On the Time-varying Correlations and Hedging Effectiveness: An Analysis of Crude Oil, Gold, and Stock Market

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ABSTRACT

In this paper, we examine the time-varying conditional correlation between international crude oil and gold prices, along with their volatilities, and Indian stock prices, using the DCC-GARCH model. The results suggest that the relationships between stock/oil, stock/gold, stock/oil volatility, and stock/gold volatility are time-varying. We then use the conditional variance and covariance derived from the DCC-GARCH model to measure the optimal portfolio weight and hedge ratios for stock/oil and stock/gold portfolios. Our findings indicate that, to minimize risk without reducing expected returns, investors should hold 37% and 49% of oil and gold, respectively, in their portfolios, relative to stocks. However, there is wide variation in the hedge ratios over time, suggesting that portfolio managers will need to rebalance portfolios frequently. Finally, we analyze correlation coefficients under various deciles of stock market returns, showing that gold can be used as a hedge against stock market declines. Given India's significant improvement in financial indicators, the results of this study will be useful for portfolio managers, risk managers, policymakers, and researchers.

Keywords: Crude Oil, Gold, Stock Price, Hedging, Portfolio
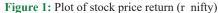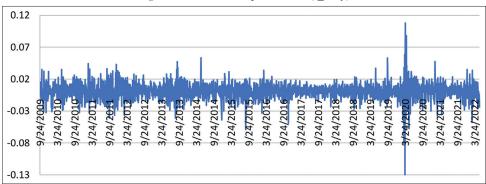**JEL Classifications:** B26, G11, Q02, Q43

1. INTRODUCTION

The global oil and gold markets have seen significant shifts recently, notably in terms of prices. The price of crude oil and gold has become much more volatile as a result of numerous crises, such as the global financial crisis of 2007, the European debt crisis of 2010, the Corona pandemic of 2019, and other economic and political events. The Indian stock market has seen tremendous volatility throughout these times, which has caused investors to become quite concerned as it raises the risk involved in managing their portfolios. Investing in different asset classes provides diversification benefits. Hence, portfolio managers and institutional investors are required to understand how the prices of different assets, especially commodities and equities, are connected in time horizon.
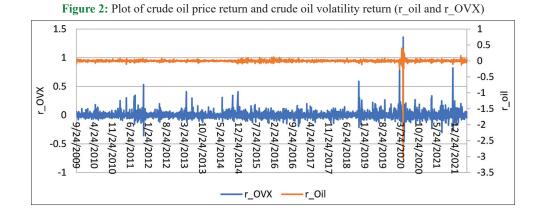
Previous studies have shown that the relationship between the price of oil and stock prices, as well as the price of gold and stock prices, is static. However, recent studies indicate that the relationship is dynamic and time-varying mainly due to the excessive co-movement of the global financial system. (Broadstock and Filis, 2014; Chang et al., 2013; Chkili, 2016; Dey and Sampath, 2018; Kumar, 2014; Mohaddes and Pesaran, 2017; Shakil et al., 2018). Historical data makes this quite obvious. The historical price series of the S and P 500, WTI crude oil price, and LBMA gold price demonstrate that these series moved in tandem at times, moved in opposition at other times, and didn't exhibit any association at other times (Figures 1 and 2 in the appendix).

The relationship between the prices of different financial assets will be dynamic or time-varying when the correlation among

Figure 1: Plot of stock price return (r_nifty)

Figure 2: Plot of crude oil price return and crude oil volatility return (r_oil and r_OVX)

such asset prices moves differently over a period of time. The difference may be in the degree or the direction and this occurs due to business cycle fluctuations, economic and financial crises, and geopolitical events (Choi and Hammoudeh, 2010). Thus, understanding the relationship between stocks and different commodity assets is of great importance for portfolio designers, risk managers, and market regulators. The time-varying link between return of a variety of commodities, including oil and gold, and stock prices was examined by many researchers (Choi and Hammoudeh, 2010; Creti et al., 2013; Singh and Sharma, 2018). They identified the fundamental reasons for this time-varying behaviour as the fluctuations in volatility of commodity prices (Choi and Hammoudeh, 2010) and different financial economic crises (Creti et al., 2013; Singh and Sharma, 2018).

2. LITERATURE REVIEW

2.1. Time-varying Relationship between Crude Oil and Stock Prices

Numerous studies demonstrate that the price of crude oil and stock prices have a dynamic or time-varying relationship. Miller and Ratti (2009) for the OECD countries; Aloui et al. (2013) for the central eastern European countries; Ji et al. (2020) for the BRICS; Filis et al. (2011) for the oil-importing and oil-exporting countries; and Broadstock and Filis (2014) for the United States and China. These studies have provided various justifications for why the link between crude oil and stock prices varies over time. Some of the justifications are from the perspective of various international events. The explanations include excessive co-movement of the global financial system (Chang et al., 2013), the global financial

crisis (Mollick and Assefa, 2013), business cycle fluctuations, economic and financial crises, and geopolitical crises, as well as the turbulence in the stock market and oil markets (Miller and Ratti, 2009). While, others explained it in terms of the sources of crude oil shocks. This includes demand shocks (Filis et al., 2011), supply-side, precautionary demand, and aggregate demand shocks (Broadstock and Filis, 2014; Degiannakis et al., 2013).

Due to the influx of high frequency models in financial econometrics, much research in recent times has analysed time-varying dependence among oil prices, gold prices and stock market prices. Miller and Ratti (2009) examined the correlation between stock prices and oil prices in OECD nations and found that, after 1999, the relationship underwent a significant alteration both in terms of direction and degrees. Aloui et al. (2013) examined the time varying movement oil price with stock market return in six Central and Eastern European (CEE) countries. Using the Copula model, they concluded that the relationship between crude oil price and stock market returns is mostly positive and persistently time varying. Ji et al. (2020) examined the dynamic dependency between oil prices and stock returns of major emerging economies. They show, using time-varying copula models, that the link between oil price and stock return is time-varying, including in India, and exhibits different behaviours depending on the type of shock.

Different authors explained the reason for the time-varying structure of the relationship between crude oil prices and stock prices. Filis et al. (2011) compared the time-varying link between stock prices and oil prices in oil-importing and exporting countries

and found that the link is largely time-varying. They also revealed that the relationship does not significantly differ for oil-importing and oil-exporting nations, and the demand-side factors of crude oil price are the primary causes of the time-varying nature of the relationship between crude oil and stock price. Degiannakis et al. (2013) investigated the correlation between the price of crude oil and the industrial sector returns in 10 European nations and found that the relationship varies significantly over time. They added that the source of shocks, such as supply-side, precautionary demand, and aggregate demand shock plays a crucial factor in determining the time-varying pattern of correlation between crude oil prices and stock market returns.

2.2. Time-varying Relationship between Gold and Stock Prices

When the stock market experiences significant negative shocks over a short period of time, investors are most inclined to search for safer investment avenues and shift their portfolios toward safe haven assets. Gold is one investment option which acts widely as a better hedge against stocks (Baur and Lucey, 2010; Chkili, 2016; Dey and Sampath, 2018; Kumar, 2014). Since 2000, gold has offered better diversification benefits to investors as it is the most defensive asset and is infamously known as a safe haven asset (Kumar, 2014).

Numerous studies demonstrate the time-varying correlation between gold and stock prices and the role of gold as a hedging tool against the stock market. The majority of such studies discover that the correlations between gold and stock markets are not constant over time and shift significantly between negative and positive values in different time periods (Chkili, 2016; Dey and Sampath, 2018; Kumar, 2014). This indicates that there is a wider scope to study how portfolio diversification benefits change over time for stock-gold investments (Kumar, 2014).

The studies that analysed the stock-gold nexus concluded that the correlation between gold and stock market prices is time-varying both in terms of direction and degree. Kumar (2014), Jain and Biswal (2016), Dey and Sampath (2018), Singh and Sharma (2018) for India; Chkili (2016) for BRICS countries. Some studies looked at the dynamic correlation of different assets and asset classes in order to compare the role of gold as an investment asset and its hedging effectiveness. They also discover that the relationship between gold and stocks shifted during times of high and low volatility (Choi and Hammoudeh, 2010; Creti et al., 2013). The main causes of the time-varying behaviour in the relationship between gold and stock markets include changes in the business cycle, geopolitical and economic events, and significant shifts in supply and demand (Choi and Hammoudeh, 2010). In contrast to these studies, some studies concluded that gold is not a viable safe haven during periods of high volatility and that the relationship between gold and stock prices is time-invariant (Yunus, 2020), as an example).

The dynamic correlations among five assets, including gold and stock prices, were examined by Choi and Hammoudeh (2010). They also compared how this relationship changed over the period of high and low volatility and found that the relationship

between different commodity prices and commodity and stock prices was time-varying. The same result was confirmed by Creti et al. (2013) and he added that the connection between stock and commodities markets changed mainly due to the financial crisis. In both studies, they evidenced the role of gold as a safe haven. Chkili (2016) investigates the time-varying correlation between gold spot prices and the stock prices of emerging economies (BRICS). He examined the efficiency of gold as a hedge against stock market risk using dynamic conditional correlation (DCC) models and concluded that the correlation coefficients greatly varied in terms of sign and values. Furthermore, he pointed out that there was a low or negative time-varying correlation between gold and stock prices in all the economies under study during various financial crises, suggesting that gold performed better as a hedge against stock market risk.

There are some specific studies that analysed the time-varying behaviour of gold-stocks in India. Kumar (2014) looked at the dynamic correlation between the price of gold and a few Indian sectoral stock indices. He also estimated optimal hedge ratios and weights to examine the ability of gold in mitigating stock market risk. He finds that correlation between gold and sectoral stock indices in India is time-varying both in terms of direction and degree. The relationship was mostly negative during crises like the dot-com bubble in the U.S (2000), the global financial crisis (2007) and the European debt crisis (2011). Whereas the relationship was primarily positive during all other normal times. This study implied that investors strive to reduce the risk attached to their investments and to improve the risk-return trade-off of asset portfolios, particularly during times of financial crisis.

Jain and Biswal (2016) examined the relationship among crude oil, gold, and the stock market in India. Their findings suggest that gold has emerged as a powerful investment asset in India and that relationship between crude oil, gold and Indian stock prices is time-varying. Dey and Sampath (2018) find that gold is a good hedge against stocks, provides a safe haven during times of unrest, and in India, a 30% weighting for gold in a gold/stock portfolio is ideal.

In order to assess the hedging and safe haven qualities of gold in various market conditions, Yunus (2020) investigated the time-varying link between gold, equities, bonds, and real estate. The main finding of his research is that gold is not a reliable safe haven during times of extreme volatility. Dar and Maitra (2017) examined relationship between gold and stock prices in the U.S, China and India and found that the relationship between gold and stock is time-invariant in India and the U.S. They observed that gold is not a better hedge against stocks and it has not given diversification benefits to investors.

2.3. The Research Gap

From a detailed review of prior literature on the time-varying relationship between crude oil and stock prices and between gold and stock prices, we identified the following gaps: First, there isn't much research that looks at how the prices of gold and crude oil fluctuate over time in relation to stock prices. When it comes to the direction and strength of the link, the available studies produce

conflicting conclusions. Secondly, many studies examined the time varying relationship between either stock and oil or gold and stock and none of the previous work tries to capture possible heterogeneous relationships between oil volatility and stock price and gold volatility with stock market over different time periods. Thirdly, only a small number of these research focused on the role that commodities like gold and crude oil in hedging the risks associated with the stock market. Some studies find that crude oil and gold provide hedging effectiveness against stock market crashes only during the crisis period.

This work will augment the existing literature in the following ways: First, we investigate the time-varying relationship between crude oil, gold and stock markets and the performance of crude and gold as hedging instruments against equity in India. Since India witnessed a significant improvement in their financial indicators, the results from this may help portfolio managers, risk managers, policy makers, and researchers. Secondly, this work has included crude oil volatility and gold volatility in addition to crude oil and gold.

Thirdly, contrary to previous studies where the volatility is estimated through GARCH-type models, we use oil and gold price volatilities readily tradable on the Chicago Board of Options Exchange. Finally, at the empirical stage, we use the DCC approach with a GARCH specification to estimate the conditional correlations, the conditional covariances, and the conditional variances. The choice of this model is motivated by several reasons. Firstly, it uses a long daily nominal series and employs GARCH-DCC for analysing dynamic interrelationships and conditional correlation, which have various advantages over the conventionally used econometric models. DCC-GARCH is highly flexible for modelling large variance–covariance matrices and explicitly accommodates cross-market co-movements through time Mensi et al. (2014).

Based on the research gap, we aimed to investigate the following two objectives. First, to examine the time-varying correlation of stock prices with international crude oil prices, gold prices, and their associated volatilities and second one is to investigate the hedging effectiveness and diversification of commodities like crude oil and gold in the Indian equity market.

3. METHODOLOGY

For examining the objectives, we have used the daily return series of the CNX Nifty-50 as the proxy for Indian equity return. The return on WTI Future Price (Contract-1) in rupee and COMEX Gold Futures (GC-1) in rupee are used as a proxy for the return on international crude oil and gold prices, respectively. The Chicago Board Option Exchange's (CBOE) Crude Oil Volatility Index (OVX) and the Gold ETF Volatility Index (GVZ) are considered as proxies for crude oil and gold volatility. The data was collected from September 24, 2009 to May 13, 2022. The holidays and the day on which all the data points were not available are removed from the analysis. So, we are considering only 2692 days of data and have removed 1923 days of data points. The selection criteria is that the day on which the information on all variables is available is only considered. The data on stock prices is extracted from the website of the National Stock Exchange (www1.nseindia.com). The crude oil prices, gold prices, and

exchange rates are extracted from the International Energy Agency's website (www.eia.gov), www.nasdaq.com, and the Federal Reserve's Economic Data Website (www.fred.stlouisfed.org), respectively. The data on crude oil and gold volatilities is extracted from the Chicago Board of Options Exchange's website (www.cboe.com).

For analysing the possible covariance and correlation structure, a GARCH-DCC model is applied to return series and obtains time-varying correlation, optimal hedge ratio, and hedging effectiveness. As a preliminary step, return series are calculated using $r_i = \dfrac{p_t - p_{t-1}}{p_{t-1}}$

The DCC is based on a two-step procedure. First, we fit a GARCH (1, 1) model of Bollerslev (1986) as it allow us to reduce the number of estimated parameters by imposing non-linear restrictions.

The mean and variance equation under the GARCH (1, 1) model looks like;

$$Y_t = \phi + \varepsilon_t \tag{1}$$

$$h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} \tag{2}$$

The mean equation is a function of constant and standardised residual which follows $\varepsilon_t \sim N(0,\sigma_t^2)$, whereas the variance equation is a function of a constant term, squared lagged residual from the mean equation to represent the volatility from the previous period and forecasted variance from the last period.

As second step, a simple bivariate DCC model for two return series (oil and stock) is constructed. The conditional variance–covariance matrix H_t of the residuals $\left[\varepsilon_t^{oil} \ \& \ \varepsilon_t^{sp} \right]$ can be defines as

$$\varepsilon_t \mid \Omega_{t-1} \sim N\left(0, H_t\right), \qquad H_t \equiv \begin{bmatrix} h_t^{oo} & h_t^{os} \\ h_t^{so} & h_t^{ss} \end{bmatrix}$$

where, ε_t is the (2×1) vector of residuals obtained from VAR model and Ω_{t-1} is all the other information set available up to (t–1). Under a two-variable case, the variance–covariance matrix of residuals is specified as

$$H_t = D_t R_t D_t \tag{3}$$

Where $D_t = diag(\sqrt{h_t^{oo}}, \sqrt{h_t^{ss}},)$ is the (2×2) diagonal matrix of the conditional standard deviations of the residuals obtained from GARCH (1, 1). $h_t = \phi + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1}$.

R_t is the matrix of time varying conditional correlations,

$$R_t = \left[\rho_t^{ij} \right] = diag(Q_t)^{-\frac{1}{2}} Q_t diag(Q_t)^{-\frac{1}{2}}$$

R_t is a (2×2) positive definite matrix depends on squared residual $\left[U_{i,t} = \varepsilon_t^i / \sqrt{h_t^{ii}} \right]$, their unconditional variance-covariance matrix (Q).

$$Q_t = R + \alpha(S_{t-1}S'_{t-1} - R + \beta(Q_{t-1} - R)$$

So we can write a Bivariate DCC-GARCH (1, 1) as

$$H_t = D_t R_t D_t = \begin{bmatrix} h_t^{oo} & \rho_t^{os}\sqrt{h_t^{oo}h_t^{ss}} \\ \rho_t^{os}\sqrt{h_t^{oo}h_t^{ss}} & h_t^{ss} \end{bmatrix}$$

The correlation coefficient between two can be written as

$$\rho_t^{og} = \frac{E_{t-1}\left[\varepsilon_t^o \varepsilon_t^s\right]}{\sqrt{E_{t-1}\left[\left(\varepsilon_t^o\right)^2\right]}\sqrt{E_{t-1}\left[\left(\varepsilon_t^s\right)^2\right]}}$$

The parameters can be estimated using a quasi-specification of MLE. Its log likelihood function looks like;

$$\log L = \frac{1}{2}\sum_{t=1}^{T}\left[k\log(2\pi) + 2\log|D_t| + \log|R_t| + \varepsilon_t' R_t^{-1} \in_t\right] \quad (4)$$

4. RESULTS

4.1. Preliminary Analysis

The daily return series of stock prices presented in Figure 1 reveals that Indian equity investors received satisfactory returns over the past decade, but the returns fluctuated significantly. The initial period (2009-2014) had high volatility, owing to the international market's greater volatility, such as the European debt crisis, China's slowdown, and uncertainty regarding the US interest rate hikes. From 2015 to 2019, the return volatility significantly decreased in comparison to the earlier period, partly because of the withdrawal of Foreign Institutional Investors (FIIs) investing in other emerging countries due to valuation and taxation issues, such as MAT notice to FIIs. The greatest ups and downs in returns occurred during the initial period of the COVID-19 outbreak. However, the market recovered quickly, regaining its earlier momentum due to the relaxation of lockdown, successful COVID-19 vaccine trials, and stimulus package announcements.

The return series of crude oil given the secondary axis of Figure 2 shows that the fluctuation in crude oil return was high compared to other study variables during the study period. It was a little bit

high in the beginning of the 2010s. The reason can be attached to demand and supply side factors such as rising global oil consumption, decreasing interest rates in several European nations, the Arab Spring, the Mideast and North African crises, and OPEC concerns, etc. During the years 2012-2014, the fluctuation in return was It can be reasoned that the steepest price drops between mid-2014 and early-2016 were the major reason for this. Due to a huge decline in economic activity during the corona epidemic in 2020, the return of crude oil declined and recorded the greatest fall in its history at −3.05%. Following a strong economic comeback in the aftermath of the lockdowns, oil prices have also started to increase. In the secondary axis of Figure 2, the return of the Crude Oil Volatility Index (OIX) is presented. It demonstrates that the oil and volatility returns are moving in opposite directions for the majority of the day.

The return of gold price (gold) and gold volatility index (GVZ) are displayed on secondary and primary axes, respectively, in Figure 3. During the research period, gold investors saw a good and consistent return on their investment. During the period from 2016 to 2017, the return was comparably low owing to the price fall in gold. Since 2017 onwards, the gold return has shown a remarkable increase, mostly due to rising geopolitical risk, the trade dispute between China and the United States, significant central bank purchases, and the corona pandemic. The return of the gold price and GVZ together shows that, most of the time, they have a negative relationship. The main factors for variations in gold prices and gold volatility returns include changes in inflation, interest rates, demand from various stakeholders, geopolitical and economic crises, and currency fluctuations.

Table 1 summarises the static correlation between return series. Even though the coefficient is very small, stock prices and oil are positively correlated. Stock prices and the crude volatility index (OVX) and stock prices and the gold volatility index (GVZ) have negative and high correlations of −0.22 and −0.12, respectively. The negative correlation between the SP and the gold price suggests that gold can be used as a hedge against stock market losses.

4.2. Unitroot Test Results

As a preliminary step, Augmented Dickey Fuller (Dickey and Fuller, 1979) and Phillips Perron test (Phillips and Perron, 1988) is
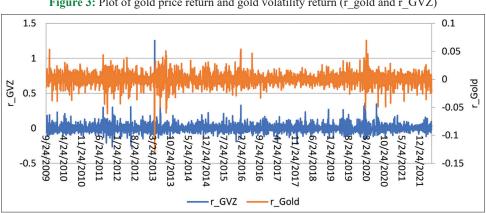
Figure 3: Plot of gold price return and gold volatility return (r_gold and r_GVZ)

conducted to determine the nature of the return series. The results are given in Table 2. The reported results for all variables under all the test and specification reveals that, the hypothesis of a unit root can be rejected even at 0.01 level of significance, suggesting that all the return series are stationary.

4.3. Dynamic Conditional Correlation- GARCH (DCC-GARCH)

To deal with time-varying covariance and correlation, the econometric analysis has switched to DCC-GARCH models. It is a useful technique for figuring out time-varying covariance between multiple variables. Due to its capacity to accurately reflect the time-varying character of asset correlation, we have used the DCC-GARCH (1,1) model. The parameters and diagnostic test results are reported in Table 3.

In all the models, ARMA (0, 0) models are used in the mean equation and GARCH (1, 1) models are used in the variance

equation, and in both cases no regressor is used in either the mean or variance equation. The sample mean of the squared residual is used for the recursion procedure. The coefficients (ϕ)of mean equations in GARCH (1 1) models are significant except for OVX and GVZ. All the estimated conditional variance (ω) coefficients are significant for all the study variables. The α and β are ARCH and GARCH term. The $\alpha + \beta$ measures short term and long-term memory respectively and short-term memory is always less than short-term memory. The magnitude of the GARCH coefficient (β) is high, suggesting the high persistence of volatility. The sum of ARCH and GARCH coefficients is above 0.9 suggesting the long persistence of volatility. Ljung-Box Q and Q^2 tests on standardised and squared standardised residuals show that the null hypothesis of no serial correlation is accepted for all the variables except OVX and GVZ.

5. DISCUSSION

5.1. Correlation Targeting Coefficients from DCC-GARCH (1,1) Models

Table 4 shows the average time-varying correlation coefficients for each possible pair of study variables. This coefficient shows that the relationship between stock prices and oil is positive, while the relationship between stock prices and gold is negative. The VIX and GVZ are negatively correlated with stock prices.

The time varying correlation coefficients for different sample period is presented in Figure 4.

The time-varying conditional correlation coefficients of four pairs of relationships, viz., stock price and oil, stock price oil volatility index, stock price and gold, and stock price and gold volatility index, are displayed in Figure 4. It shows that correlations between all the pairs are clearly time-varying. From the figure, one can observe that the relationship between crude oil and stock prices in India is predominantly positive, whereas the stock price and oil volatility index is completely negative with a few exceptions (41 days). In both cases, we can observe significant time-varying changes in the degree of correlation. This result was supported by Broadstock and Filis (2014) for the U.S. and China. In the case of

Table 1: Correlation matrix

Variable→↓	r_Sp	r_Oil	r_OVX	r_Gold	r_GVZ
r_Sp	1				
r_Oil	0.03394	1			
r_OVX	−0.2204	−0.5052	1		
r_Gold	−0.1475	0.0423	0.0475	1	
r_GVZ	−0.1274	−0.0649	0.2916	−0.0223	1

Table 2: Unit root test results: Augmented Dickey Fuller and Phillips Perron test

Test	Specification→	Constant	Constant and Trend
	Variable↓	t-Statistic	t-Statistic
Phillips	r_sp	−52.2164	−52.2101
Perron Test	r_oil	−37.2718	−37.2583
	r_OVX	−51.6575	−51.7269
	r_gold	−54.5009	−54.4939
	r_GVZ	−57.8044	−57.7932
Augmented	r_sp	−52.2173	−52.2109
Dickey	r_oil	−21.0912	−21.1012
Fuller test	r_OVX	−33.4245	−33.4455
	r_gold	−54.4955	−54.4883
	r_GVZ	−56.5394	−56.5292

Table 3: Results from DCC-GARCH (1, 1) model

Parameters	r_Sp	r_Oil	r_OVX	r_Gold	r_GVZ
Mean Equation					
ϕ	0.000857 (4.252)***	0.001212 (3.206)***	0.002072 (1.293)	0.000274 (2.420)***	0.000564 (0.5180)
Variance Equation					
ω	0.031029 (3.151)***	0.04654 (2.5041)	6.814786 (1.898)*	0.042517 (2.7202)***	2.580399 (2.671)***
α (ARCH)	0.078475 (5.949)***	0.272199 (4.020)***	0.294127 (1.717)*	0.05633 (1.161)	0.196004 (1.889)*
β (GARCH)	0.899132 (58.86)***	0.813412 (59.83)***	0.6290 (3.967)***	0.911209 (10.34)***	0.775326 (18.46)***
Model Fitness					
Ljung-Box Q (5)	3.65372 [0.6002656]	5.45407 [0.3630086]	8.64414 [0.1241283]	3.96543 [0.55440]	22.28 [0.000463]
Ljung-Box Q (10)	5.89316 [0.8241583]	7.86558 [0.6419649]	17.8963 [0.0567380]	8.9934 [0.53273]	25.5711 [0.0043620]
Ljung-Box Q (20)	18.3574 [0.5638768]	12.6922 [0.8901983]	33.5129 [0.0296151]	28.3438 [0.10150]	38.6755 [0.0073145]
Ljung-Box Q^2 (5)	4.12293 [0.5318564]	0.319689 [0.9972563]	0.966834 [0.9651962]	0.962850 [0.9655062]	0.346194 [0.9966830]
Ljung-Box Q^2 (10)	15.1672 [0.1260841]	22.6597 [0.120748]	2.62659 [0.9888997]	1.70109 [0.9981603]	0.576032 [0.9999870]
Ljung-Box Q^2 (20)	23.0156 [0.2880276]	23.1017 [0.2838291]	6.53075 [0.9979640]	3.54977 [0.9999828]	1.07535 [1.0000000]
Log Likelihood	8407.806	6165.055	3683.436	8388.424	3860.989
Jarque-Bera	679.11	3.52E+05	4190.3	1.08E+05	1.11E+05

Parenthesis [ ]are t values

Table 4: Correlation targeting coefficients from DCC-GARCH (1,1) models

Variables	Coefficient	Standard error	t value
$\rho_t^{sp\_oil}$	0.037113	0.037668	4.48***
$\rho_t^{sp\_ovx}$	−0.16348	0.03659	−4.467***
$\rho_t^{sp\_gold}$	−0.14366	0.042102	−3.412***
$\rho_t^{sp\_gvz}$	−0.12855	0.033181	−3.874***
$\rho_t^{ovx\_oil}$	−0.43384	0.053381	−8.127***
$\rho_t^{gold\_oil}$	0.13952	0.043588	3.201***
$\rho_t^{gvz\_gold}$	−0.01784	0.09051	−0.1972
$\rho_t^{gold\_ovx}$	0.037535	0.049537	0.7577
$\rho_t^{gvz\_oil}$	−0.12159	0.040739	−2.985***
$\rho_t^{gvz\_ovx}$	0.296193	0.053541	5.532***

crude oil-stock price relations, we observed negative coefficients of correlation in 827 days while positive values were observed in 1865 days. The correlation between gold-stock prices and gold volatility index-stock prices is predominantly negative except on 175 and 287 days, respectively.

According to the definition of diversifier and hedge given by Baur and Lucey (2010), if the prices of two assets or portfolios are positively but not perfectly correlated on an average, then such assets can be considered as diversifiers. On the other hand, if it is uncorrelated or negatively correlated, then such an asset can be used to hedge. In our case, the correlation between oil and stock prices is positive even though the coefficients are low, suggesting that crude oil provides diversification benefits. Regarding the gold price, it is negatively correlated with stock prices, indicating that gold can be used to hedge against stock market loss.

5.2. Decile-wise Conditional Correlation

For getting better insight, the average correlation at various deciles of stock market return is calculated and presented in Table 5.

It shows that stock–gold correlation is negative throughout the decile, the coefficients are very low and it varies between−0.1263 and −0.1389. This indicates that the correlation between gold and stock prices is not varying much with the variation in stock returns. Surprisingly, we find that the negative correlation between stocks and gold is higher when the equity returns are low and the coefficient is very low. This gives us the conclusion that gold is not a strong hedge but only a weak hedge against equities. The finding is in tandem with Hood and Malik (2013) and Dar and Maitra (2017) who reported that gold serves as a weak safe-haven asset for stock returns. The decile-wise oil price and stock price correlation shows that, during the low returns, the relationship is positive and the coefficient is very low, which is 0.0526. In all other deciles, it is negative and varies between −0.16 and −0.18. This indicates that crude oil can be used as a diversifier only during the extreme stock market falls. The major practical implication of this finding is that crude oil and gold are still not potential investment avenues for investors. Crude oil diversification is limited and only available on a very small scale. As indicated by small correlation

coefficients throughout different phases of stock return, hedging is also limited in the case of gold. Investors are only considering oil as a diversifier and gold as a hedging instrument during extreme stock market falls.

5.3. Optimal Portfolio Weights and Hedging Effectiveness of Oil and Gold

To illustrate the implications of our findings from the DCC-GARCH model, we can use optimal portfolio weights and risk hedging strategies. We consider a hedged portfolio of stocks and oil and gold in which an investor tries to hedge exposure to crude oil and gold price movements. An investor's ultimate aim is to minimise the risk of stock/oil or stock/gold portfolios without reducing their expected return. To construct optimal portfolio weight of stock/oil and stock/gold, we used Kroner and Ng (1998) methods. For that, we utilised the variance and covariance estimates from the DCC-GARCH (1,1) model. The optimal weight between two assets i and j is given as.

$$W_{ij,t} = \frac{h_{i,t} - h_{ij,t}}{h_{j,t} - 2h_{ij,t} + h_{i,t}}$$

$W_{ij,t} = 0, if\ W_{ij,t} < 0,\ W_{ij,t} = 1, if\ W_{ij,t} > 0\ and\ W_{ij,t} = W_{ij,t}, if\ 0 < W_{ij,t} < 0$
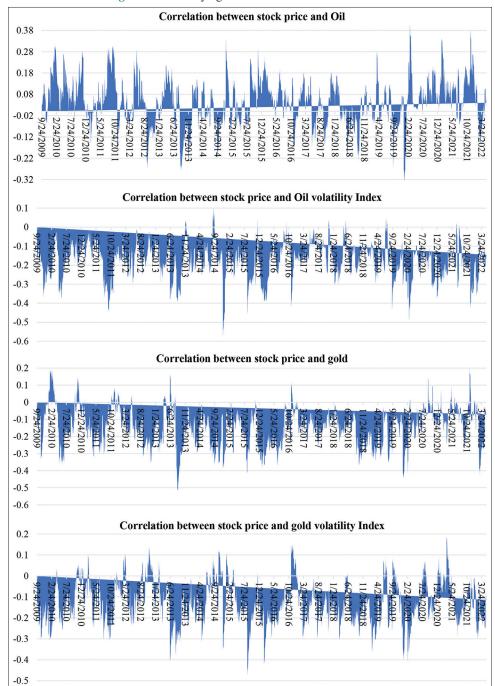
Where, $h_{i,t}$ is conditional variance of stock price and $h_{j,t}$ is conditional variance of oil and gold. $h_{ij,t}$ is conditional covariance between stock price and oil and between stock price and gold.

By design, the weight of the stock price in the stock/oil portfolio is equal to $(1–W_{ij,t})$.

This method is used by many research to find the optimal portfolio holding (e.g., Chkili, 2016; Choi and Hammoudeh, 2010; El Hedi Arouri et al., 2011; Kumar, 2014). We examine the optimal portfolio weights for two different asset classes, namely stocks and oil and stocks and gold. The descriptive statistics of the optimal portfolio weights for stock/oil and stock gold are presented in Table 6.

The mean value of the optimal weights for stock/oil and stock/gold pairs is 0.37 and 0.49, respectively. It indicates that a 1-rupee long position in stocks can be hedged with 37 paise of a short position in oil and 49 paise of a short position in gold. This implies that the optimal allocation for oil in a one-rupee stock/oil portfolio is 37%, while only 63% should be invested in the stock market. The optimal allocation for gold in a one-rupee stock/gold portfolio is 49%, while only 51% should be invested in the stock market.

From this, we can conclude that, in order to minimise risk without lowering the expected return, investors should have 37% and 49% more oil and gold, respectively, in their portfolio. The result regarding oil is in line with the finding of El Hedi Arouri et al. (2011), who found that investors in Europe and the US should have more stocks than oil in their portfolios. Since the gold market is characterised by lower volatility than the stock market, this result is not surprising. Moreover, various crises that occurred during the study period, such as the European debt crisis and the Corona outbreak, had little effect on the gold price. This result is in line

Figure 4: Time varying conditional correlation coefficients

Table 5: Decile-wise conditional correlation

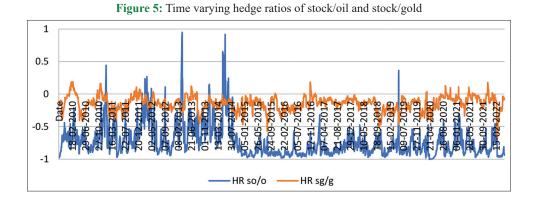| Decile of stock market return	Sp and Oil	Sp and OVX	Sp and gold	Sp and GVZ
Lowest	0.0648	−0.1984	−0.1389	−0.1367
2	−0.1689	−0.0089	−0.1236	−0.1263
3	−0.1716	−0.0046	−0.1280	−0.1280
4	−0.1606	−0.0017	−0.1309	−0.1300
5	−0.1670	0.0008	−0.1264	−0.1269
6	−0.1687	0.0033	−0.1268	−0.1260
7	−0.1648	0.0061	−0.1228	−0.1228
8	−0.1837	0.0100	−0.1340	−0.1335
Highest	−0.1526	−0.1976	−0.1370	−0.1438

Figure 5: Time varying hedge ratios of stock/oil and stock/gold

Table 6: Descriptive statistics of optimal portfolio weights

Portfolio Pairs→Statistics↓	Stock and oil	Stock and gold
Mean	0.37025	0.4971
Standard Error	0.00088	0.0017
Median	0.35754	0.4845
SD	0.04585	0.0909
Range	0.61741	0.7495
Minimum	0.33339	0.0765
Maximum	0.95081	0.8260

with the finding of Kumar (2014), who found that in order to minimise risk without lowering the expected return, investors in India should have 6% to 33% of gold in their portfolio. A similar result was also observed by Chkili (2016) for BRICS countries. El Hedi Arouri et al. (2011) also find that possessing gold improves the risk-adjusted performance of a diversified portfolio of stocks in China.

Hedge ratio is the amount of investment that an investor can hold in order to minimize the risk without reducing the portfolio returns (Chkili, 2016). Investors and market participants try to minimize the risk of the hedged portfolio and to reduce the cost risk. Hedge ratio is the most widely used method in determining the hedging behaviour, based on the information available at time t (Chkili, 2016; El Hedi Arouri et al., 2011; Kumar, 2014; Soytas et al., 2009).

$$\beta_{ij,t} = \frac{h_{ij,t}}{h_{j,t}}$$

In order to minimize the risk, the long position in one rupee in asset i can be hedged by a short position in $\beta_{ij,t}$ rupee of asset j.

In order to calculate the hedge ratio, we used conditional variance and covariance derived from DCC-GARCH (11) model. The estimates of the conditional variance and covariance from DCC can be used to compute the optimal hedge ratio which is based on the minimisation of the variance of the portfolio return (Kroner and Sultan, 1993). The calculated time varying hedge ratios for stock/oil and stock/gold pairs are given in Figure 5.

The hedge of stock with oil (blue colour line) means that a long position in stock can be hedged with a short position in oil. Similarly, in orange, the orange colour line indicates that a long position in stock can be hedged with a short position in gold. In both cases, there is wide variation in the hedge ratio over time. This implies that the portfolio managers have to rebalance the portfolio more often.

6. CONCLUSION

This paper analyzed the correlation between crude oil, stocks, and gold using the DCC-GARCH model. The study found that stock prices have the highest daily average returns followed by gold and oil. Oil and its volatility index, OVX, are highly volatile, making crude oil investment risky. Stocks have slightly higher volatility than gold but are more profitable on a return-risk basis. Gold can be used as a hedge against stock market losses.

The correlation between crude oil and stock prices is low but predominantly positive. Gold can be used to hedge against stock market loss, as the correlation between stock prices and the gold volatility index is negative. The correlation coefficients vary with different stock market returns, indicating that gold can be used as a hedge against stock market falls. The diversification benefits of oil are low, and crude oil diversification is limited. The mean value of the optimal portfolio weights for stock/oil and stock/gold pairs is 0.37 and 0.49, respectively, indicating that investors should have 37% and 49% more oil and gold in their portfolio to minimize risk without lowering the expected return.

Since India witnessed a significant improvement in their financial indicators, the results from this may help portfolio managers, risk managers, policy makers, and researchers. This study informs investors that the stock market outperforms commodities in terms of average daily return, and that investing in crude oil carries a high level of risk. Investing in volatility indices carries a high level of risk and reward. The study also gives insight to portfolio managers that, in India, the diversification benefits of commodities like oil are very low. Similarly, the hedging benefits of gold are available only against extreme stock market falls. This study also gives some implication to financial analysts that, to minimise risk without lowering the expected return, investors should have 37% and 49% of oil and gold, respectively, in their portfolio. This implies that the portfolio managers have to rebalance the portfolio more often. Our findings have important implications for implementing trading strategies and in the evaluation of investment and asset allocation decisions by portfolio managers, financial analysts, and institutional investors such as pension funds. This study highlights

the need for dynamic policy making in India to contain oil and gold price fluctuations and stock market volatility using gold and oil prices as instruments.

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APPENDICES

Appendix Figure 1: S and P 500 and WTI crude oil price

Appendix Figure 2: S and P 500 and LBMA gold price