes is significant at the 1% level, while the values are similar ($\sigma_1 = -4.35$ and $\sigma_2 = -3.11$). The transition matrix parameters (P11 = 3.23 and P21 = −1.17) are also significant at the 1% significance level. Therefore, the Markov Switching process and the existence of two regimes are confirmed.

Also, the transition probabilities are presented in Table 4. Probability 1|1 (P11) denotes the probability of being in regime 1, given that the system was in the same regime during the previous period (0.962), while Probability 2|2 (P22) indicates the probability of being in regime 2, given that the system was in regime 2 during the previous period (0.764). Probability 1|2 (P12) shows the probability of being in regime 1, given that the system was in regime 2 (0.236), while Probability 2|1 (P21) shows the probability of being in regime 2, given that the system was in regime 1 (0.038). Therefore, regime 1 is the most persistent of the two. In addition, the expected duration in regime 1, given that the system was in regime 1, is 26.31 days, while the expected duration in regime 2, given that the system was in regime 2, is 4.24 days.

Table 4: Transition probabilities

Transition	Prob.
P11	0.962
P12	0.236
P21	0.038
P22	0.764

3.3. ARCH and GARCH Models

ARCH(1) and ARCH(3) models are estimated for the gasoline return series, assuming again that the series depends on the crude oil returns. The results indicate that between ARCH(1) and ARCH(3), the ARCH(3) is preferable, based on the Schwarz Information Criterion (SIC)[5]. However, the ARCH models cannot capture the dynamics of volatility, and for this reason, a GARCH(1,1) is also estimated. In particular, using the GARCH(1,1) instead of the ARCH(3) model, greater parsimony can be achieved, which is important for forecasting. In addition, based on the SIC, the GARCH(1,1) is preferable (SIC for ARCH[3] is −5.121 and for GARCH[1,1] is −5.220). The estimation results for ARCH(3) and GARCH(1,1) models are presented in Table 5.

Based on the diagnostic tests for the GARCH(1,1) model, there are non-normal dynamics, as the Jarque-Bera $P < 0.01$, while linear dependence is found from the Q-correlograms (autocorrelation exists). However, no evidence of non-linear dependence exists based on the correlogram of squared residuals and ARCH test (ARCH[10] = 0.973 > 0.05 and ARCH[20] = 0.755 > 0.05). The Jarque-Bera test and the correlograms are presented in Figures B6-B8.

To account for the linear dependence detected in the GARCH(1,1) model, a GARCH(1,1) with a lagged gasoline return, $GARCH(1,1)_{GASt-1}$, is estimated, and the results are reported in Table 5. Based on the SIC, GARCH (1,1) and $GARCH(1,1)_{GASt-1}$ can be considered equally preferable[6]. However, the linear dependence observed in GARCH(1,1) must be addressed, and therefore

3 The series mean and variance structure is stable over time.

4 A regression model is estimated for gasoline return (GASt) assuming that the constant is the conditional mean, and the ARCH test is performed. For the ARCH test, 10 lags are used (2 weeks in the sample) and the null hypothesis, Ho: no ARCH effect exists, is tested. Ho is rejected as Prob. Chi-square (10) is 0.00<0.05, indicating that the gasoline return shows evidence of ARCH effect.

5 SIC for the ARCH(1) model is -5.017, while for the ARCH(3) model is -5.121. Therefore, ARCH(3) model is preferable.

6 SIC for GARCH(1,1) is -5.220, while for GARCH(1,1)GASt-1 is -5.219.

Figure 3: Out-of-sample forecasts

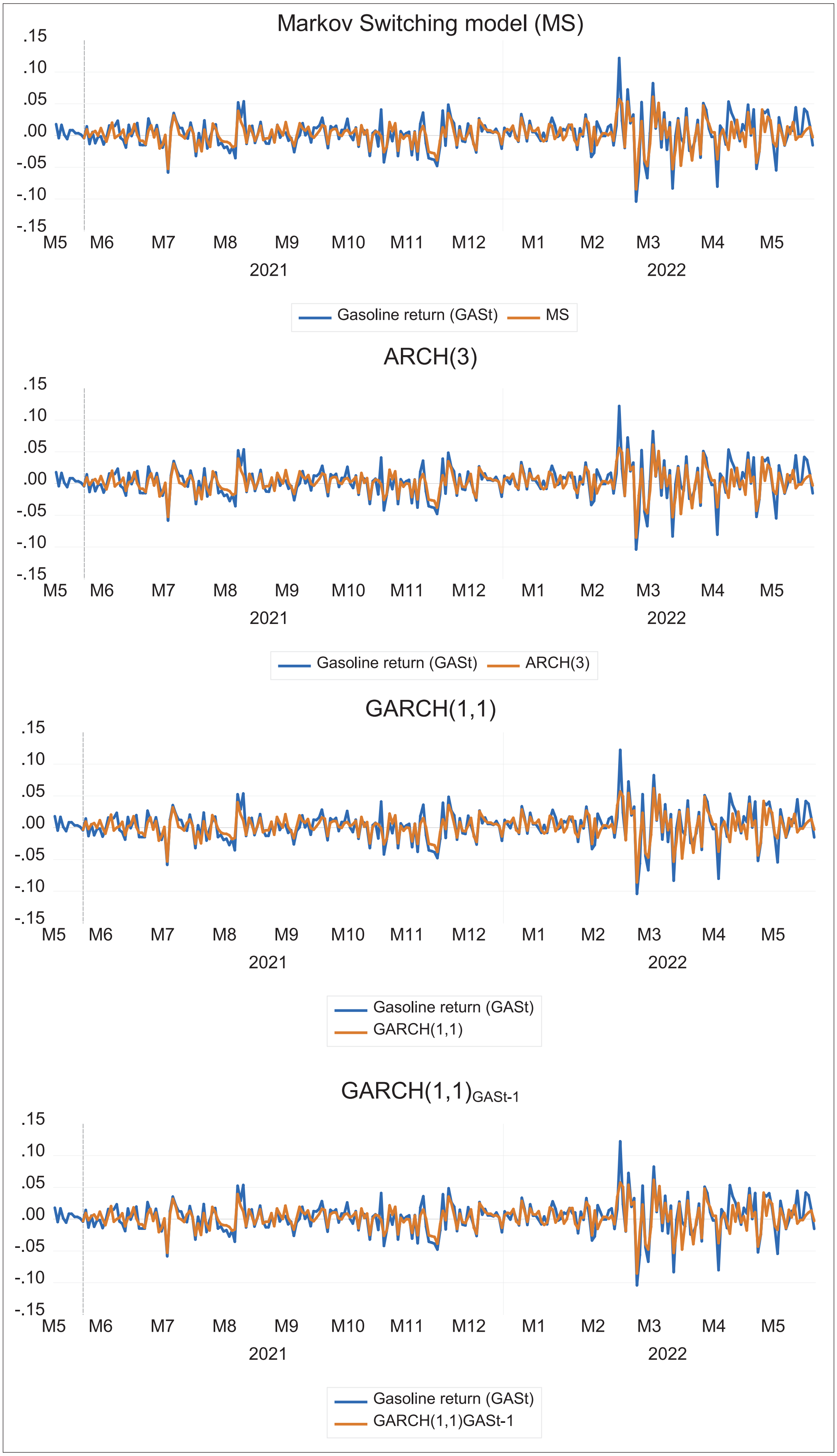


Figure 3: (Continued)

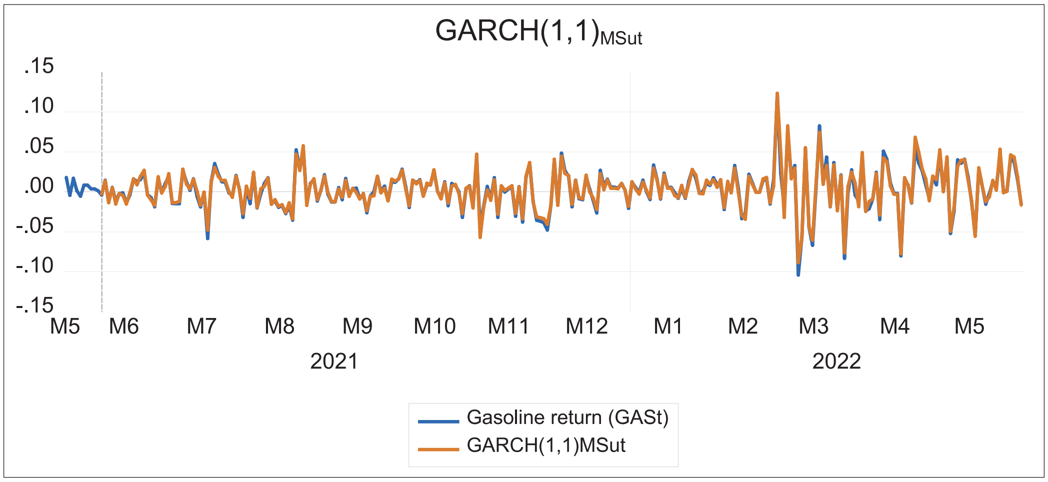


Table 5: ARCH (3), GARCH (1,1) GARCH $(1,1)_{GASt-1}$ and GARCH $(1,1)_{MSut}$ estimates

Model	ARCH (3)		GARCH (1,1)		GARCH $(1,1)_{GASt-1}$		GARCH $(1,1)_{MSut}$	
	Coefficient	z-stat	Coefficient	z-stat.	Coefficient	z-stat	Coefficient	z-stat
μ_{t1}	0.00**	2.36	0.00**	2.12	0.00**	2.06	0.00***	26.93
γ_1	0.71***	153.7	0.72***	115.29	0.72***	114.72	0.65***	984.31
ω	0.00***	44.64	0.00***	21.31	0.00***	21.32	0.00***	49.24
α_1	0.74***	135.31	0.13***	34.08	0.13***	33.85	3.40***	145.52
α_2	0.18***	19.15	-	-	-	-	-	-
α_3	0.17***	16.5	-	-	-	-	-	-
β_1	-	-	0.87***	369.32	0.87***	366.16	0.052***	19.41
δ_1	-	-	-	-	0.02**	2.48	-	
ζ_1	-	-	-	-	-	-	1.13***	1619.8
SIC	−5.121		−5.220		−5.219		−7.604	
Diagnostic test								
Q (10)	3.341		3.386		3.426		0.214	
	[0.972]		[0.971]		[0.970]		[1.000]	
Q (20)	16.017		15.523		15.355		0.321	
	[0.716]		[0.746]		[0.756]		[1.000]	
ARCH (10)	0.334		0.331		0.335		0.021	
	[0.972]		[0.973]		[0.972]		[1.000]	
ARCH (20)	0.788		0.768		0.760		0.0160	
	[0.731]		[0.755]		[0.764]		[1.000]	

Q (10) and Q (20) are the Ljung and Box (1978) Q-statistics of orders 10 and 20, respectively. ARCH (10) and ARCH (20) are the non-heteroskedasticity statistics of orders 10 and 20, respectively. P-values of the statistics are reported in [ ]
** and *** denote rejection at 1% and 5% respectively

GARCH$(1,1)_{GASt-1}$ is considered more appropriate specification. The results of the GARCH$(1,1)_{GASt-1}$ show that the previous day's residual variance (the GARCH term) and the crude oil return volatility are both significant at the 1% level, indicating that previous day's gasoline return volatility and crude oil return volatility influence today's gasoline return volatility. In addition, the lagged gasoline return is significant at the 5% level, confirming the existence of linear dependence.

Also, to capture both the volatility in the conditional mean and in the conditional variance, a GARCH(1,1) is fitted on the Markov switching residual (GARCH$[1,1]_{MSut}$). The results show that GARCH$(1,1)_{MSut}$ is preferable, as it minimizes the SIC (−7.604). The results are presented in Table 5.

4. FORECASTING PERFORMANCE EVALUATION OF MODELS

As the out-of-sample performance is crucial for the market participants, the forecasting performance of the MS, ARCH(3), GARCH(1,1), GARCH$(1,1)_{GASt-1}$, and GARCH$(1,1)_{MSut}$ models is examined. The sample data covers the period from January 2, 1992, to June 6, 2022, while the out-of-sample period used for forecasting evaluation covers the period Jun 7, 2021, to June 6, 2022. The RMSE, MAE, SMAPE and Theil U1 are used as evaluation criteria, with lower values indicating better predictive accuracy of the model. Table 6 shows that the GARCH$(1,1)_{MSut}$ displays the best out-of-sample performance.

Table 6: Evaluation of the forecasting performance (average return forecast)

Model	RMSE	MAE	SMAPE	Theil U1
Markov Switching	0.014996	0.010765	91.51557	0.334175
ARCH (3)	0.015024	0.010750	91.17665	0.335569
GARCH (1,1)	0.014981	0.010697	90.76971	0.332766
GARCH $(1,1)_{GASt-1}$	0.014967	0.010726	91.18358	0.332375
GARCH $(1,1)_{MSut}$	0.004272	0.002945	31.52062	0.008265

Table 7: Evaluation of the forecasting performance (conditional variance forecast)

Model	RMSE	MAE	SMAPE	Theil U1
ARCH (3)	1130.278	409.3610	186.3841	0.999974
GARCH (1,1)	0.025741	0.018653	171.0630	0.912042
GARCH $(1,1)_{GASt-1}$	0.025726	0.018625	168.7034	0.903785
GARCH $(1,1)_{MSut}$	2.9E+133	2.4E+132	199.8561	1.00000

In addition, Figure 3 presents the average return forecast obtained from the five estimated models. As it can be seen, the GARCH$(1,1)_{Must}$ model displays the greatest accuracy, outperforming the other models.

In addition, the forecasts of the conditional variance are generated using the square returns, and the same criteria are used to evaluate the forecasting performance of the models, except from the Markov Switching[7]. The results show that GARCH$(1,1)_{GASt-1}$ outperforms ARCH(3), GARCH(1,1) and GARCH$(1,1)_{MSut}$. Therefore, the GARCH$(1,1)_{GASt-1}$ is the best model for forecasting the volatility of gasoline returns. The results are presented in Table 7.

5. CONCLUSION

In this paper, the short-term dynamics of the gasoline price volatility were modelled using non-linear models, particularly the Markov Switching model and the ARCH/GARCH models, as regimes and non-linear dependence are present in the energy price returns. In particular, the Markov Switching model was used to capture the volatility in the conditional mean, describing the series' behaviour in different regimes, while the ARCH and GARCH models were used to capture the volatility in the conditional variance. In addition, this study also fitted a GARCH on the Markov Switching residual to combine both conditional mean and variance. Finally, we evaluated the forecasting performance of the estimated models using the root mean squared error (RMSE), mean absolute error (MAE), symmetric mean absolute percentage error (SMAPE) and Theil U1 criteria.

The evidence shows that GARCH(1,1) fitted on the Markov switching residual, GARCH$(1,1)_{Must}$, outperforms the other models for the average returns forecast. However, the GARCH(1,1) with linear dependence, GARCH$(1,1)_{GASt-1}$, is the best model for forecasting the conditional variance of gasoline returns.

Identifying the best time series model is very important for the market participants, especially for oil companies to evaluate the market conditions. In particular, adopting a GARCH$(1,1)_{Must}$ can provide more accurate forecasts for average gasoline returns by capturing both volatility and regime shifts in the market. Additionally, utilizing GARCH$(1,1)_{GASt-1}$ to forecast volatility of gasoline returns contributes to improved pricing accuracy and informs policy decision-making in energy market uncertainty.

7 Markov Switching model captures the non-linearity in the conditional mean but does not account for the conditional variance.

REFERENCES

Ambya, A., Gunarto, T., Hendrawaty, E., Kesumah, F.S.D., Wisnu, F.K. (2020), Future natural gas price forecasting model and its policy implication. International Journal of Energy Economics and Policy, 10(5), 64-70.

Bollerslev, T. (1986), Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics, 31(3), 307-327.

Cheong, C.W. (2009), Modeling and forecasting crude oil markets using ARCH-type models. Energy Policy, 37, 2346-2355.

Chkili, W., Hammoudeh, S., Nguyen, D.K. (2014), Volatility forecasting and risk management for commodity markets in the presence of asymmetry and long memory. Energy Economics, 41, 1-18.

Chung, S. (2024), Modelling and Forecasting Energy Market Volatility Using GARCH and Machine Learning Approach. arXiv preprint arXiv:2405.19849. Doi: 10.48550/arXiv.2405.19849

Dickey, D.A., Fuller, W.A. (1979), Distribution of the estimators for autoregressive time series with a unit root. Journal of the American Statistical Association, 74(366), 427-431.

Dolado, J., Jenkinson, T., Sosvilla-Rivero, S. (1990), Cointegration and unit roots. Journal of Economic Surveys, 4(3), 33-49.

Enders, W. (1995) Applied Econometric Time Series. New York: Wiley.

Energy Information Administration EIA. (2022). Available from: https://www.eia.gov

Engle, R.F. (1982), Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. Econometrica, 50, 987-1007.

Gujarati, D.N. (2003), Basic Econometrics. 4th ed. Boston: McGraw-Hill.

Hamilton, J.D. (1989), A new approach to the economic analysis of non-stationary time series and the business cycle. Econometrica, 57, 357-384.

He, X.J. (2023) Forecasting gasoline price with time series models. Communications of the IIMA, 21(1), 1.

Hendrawaty, E., Azhar, R., Kesumah, F.S.D., Sembiring, S.I.O., Metalia, M. (2021), Modelling and forecasting crude oil prices during COVID-19 pandemic. International Journal of Energy Economics and Policy, 11(2), 149-154.

Jarque, C.M., Bera, A.K. (1980), Efficient tests for normality, homoscedasticity and serial independence of regression residuals. Economics Letters, 6(3), 255-259.

Kwiatkowski, D., Phillips, P.C.B., Schmidt, P., Shin, Y. (1992), Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? Journal of Econometrics, 54(1-3), 159-178.

Lin, Y., Xiao, Y., Li, F. (2020), Forecasting crude oil price volatility via a HM-EGARCH model. Energy Economics, 87, 104693.

Ljung, M., Box, G. (1978), On a measure of lack of fit in time series models. Biometrika, 65, 297-303.

Lv, X., Shan, X. (2013), Modeling natural gas market volatility using GARCH with different distributions. Physica A Statistical Mechanics and its Applications, 392, 5685-5699.

Nomikos, N.K., Pouliasis, P.K. (2011), Forecasting petroleum futures markets volatility: The role of regimes and market conditions. Energy Economics, 33, 321-337.

Osho, G.S., Oloyede, B. (2024), A generalized autoregressive conditional heteroscedasticity GARCH for forecasting and modeling crude oil price volatility. Journal of Applied Business and Economics, 26(6), 39-53.

Phillips, P.C.B., Perron, P. (1988), Testing for a unit root in time series regression. Biometrica, 75(2), 335-346.

Roshanpour, R., Parsanejad, M., Asgari, S., Mahmoodi, F.H. (2025), Evaluating and forecasting conventional gasoline price fluctuations using Garch models with two distributions and machine learning methods. Transactions on Quantitative Finance and Beyond, 2(1), 30-42.

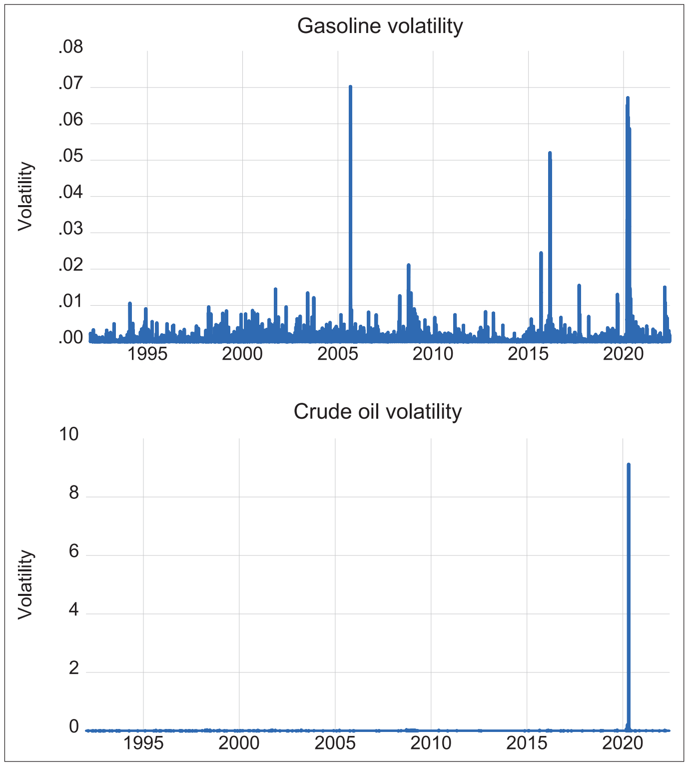
Sadorsky, P. (2006), Modelling and forecasting petroleum futures volatility. Energy Economics, 28, 467-488.

Verbeek, M.J.C.M. (2012), A Guide to Modern Econometrics. 4th ed. Chichester: John Wiley and Sons.

Wang, Y., Wu, C. (2012), Forecasting energy market volatility using GARCH models: Can multivariate models beat univariate models? Energy Economics, 34, 2167-2181.

APPENDIX

Figure A1: Plots of gasoline and crude oil volatility



Source: Energy Information Administration (EIA)

Figure B1: Correlogram of gasoline return (GAS_t)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.024	0.024	4.6334	0.031
		2	0.007	0.006	5.0068	0.082
		3	-0.029	-0.030	11.823	0.008
		4	0.002	0.003	11.842	0.019
		5	0.000	0.001	11.844	0.037
		6	-0.010	-0.011	12.580	0.050
		7	-0.015	-0.014	14.290	0.046
		8	0.018	0.019	16.804	0.032
		9	-0.018	-0.019	19.281	0.023
		10	0.022	0.021	22.996	0.011
		11	0.009	0.009	23.608	0.014
		12	0.014	0.012	25.223	0.014
		13	0.017	0.018	27.629	0.010
		14	0.036	0.036	37.832	0.001
		15	-0.002	-0.003	37.851	0.001
		16	0.018	0.018	40.476	0.001
		17	-0.009	-0.006	41.098	0.001
		18	0.005	0.004	41.280	0.001
		19	-0.015	-0.013	43.012	0.001
		20	0.018	0.019	45.645	0.001
		21	-0.001	-0.001	45.648	0.001
		22	-0.031	-0.033	53.302	0.000
		23	0.012	0.016	54.541	0.000
		24	0.005	0.002	54.753	0.000
		25	0.014	0.011	56.268	0.000
		26	0.011	0.009	57.175	0.000
		27	-0.024	-0.024	61.711	0.000
		28	0.002	-0.000	61.730	0.000
		29	-0.013	-0.012	63.075	0.000
		30	-0.001	-0.002	63.083	0.000
		31	-0.010	-0.011	63.887	0.000
		32	-0.011	-0.010	64.897	0.001
		33	-0.034	-0.033	73.982	0.000
		34	0.003	0.002	74.033	0.000
		35	-0.012	-0.011	75.199	0.000
		36	-0.022	-0.024	78.916	0.000

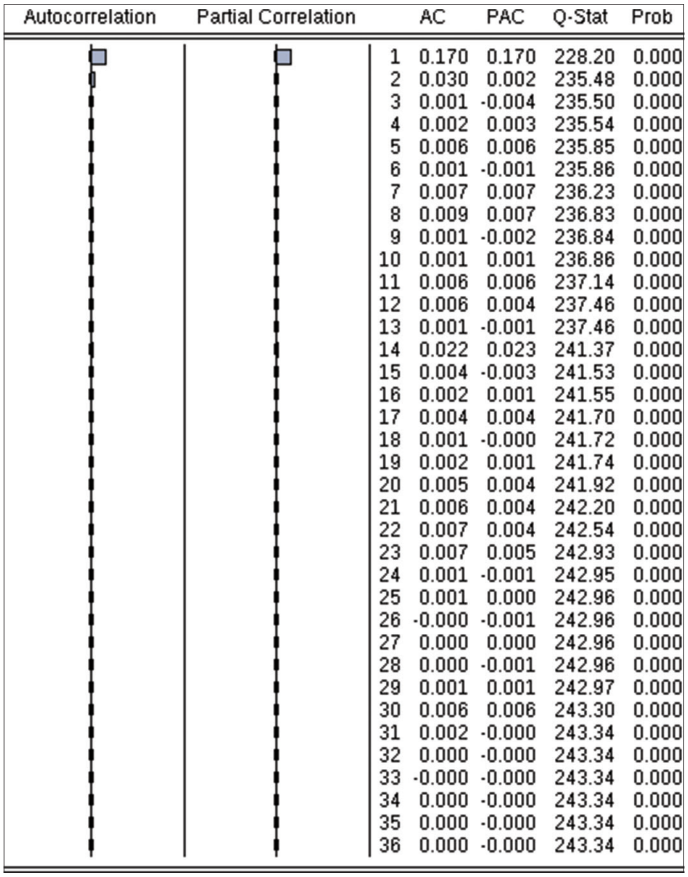
Figure B2: Correlogram of crude oil return (WTI_t)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.204	0.204	331.63	0.000
		2	-0.095	-0.143	403.68	0.000
		3	-0.038	0.014	415.24	0.000
		4	0.035	0.028	424.74	0.000
		5	0.042	0.025	439.04	0.000
		6	-0.005	-0.014	439.25	0.000
		7	-0.053	-0.043	461.85	0.000
		8	-0.054	-0.037	485.31	0.000
		9	0.011	0.020	486.32	0.000
		10	-0.005	-0.026	486.49	0.000
		11	-0.063	-0.055	518.18	0.000
		12	-0.052	-0.024	539.33	0.000
		13	-0.018	-0.015	541.90	0.000
		14	-0.072	-0.084	583.24	0.000
		15	-0.006	0.026	583.53	0.000
		16	0.018	0.001	586.18	0.000
		17	0.041	0.041	599.43	0.000
		18	-0.002	-0.020	599.46	0.000
		19	-0.008	0.002	600.01	0.000
		20	-0.041	-0.049	613.65	0.000
		21	0.028	0.041	619.96	0.000
		22	-0.018	-0.060	622.45	0.000
		23	0.025	0.054	627.56	0.000
		24	0.035	0.013	637.24	0.000
		25	0.008	-0.002	637.77	0.000
		26	0.009	0.007	638.46	0.000
		27	0.013	0.012	639.87	0.000
		28	0.003	-0.009	639.95	0.000
		29	-0.027	-0.020	645.88	0.000
		30	0.037	0.049	656.81	0.000
		31	0.024	0.006	661.45	0.000
		32	0.007	0.008	661.79	0.000
		33	-0.009	-0.008	662.43	0.000
		34	-0.010	-0.012	663.21	0.000
		35	-0.012	0.002	664.36	0.000
		36	-0.001	-0.009	664.38	0.000

Figure B3: Correlogram of squared gasoline return (GAS_t)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.257	0.257	523.98	0.000
		2	0.245	0.192	1001.5	0.000
		3	0.174	0.082	1241.4	0.000
		4	0.083	-0.016	1296.3	0.000
		5	0.204	0.157	1628.0	0.000
		6	0.049	-0.050	1647.0	0.000
		7	0.188	0.136	1926.9	0.000
		8	0.113	0.020	2029.3	0.000
		9	0.126	0.054	2154.4	0.000
		10	0.179	0.079	2408.0	0.000
		11	0.103	0.027	2492.0	0.000
		12	0.155	0.038	2682.1	0.000
		13	0.048	-0.040	2700.6	0.000
		14	0.051	-0.026	2721.6	0.000
		15	0.101	0.049	2802.5	0.000
		16	0.046	-0.008	2819.0	0.000
		17	0.083	-0.000	2874.0	0.000
		18	0.064	0.020	2906.3	0.000
		19	0.086	0.025	2965.7	0.000
		20	0.157	0.099	3160.8	0.000
		21	0.082	0.001	3214.4	0.000
		22	0.118	0.025	3324.8	0.000
		23	0.033	-0.043	3333.5	0.000
		24	0.019	-0.032	3336.5	0.000
		25	0.045	-0.002	3352.9	0.000
		26	0.069	0.063	3391.1	0.000
		27	0.127	0.051	3520.6	0.000
		28	0.070	0.005	3559.5	0.000
		29	0.069	-0.017	3596.9	0.000
		30	0.051	-0.019	3617.4	0.000
		31	0.062	0.020	3648.5	0.000
		32	0.086	0.025	3708.0	0.000
		33	0.034	-0.011	3717.1	0.000
		34	0.031	-0.024	3724.9	0.000
		35	0.046	0.020	3741.8	0.000
		36	0.052	0.015	3763.2	0.000

Figure B4: Correlogram of squared crude oil return (WTI_t)



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.170	0.170	228.20	0.000
		2	0.030	0.002	235.48	0.000
		3	0.001	-0.004	235.50	0.000
		4	0.002	0.003	235.54	0.000
		5	0.006	0.006	235.85	0.000
		6	0.001	-0.001	235.86	0.000
		7	0.007	0.007	236.23	0.000
		8	0.009	0.007	236.83	0.000
		9	0.001	-0.002	236.84	0.000
		10	0.001	0.001	236.86	0.000
		11	0.006	0.006	237.14	0.000
		12	0.006	0.004	237.46	0.000
		13	0.001	-0.001	237.46	0.000
		14	0.022	0.023	241.37	0.000
		15	0.004	-0.003	241.53	0.000
		16	0.002	0.001	241.55	0.000
		17	0.004	0.004	241.70	0.000
		18	0.001	-0.000	241.72	0.000
		19	0.002	0.001	241.74	0.000
		20	0.005	0.004	241.92	0.000
		21	0.006	0.004	242.20	0.000
		22	0.007	0.004	242.54	0.000
		23	0.007	0.005	242.93	0.000
		24	0.001	-0.001	242.95	0.000
		25	0.001	0.000	242.96	0.000
		26	-0.000	-0.001	242.96	0.000
		27	0.000	0.000	242.96	0.000
		28	0.000	-0.001	242.96	0.000
		29	0.001	0.001	242.97	0.000
		30	0.006	0.006	243.30	0.000
		31	0.002	-0.000	243.34	0.000
		32	0.000	-0.000	243.34	0.000
		33	-0.000	-0.000	243.34	0.000
		34	0.000	-0.000	243.34	0.000
		35	0.000	-0.000	243.34	0.000
		36	0.000	-0.000	243.34	0.000

Figure B5: GAS_t residual plot

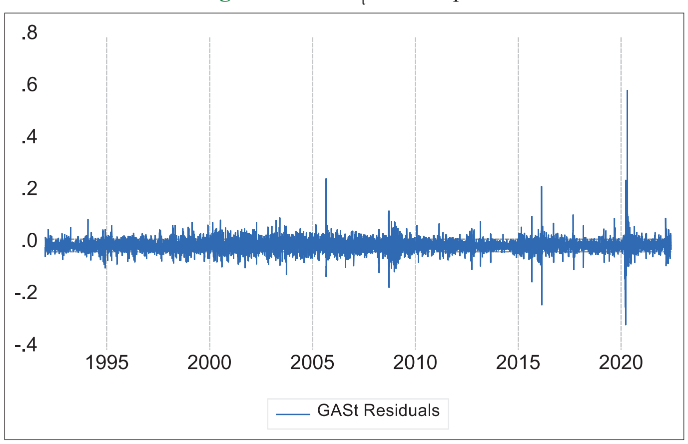


Figure B6: Jarque-Bera test to check for non-normal dynamics in the GARCH(1,1)

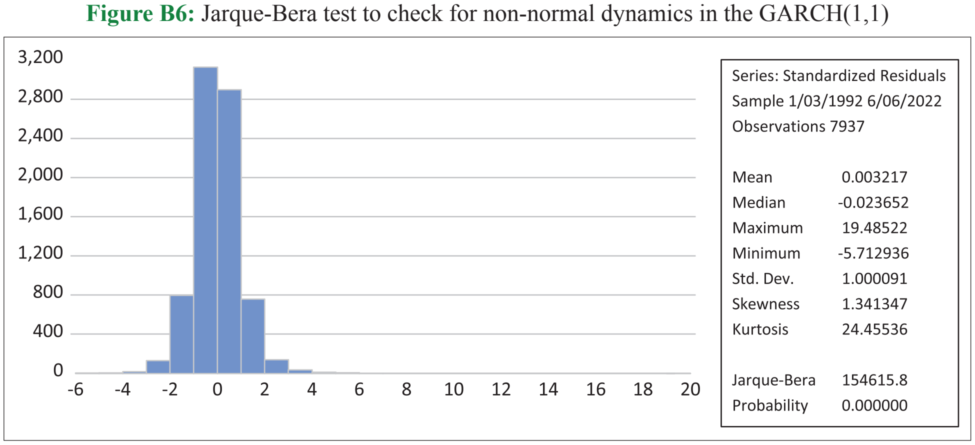


Figure B7: Q- Correlogram to detect the existence of linear dependence in the GARCH(1,1)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.021	0.021	3.6285	0.057
		2	0.007	0.007	4.0702	0.131
		3	-0.038	-0.038	15.268	0.002
		4	-0.019	-0.018	18.206	0.001
		5	-0.007	-0.005	18.570	0.002
		6	-0.031	-0.032	26.081	0.000
		7	-0.014	-0.014	27.605	0.000
		8	-0.002	-0.002	27.647	0.001
		9	-0.008	-0.011	28.184	0.001
		10	0.019	0.017	30.995	0.001
		11	-0.001	-0.003	31.013	0.001
		12	-0.002	-0.004	31.034	0.002
		13	0.005	0.006	31.263	0.003
		14	-0.019	-0.019	34.135	0.002
		15	-0.006	-0.006	34.441	0.003
		16	-0.013	-0.012	35.874	0.003
		17	-0.009	-0.009	36.489	0.004
		18	-0.007	-0.007	36.829	0.006
		19	0.014	0.014	38.393	0.005
		20	0.012	0.009	39.499	0.006
		21	-0.012	-0.014	40.566	0.006
		22	-0.010	-0.010	41.420	0.007
		23	0.008	0.009	41.964	0.009
		24	-0.006	-0.007	42.252	0.012
		25	-0.021	-0.021	45.643	0.007
		26	-0.006	-0.003	45.893	0.009
		27	-0.024	-0.024	50.603	0.004
		28	0.009	0.007	51.189	0.005
		29	-0.017	-0.018	53.421	0.004
		30	-0.009	-0.012	54.115	0.004
		31	-0.021	-0.022	57.599	0.003
		32	-0.027	-0.028	63.239	0.001
		33	-0.017	-0.019	65.447	0.001
		34	-0.008	-0.009	65.945	0.001
		35	-0.001	-0.004	65.951	0.001
		36	-0.006	-0.010	66.224	0.002

Figure B8: Correlogram of squared residuals to detect the existence of non-linearity in the GARCH(1,1)

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.012	0.012	1.0717	0.301
		2	0.001	0.001	1.0806	0.583
		3	-0.004	-0.004	1.1850	0.757
		4	-0.005	-0.005	1.3856	0.847
		5	-0.003	-0.003	1.4579	0.918
		6	-0.007	-0.007	1.8638	0.932
		7	-0.005	-0.005	2.0648	0.956
		8	-0.008	-0.008	2.5926	0.957
		9	-0.009	-0.009	3.2926	0.952
		10	-0.003	-0.003	3.3861	0.971
		11	-0.007	-0.007	3.8189	0.975
		12	-0.007	-0.007	4.1619	0.980
		13	-0.005	-0.005	4.3432	0.987
		14	0.002	0.001	4.3619	0.993
		15	-0.004	-0.005	4.5060	0.996
		16	-0.005	-0.005	4.6973	0.997
		17	-0.006	-0.006	4.9508	0.998
		18	0.002	0.002	4.9747	0.999
		19	-0.007	-0.007	5.3592	0.999
		20	0.036	0.036	15.522	0.746
		21	-0.003	-0.004	15.592	0.792
		22	0.002	0.001	15.616	0.834
		23	0.001	0.001	15.620	0.871
		24	-0.001	-0.001	15.632	0.901
		25	0.016	0.016	17.765	0.852
		26	-0.005	-0.005	17.947	0.878
		27	0.059	0.059	45.674	0.014
		28	-0.000	-0.001	45.674	0.019
		29	0.005	0.006	45.913	0.024
		30	-0.006	-0.006	46.214	0.030
		31	-0.004	-0.002	46.323	0.038
		32	-0.005	-0.004	46.530	0.047
		33	-0.000	0.001	46.530	0.059
		34	-0.003	-0.003	46.618	0.073
		35	-0.003	-0.002	46.708	0.089
		36	0.001	0.002	46.710	0.109