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Forecasting Uncertainty Intervals for Return Period of Extreme Daily Electricity Consumption

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ABSTRACT

The use of extreme value theory (EVT) is usually aimed at quantifying the asymptotic behaviour of extreme quantiles. The generalised Pareto distribution (GPD) with peaks-over-threshold approach is applied to bootstrap uncertainty intervals for the return periods of extreme daily electricity consumption in South Africa (SA) for the period of 02 January 2014–29 March 2021. The leeway of extremes in daily electricity consumption studied here is the impetus behind this study. To examine the effect of a time-based and extreme non-stationary trend in a dataset, a non-stationary GPD is cast-off in computing the shape parameter ξ and, this resulted in the establishment of a type III GPD known as a Weibull class for the SA electricity sector. Results of this study revealed a non-stationary trend with a prediction power of 89.6% for the winter season and 85.65% non-winter season. This means that EVT provides a robust basis for statistical modelling of extreme values. Furthermore, a base for future researchers for conducting studies on emerging markets, more specifically in the SA context has also been contributed.

Keywords: Bayesian; Extreme Value Theory; Generalised Pareto Distribution; Markov-chain-Monte-Carlo; Peaks-Over-Threshold
JEL Classifications: C1, C4, C5

1. INTRODUCTION

South Africa (SA) is considered to be the highest electricity producer and consumer in Africa. It has been reviewed that, more than 50% of engendered electricity in Africa comes from SA. Likewise, the nation-state is archaeologically well-thought-out to have one of the peak electricity reserve margins Sigauke (2014). These margins decreased from 25% in 2002 to 20% in 2004 and thereafter to 16% in 2006. Meanwhile, in 2008, the reserve margins were projected to be 8-10% which was extremely below the target margin of 15%. Since the year 2013, electricity consumption has increased remarkably in the residential sector and this increase was from 18200 kWh in 2013 to 19000 kWh in 2016.

The combined effects of all these changes in demography, and economy can be investigated using historical patterns, but contribute to uncertainties when forecasting future electricity consumption. The infiltration of different sources of electricity, for example, renewables, sunlight based and wind, could equally have added to a decrease in electricity demand. Due to the absence of capacity experienced by Eskom by 2007 to generate enough electricity, some organisations and families had to find different ways of power sources and Mokilane (2018) emphasised that this would have brought about a decrease in demand for electricity. Shockingly, the genuine size of the power consumption market is yet obscure because of the inaccessibility of sustainable power sources and different types of power generation. The joined impact of everyone on adjustments in demography, economy and energy consumption patterns are explored utilising chronicled designs; thus far add to vulnerabilities when attempting to predict and forecast future electricity consumption. Ever since the year 2008, different interventions toward electricity supply were implemented because of high electricity demand. This fused a national awareness campaign among others (Khobai, 2018). In some instances, load shedding had to be used as the last resort to prevent a system-wide blackout. These interventions slightly brought electricity demand close to its supply, while at the same time reasonable reserve margins were being maintained. In energy sectors risk denotes a probability distribution of future returns; hence, uncertainty is considered as a broader concept that incorporates indistinctness about the parameters of this probability distribution Babatunde et al. (2020) and Anwar et al. (2020).

There are various types of measures seeking to estimate risk and uncertainty: (1) realised and derivatives-implied distributions of returns across assets, (2) news-based measures of policy and political insecurity, (3) survey-based indicators, (4) econometric measures, and (5) ambiguity indices. The benefits of macro trading are threefold. First, uncertainty measures provide a basis for comparing risk assessment of the prevailing market with private information and research. Second, changes in economic indicators often forecast near-term flows in and out of risky asset classes. Third, the level of public and any market uncertainty is indicative of risk premia offered across asset classes.

However, daily return periods necessitate planning under uncertainty, and one must contend with operational, tactical, and strategic considerations. The energy sector planning under uncertainty entails determining the appropriate location of a sector, its size, transmission, and distribution (returns flow analysis, analysis of the frequency, and occurrence of extreme losses and scheduling of risk factors). Uncertainties in forecasting extreme daily losses may arise because of increased technology allowing energy fraud systems; resulting in a stock market crash, population growth, and general randomness in individual energy market participants, as well as current economic insecurity and political conditions Sigauke et al. (2014). The inborn vulnerabilities in predictions suggest that forecasts ought to be probabilistic in an ideal world. In other words, they should take the form of probability distributions over future quantities or events (Gneiting and Katzfuss, 2014). Probabilistic forecasting of electricity uncertainty and risk promotes the management of electricity use and planning. The use of VaR and or ES is a measure that aims at lowering risk effects such as that of credit risk, exchange rate risk, and interest rate risk—to mention a few that harm the economy and some other sectors of the economy. This also gives a positive impact on market values that are associated with the use of other portfolios such as an aggressive portfolio. Short-term forecasting has a superior impact on the safety and financial implication of the economic network. Since the energy sector has a stochastic and uncontrollable nature, the current study makes use of generalised Pareto distribution (GPD) via peaks-over-threshold (POT) through the Bayesian procedure. This approach helps to obtain more robust credible estimates and solve uncertainty problems.

To achieve this objective, a more robust methodology for modelling uncertainty is considered, and Beutner et al. (2018) defined this method as a bootstrap to probability forecasting. These authors emphasised that different bootstrap procedures have been studied and are based only on generalized autoregressive conditional heteroscedasticity (GARCH) estimates. Therefore, this study extends the GARCH bootstrap estimates to an extreme approach by utilising the GPD and comparing the performance of four risk measures namely Value—at—Risk (VaR), Expected shortfall (ES), Conditional tail expectation (CTE) and Glue—Value—at—Risk (GVaR) in estimating associated risk on daily electricity uncertainty, and currently, no other study has taken this approach. Bootstrapping of extreme intervals for return periods with GPD estimates gives precise and accurate extreme return or loss periods.

Since the number of recent contributions related to forecasting returns and loss periods is extremely large, the main contribution of this study is forecasting and bootstrapping extreme returns periods on losses of daily electricity consumption by the use of GPD. This approach takes into consideration real-time forecasting and it improves the accuracy of the forecasts, and one to identify extreme changes in the immediate, especially when dealing with economic conditions that are constantly changing. Most researchers when forecasting extreme daily return periods overlook this feature. The second contribution is the development of a truncated Wild bootstrap (TWB). However, Siegl and West (2001) used a Monte-Carlo (MC) bootstrapping method which refined the computational results in different ways through resampling. But, with the proposed truncated Wild bootstrap, the procedure accounts for asymmetric distributions; moreover, under further assumptions, if the observations belong to the domain of attraction of a symmetric stable law, it performs equally well in terms of average coverage, and yet, shorter intervals in smaller samples as compared to recursive bootstrap through the MC procedures. The last contribution is the estimation of risks associated with losses on daily electricity consumption and comparing the prediction performance of these risk measures.

2. METHODOLOGY

The data that is used in this paper comprises of total daily electricity consumption of SA for the period of 02 January 2014 to 29 March 2021. Data for the period 01 August to 30 April of each year are defined as the non-winter season while the remaining period of 01 May to 31 July is defined as the winter season period. In total there are 2645 observations from which 2644 daily consumption is calculated. Let X_t be daily electricity consumption on day t and X_{t-1} be daily electricity consumption on day t-1, then, daily consumption is defined as a day-to-day change in electricity consumption. The South African power utility company (ESKOM) provided this data. Assuming that X_t has a density function of F; an access distribution over according to Sigauke (2014) has the following density function

$$F_u(x) = P(X - u \le x | X > 0) = \frac{F(x+u) - F(u)}{1 - F(u)} \quad (1)$$

for $0 \le x < xF - u$, where $F \le \infty$ is the right endpoint of F. According to Fotouhi (2019) and Sigauke et al. (2013) the right endpoint is understood as xF = sup (x ∈ R: F(x) < 1).

2.1. The GPD and POT

The GPD plays a role in a natural distribution of excesses over a reasonably high threshold Maposa et al. (2016). According to Arnold (2014), the shape parameter ξ in a GPD serves the same purpose as in the generalised extreme value distribution (GEVD). Nortey et al. (2015) and Scarrott and MacDonald (2012) disclosed that for a sequence of observations say $X_1,\ldots,X_n$, the extremes x-u of some suitably high threshold u can be well approximated by GPD (μ; σ; ξ); while Chinhamu et al. (2015) revealed that a three parameter GPD with σ as a scale parameter and ξ as a shape parameter is given by

$$G_{\xi,\beta}(x)=\begin{cases}1-\left(1+\dfrac{\xi(x-\mu)}{\sigma}\right)^{-\frac{1}{\xi}}, & \xi\neq 0\\ 1-\exp-\left(\dfrac{-(x-\mu)}{\sigma}\right), & \xi=0\end{cases} \quad (2)$$

where, $x>0$ when $\xi\leq 0$, Tsay (2014) and Huang et al. (2015). For the selection of a threshold value, the Hill's and mean excess life plots are used in this study. If the behaviour of both plots increases, these results provide a piece of evidence that the tails of a series studied are heavier than the exponential distribution. The point from which an increasing and linear behaviour starts can be a rough indication of an appropriate threshold value.

2.2. Bayesian Inferences to Parameter Estimates

In the Bayesian methodology, all obscure quantiles are considered as irregular factors and vulnerabilities over those quantiles that are addressed utilizing the likelihood contingent of the accessible data. When estimating any parameter using classical or frequentist methods, the sampling distribution of a parameter is most likely assumed to be normal or Gaussian. This methodology is very unrefined in the sense that in real situations the sampling distributions of parameters can deviate from normality. With Bayesian analysis, reasonable approximations to the sampling distribution are thought of; and their inferences are arrived at utilizing non-exclusive procedures and observed data. The fundamental standard behind Bayesian statistics is as follows. Some prior thoughts regarding any parameter or data set can neither be acquired from prolonged, some detailed observations nor by comparing them with similar conditions Pu et al. (2021).

The Bayesian approach also allows for an additional source of variation, which implies that the parameters now have probability distributions with hyper-parameters giving small standard errors. This is achieved through the naive standard errors, which are computed by dividing the actual standard deviation by the number of iterations just as Maposa et al. (2016) has suggested. Furthermore, Droumaguet (2012) also emphasized that Bayesian methods provide densities of the model parameters, which solves the problem of a confidence interval, and finally Bayesian shrinkage techniques allow models to be estimated with higher dimensions and these would have complex shapes of the likelihood function and be more difficult to estimate with classical algorithms. Sigauke et al. (2012) declared that ambiguity about the parameters is very minimal.

2.3. The Likelihood Function

For the GPD, a parameter vector $\theta=(\mu, \sigma, \xi)$ and its Bayes estimates for discrete and continuous functions are given by

$$f(\theta_i|x)=\frac{f(\theta_i)f(x|\theta_i)}{f(x)}=\frac{f(\theta_i)f(x|\theta_i)}{\sum_j f(\theta_j)f(x|\theta_j)} \quad (3)$$

and,

$$f(\theta|x)=\frac{f(\theta)f(x|\theta)}{f(x)}=\frac{f(\theta)f(x|\theta)}{\int f(\theta)f(x|\theta)d\theta} \quad (4)$$

where, $f(\theta), f(x|\theta), f(x|\theta) f(x)$ are the prior, posterior, likelihood, and normalization constant respectively. In addition to that, Vidal (2014) has indicated that posterior information is a combined sum of prior and sample information. With these computations, model (4) is further modified to

$$P(\theta|x)=\frac{p(\theta)\times p(x|\theta)}{p(x)}=\frac{p(\theta)\times p(x|\theta)}{\int_{¦}^{1} p(\theta)\times p(x|\theta)d\theta}\propto p(\theta)\times p(x|\theta) \quad (5)$$

where θ is the vector parameters of the generalized Pareto distribution, $P(\theta|x)$ is the posterior distribution, x is a vector of observations; Φ is the space parameter, p(θ) is the prior distribution, and $p(x|\theta)$ is the likelihood function of the GPD.

Therefore, three parameters are estimated following a joint posterior distribution of μ, σ and ξ and this joint distribution is given by

$$\pi(\mu,\sigma,\xi|y)\propto\prod_{i=1}^{N_u}\frac{1}{\mu}\left[1+\frac{\xi(y_i-\mu)}{\sigma}\right]^{-1/\xi-1} \quad (6)$$

where ϖ (μ, σ, ξ) $\propto$ (1/σ) exp $-\xi$ is the maximal data information(MDI) before Jonathan et al. (2021), N_μ is the number of observations above the threshold. The three parameters are estimated by simulating a large number of μ' s, σ' s, and ξ' s values from the posterior distribution and taking the mean of the simulated values to obtain estimates. To simulate a set of (μ, σ, ξ)'s from the posterior Metropolis-Hastings algorithm by simulating alternatively μ and, σ from their conditional density function given a fixed ξ. The parameter ξ is then simulated from its conditional density given the selected σ. This process is repeated numerous times. Future posterior predictive tail probabilities of a future observation Y_0, can be predicted through the following posterior predictive density

$$p(Y_0>y_0|y,\mu)\propto\int\int\int\pi(\mu,\sigma,\xi|y)\left\{1+\frac{\xi}{\sigma}(y_0-\mu)\right\}^{-\frac{1}{\xi}}, \quad -\infty<\xi<\infty \quad (7)$$

According to Jonathan et al. (2021), equation (7) cannot be computed analytically but can be approximated easily by simulation. Hence, equation (6) is used to simulate the values of μ, σ, and ξ which are then substituted into equation. (7). The average over all the tail probabilities is then used to estimate the posterior predictive tail probability.

2.4. Risk Measures

Having obtained estimators for ξ, σ and μ the conditional VaR, CTE, GVaR and ES for a one-period ahead are estimated at the α level. Employing the proposed GPD, the conditional VaR according to Anjum and Malik (2020) is estimated as

$$\widehat{VaR}_{\pi} = \begin{cases} \hat{u} + \frac{\hat{\beta}}{\hat{\xi}}\left\{\left(\frac{n}{N_u}\pi\right)^{-\hat{\xi}} - 1\right\}, \hat{\xi} \neq 0 \\ \hat{u} - \hat{\beta}\log\left[\frac{n}{N_u}(1-\pi)\right], \ \hat{\xi} = 0 \end{cases} \tag{8}$$

$\hat{\beta}$and $\hat{\xi}$ are the estimates of the GPD parameters, and N_u is the number of observations above the threshold μ in a given sample see, for instance, Pfaff (2016). Expected shortfall considers a loss beyond Value-at-Risk level and is shown to be sub-additive, while VaR disregards a loss beyond the percentile and is not sub-additive. Nadarajah et al. (2014) provided a general computation of ES with a given probability π which is defined as

$$ES_{\pi} = \frac{1}{\pi}\begin{bmatrix} E\left(XI\left\{X \le VaR_{\pi}(X)\right\}\right) + \pi VaR_{\pi}(X) - \\ VaR_{\pi}(X)pr\left(X \le VaR_{\pi}(X)\right) \end{bmatrix}, \tag{9}$$

where I(.) denotes the risk indicator function. Adopting theorem 3 of Yang et al. (2015), the conditional tail expectation is computed as

2.5. Theorem 3 of Yang et al. (2015)

Let $\{X_1, X_2,\ldots, X_n\}$ be a real-valued independent random variable with the following density function $\{F_1, F_2,\ldots, F_n\}$ and the row vector $\{\theta_1, \theta_2,\ldots,\theta_n\}$ to be nonnegative and nondegenerate zero random variables which is independent of $\{X_1, X_2,\ldots, X_n\}$ but subjective on each other. If $F_k \in \mathcal{L}\cap\mathcal{D}, \mathbb{E}\theta_k^{\beta k} < \infty, \beta_k > J_k^+$ and $\mathbb{P}(\theta_k X_k > x) = 0\left(\mathbb{P}(\theta_k X_k > x)\right)$ for all $k \in \{1,2,\ldots, n\}$, then,

$$\mathbb{E}S_n^{\theta}\mathbb{I}_{\{S_n^{\theta}\}_{x\to\infty}} \sum_{i=1}^{n}\mathbb{E}\theta_1 X_1 \mathbb{I}_{\{\theta_1 X_1 > x\}} \tag{10}$$

Under the additional condition $\mathbb{P}(\theta_k X_k > x) \asymp \mathbb{P}(\theta_1 X_1 > x), k \in \{1,2,\ldots,n\}$ theorem 3 implies that

$$\mathbb{E}S_n^{\theta}\mathbb{I}_{\{S_n^{\theta}\}_{x\to\infty}} \sum_{i=1}^{n}\mathbb{E}\theta_i X_i \mathbb{I}_{\{\theta_i X_i > x\}} \tag{11}$$

if $\theta_1 = \theta_2,\ldots,\theta_n \equiv 1$. Tang and Yuan (2014) publicised that asymptotic CTE of level q is

$$CTE_q\left(S_n^{\theta}\right) = \mathbb{E}\left(S_n^{\theta} \mid S_n^{\theta} > x_q\right), \tag{12}$$

where, $x_q = VaR_q\left(S_n^{\theta}\right) = \inf\left\{y \in \mathbb{R} : \mathbb{P}\left(S_n^{\theta} \le y\right) \ge q\right\}$. Equation (12) has been later modified by Yang et al. (2015) to

$$CTE_q\left(S_n^{\theta}\right) x \underset{q\uparrow 1}{\sim} \alpha^{-(1-\alpha)}\left(\sum_{i=1}^{n} c_k \mathbb{E}\theta_k^{\alpha}\right)^{\frac{1}{\alpha}} VaR_q(x) \tag{13}$$

Given a confidence level α, the distortion function for Glue-VaR is

$$K_{\beta,\alpha}^{h_1,h_2}(u) = \begin{cases} \frac{h_1}{1-\beta}.u, & 0 \le u < 1-\beta \\ h_1 + \frac{h_1 - h_1}{\beta - \alpha}.\left[u - (1-\beta)\right], & 1-\beta \le u < 1-\alpha \\ 1, & 1-\alpha \le u < 1 \end{cases} \tag{14}$$

where, $\alpha, \beta \in [0,1]$ so that $\alpha \le \beta$, $h_1 \in [0,1]$ and $h_2 \in [h_1, 1]$, β is the additional confidence parameter in addition to α. Belles-Sampera et al. (2014), showed that the shape of the Glue-VaR is determined by distorted survival probabilities h_1 and h_2 and levels 1-β and 1-α respectively. h_1 and h_2 are known as the distortion function heights. Therefore, Glue-VaR is computed by

$$GlueVaR_{\beta,\alpha}^{h_1,h_2}(X) = \omega_1.CTE_{\beta}(X) + \omega_2.CTE_{\alpha}(X) + \omega_3.VaR_{\alpha}(X), \tag{15}$$

where, ω_1, ω_2 and ω_3 are from the notation

$$\begin{cases} \omega_1 = h_1 - \frac{(h_2 - h_1).(1-\beta)}{\beta - \alpha} \\ \omega_2 = \frac{h_2 - h_1}{\beta - \alpha}.(1-\alpha) \\ \omega_3 = 1 - \omega_1 - \omega_2 = 1 - h_2 \end{cases}$$

The choice of this measure is based on the Omega ratio as a tool for effective measurement and it also helps in dividing a set of investment outcomes into two groups: the area of profits and the area of losses. Furthermore, no other study has applied this measure for risk assessment for financial market risk.

For a continuous random variable, Omega is computed by

$$\Omega(X) = \frac{\int_{\tau}^{\infty}\left(1 - F(x)\right)dx}{\int_{-\infty}^{\tau} F(x)dx}, \tag{16}$$

where τ is a given threshold.

2.6. Return Level Periods

The return level is a common and relatively simple measure of extreme events. The return level for the first year is the quantile that in a particular year has a 1 $\frac{1}{\pi}$ probability to be exceeded (Hu and Scarrott, 2018). Assuming that a two-parameter GPD is appropriate to model a variable X_t for the exceedances of u, Coles et al. (2001) revealed that the return level for $x > u$ is as follows

$$\Pr\left\{X > x \middle| X > u\right\} = \left[1 + \xi\left(\frac{x-u}{\sigma}\right)\right]^{-\frac{1}{\xi}} \tag{17}$$

It follows that:

$$\Pr\left\{X > x \middle| X > u\right\} = \left[1 + \xi\left(\frac{x-u}{\sigma}\right)\right]^{-\frac{1}{\xi}} \tag{18}$$

where, $\zeta_u = \Pr\{X > x\}$. The level x_m that exceeded observation is the solution of

$$\frac{1}{m} = \varsigma_u \left[1 + \xi\left(\frac{x-u}{\sigma}\right)\right]^{-\frac{1}{\xi}} \quad (19)$$

When rearranged it simplifies to

$$x_m = u + \frac{\sigma}{\xi}\left[\left(m\varsigma_u\right)^{\xi} - 1\right], \quad (20)$$

provided m is sufficiently large to ensure that $x_m > u$. This all assumes that $\xi \neq 0$. If working with $\xi = 0$, showed that equation (18) leads to

$$x_m = u + \sigma \log\left(m\varsigma_u\right) \quad (21)$$

By definition, x_m is a return level for m– observation. Hu and Scarrott (2018) pointed out that plotting x_m against m on a logarithmic scale produces the same qualitative characteristics as return plots based on a GPD distribution when $\xi = 0$, it is concave when $\xi > 0$ and it is convex when $\xi < 0$. Coles et al. (2001). It is quite convenient to present the return level on the annual scale so that the N year return level is the level expected to be exceeded once every N year. Hence, the N-year return level is defined by

$$Z_N = u + \frac{\sigma}{\xi}\left[\left(Nn_y\varsigma_u\right)^{\xi} - 1\right] \quad (22)$$

But, if $\xi = 0$, then (20) becomes

$$x_m = u + \sigma \log\left(mn_y\varsigma_u\right) \quad (23)$$

2.7. Bootstrapping Predictive Uncertainty Intervals

The Truncated Wild Bootstrap (TWB) is a bootstrap algorithm that is similar in construction to the Wild Bootstrap (WB) of Cavaliere and Georgiev (2013) but is only limited to several observations that are truncated based on some critical values around an axis of symmetry, hence its name. The TWB is more general than the WB because it accounts for asymmetric distributions. Moreover, under further assumptions, namely, if the observations belong to the domain of attraction of a symmetric stable law, the TWB is the WB of Cavaliere and Georgiev (2013). The truncated Wild bootstrap for testing the following hypothesis

$H_0: \theta = \theta_0$
$H_1: \theta \neq \theta_0$ that is propose here is described by Algorithm 1 below.

Algorithm 1: Truncated Wild Bootstrap

Generate a sample $\mathcal{H} := \{X_i\}_{(i=1)}^{n}$ of i.i.d[1] random variables from model 2

For $F_{\varepsilon 1} \epsilon\ \mathfrak{D}\mathfrak{A}(\alpha), \alpha\ \epsilon(0,2)$ estimate

$\widehat{\beta}, \hat{\sigma}, \hat{\alpha}$ from $\mathcal{H}$.

Select a per cent data to be truncated based on $\widehat{\beta},$ and $\hat{\alpha}$. Hence, select v_n

Choose the mode m based on $\widehat{\beta},$ and $\hat{\alpha}$

end

Procedure: Generate bootstrap samples $\left\{\left\{x_{i,j}^{*}\right\}_{i=1}^{n}\right\}_{j=1}^{\beta}$ as follows

$$x_{i,j}^{*} = \left[\left(\left(K_i - m\right)\tau_j + m\right)\widehat{\sigma} + \theta_0\right]1\left(\left|K_i - m\right| \le v_n\right)$$
$$+X_i 1\left(\left|K_i - m\right| > v_n\right)$$

Where $K_i := \hat{\sigma}^{-1}(X_i - \theta_0)$ and B represents the number of bootstrap samples created based on a single sample $\mathcal{H}$.

Return: bootstrap P value which is equal to the proportion of bootstrap statistics S_n^{*} more extreme than S_n

$$P_{B,n,\hat{\alpha}_n,\hat{\beta}_n}^{*} = \frac{1}{B}\sum_{i=1}^{B} 1\left(S_{n,i}^{*} \le S_n\right)$$

Where $S_{n,j}^{*} := \alpha_n^{-1}\sum_{i=1}^{n}\left(X_{i,j}^{*} - \theta_0\right), 1(.)$ is an indicator function, and

$S_n := \alpha_n^{-1}\sum_{i=1}^{n}(X_i - \theta_0)$.

3. EMPIRICAL ANALYSIS AND DISCUSSION

A GPD is fitted to the daily electricity consumption of SA using a POT approach. A non-stationary GPD denoted GPD $(\Phi_0\}$ is fitted. Various R packages, such as MCMC4extremes of e Silva and do Nascimento (2022), extRemes' version 2.0-11 of Gilleland and Katz (2016), ismev of Gilleland (2018), evdBayes' version 1.1-1 of Stephenson and Ribatet (2015) among others are used to execute the main analysis. The distribution of electricity consumption is not normally distributed as evidenced in Figure 1. The first panel shows some upward and downward trends in conjunction with seasonality. This is also confirmed by the logarithmic returns on the middle panel as the series shows volatile patterns. Regarding the marginal distribution, the quantile-quantile (Q-Q) plot in the right panel reveals a strong departure from linearity in the tails. This evidence is seen in Table 1 because the reported kurtosis is greater than three and the skewness is less than zero indicating that electricity consumption is asymmetry with negatively skewed innovations.

3.1. Winter and Non-Winter Monthly Electricity Consumption over a Specified Threshold

Using the mean residual life and Hills plots, a threshold value of 19671 kWh for the winter season is selected and for the non-winter season, 19752kWh is selected as a threshold value. Initially, 75 and 104 data points are respectively collected for both winter and non-winter seasons. Unlike Thevaraja and Sanjel (2016) who used a maximum likelihood method, this study makes use of Bayesian MCMC methods for the proposed non-stationary GPD. According

Table 1: Descriptive statistics

Electricity	Kurtosis	Skewness
	69.36	−17.83

1. i.i.d is referenced independently and identically distributed

to Stroud et al. (2017), the MCMC approach provides accurate and full probabilistic inference for the parameter estimates. As revealed in Tables 2 and 3 the estimated shape parameter of a non-stationary GPD which is denoted by $\hat{\xi}$ is negative implying that the fitted GPD is a type III GPD known as a Weibull-GPD. The standard errors are calculated by dividing the actual standard deviation by the number of iterations. As Stephenson (2016) has indicated, this is because the Bayesian approach allows for an additional source of variation, which implies that the parameters now have a probability distribution with hyper-parameters that gives small standard errors; showing that ambiguity about the parameters is very minimal. Since for both winter and non-winter extremes, the shape parameter is high above the threshold estimate of 10, the implication is that $\hat{\xi}$ roughly plays the role of a scale parameter under the exponentiated GPD (Lee and Kim, 2019).

In addition, the 95% confidence interval for $\hat{\xi}$ is estimated for both winter and non-winter seasons by the following formula $\hat{\xi} = \pm Z_{\alpha/2} \times se(\hat{\xi})\hat{\xi}$. Note that these intervals for both seasons have negative limits which enclose $\hat{\xi}$ endorsing the appropriateness of a Weibull class of distributions. The estimates of the slope, μ are positive values for all seasons, indicating that daily extreme electricity consumption had an increasing trend over the past decade in SA. The same results of negative shape parameter and positive slope were found by Gagaza et al. (2019) in their study of modelling non-stationary temperature extremes in KwaZulu-Natal using the generalised extreme value distribution. Furthermore, it can be seen that the slope of distribution for both winter and non-winter seasons falls faster near zero and produces infinitely long and thick tails.

Nevertheless, the four risk measures that are discussed in the previous section are used to compute the risk of losses in the energy sector in SA for daily electricity consumption and the results are presented in Table 4.

Table 2: Winter daily electricity consumption MCMC Estimates

Threshold	Proportion	GPD						95% CI for
u	P	$\hat{\xi}$	Se ($\hat{\xi}$)	$\hat{\beta}$	Se ($\hat{\beta}$)	$\hat{\mu}$	Se ($\hat{\mu}$)	
Electricity 19671	0.845	−0.076	0.019	443.637	192.818	0.207	0.068	(−0.416; −0.090)

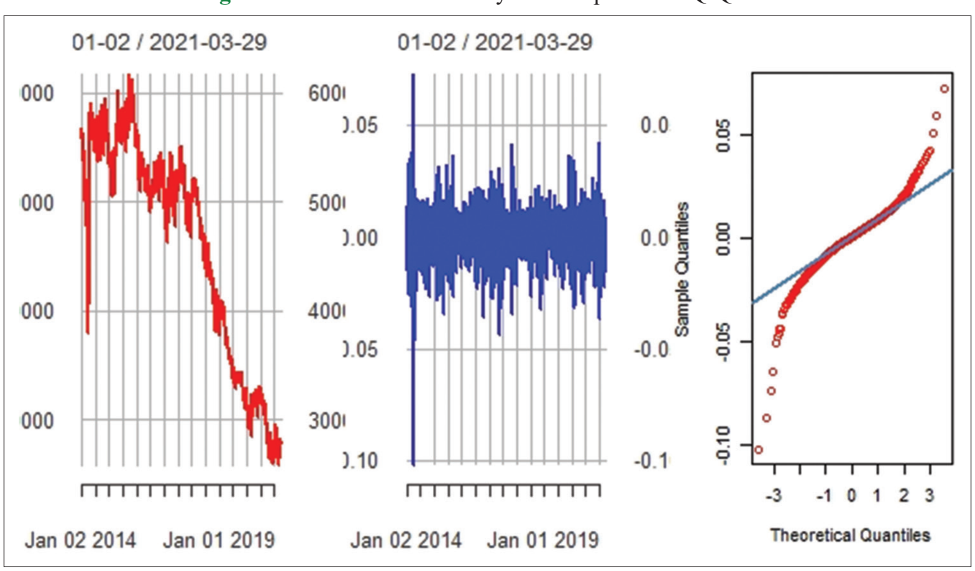
Table 3: Non-Winter daily electricity consumption MCMC Estimates

Threshold	Proportion	GPD						95% CI for
u	P	$\hat{\xi}$	Se ($\hat{\xi}$)	$\hat{\beta}$	Se ($\hat{\beta}$)	$\hat{\mu}$	Se ($\hat{\mu}$)	
Electricity 19752	0.876	−0.253	0.083	648.280	358.495	0.168	0.074	(−0.416;-0.090)

Table 4: Computation risk measures on daily losses in electricity consumption

	p	Winter Season				Non-Winter Season			
		VaR	ES	CTE	Glue VaR-Risk	VaR	ES	CTE	Glue VaR-Risk
Electricity Consumption	0.95	1.738	1.413	3.211	1.572	1.140	1.370	8.479	0.842
	0.99	1.206	0.984	5.351	3.074	1.140	1.890	10.719	0.953

Figure 1: Returns on Electricity Consumption and Q-Q Plot



For the winter season, the extreme losses at 99% for VaR 1.206. This indicates that 99% of the time Eskom is expected not to lose more than 1.206% while the expected loss using ES at 99% is 0.984%. Nevertheless, while using CTE, the extreme losses are estimated at 5.35%. The implication here is that the highest losses expected by daily electricity consumption in the winter season are estimated at 5.351%. But in the non-winter season, the highest extreme losses are estimated at 10.719% when using CTE which is surprisingly shocking because in winter more electricity is consumed by every sector of the economy. Moreover, Huang et al. (2015a), revealed that a one-day change in the financial market's value would not decrease. To take into account market liquidity constraints and Basel regulations, 5-day risk horizons in addition to the more typical 1-day horizon were being considered.

The computation of economic capital using the glue-VaR measure is more conservative than using other risk measures under the winter season. Therefore, the conclusion made is that winter and non-winter estimates of Glue-VaR risk under different confidence levels exhibit analogous characteristics as observed from VaR, ES and CTE. Generally, it can be noticed from Table 5 that the non-winter season has fewer bias estimates for all the risk measures as opposed to the winter seasons.

Table 5: Average computation of risk measures

Model	CI	VaR	ES	CTE	Glue-VaR Risk
Winter	95%	3.933	3.611	5.822	1.572
	99%	3.492	3.217	8.562	3.074
Non-winters	95%	1.460	1.831	8.241	0.988
	99%	1.460	2.311	10.549	2.226

Table 6: Posterior predictive tail probabilities

	Winter	Non-Winter
y0 (kwh)	$p\left(Y_0 > y_0 \vert y, \tau\right)$	
4000	0.0081	0.0261
4500	0.0280	0.0060
5000	0.0688	0.0106
5500	0.0150	0.1966

4. COMPARISON ANALYSIS

Some of the posterior predicted tail probabilities for various extreme daily increases in peak electricity demand are given in Table 6. ξ's, σ's and μ's are simulated as discussed in the section on methodology and the empirical results show the GPD is a good distribution show that both the GSP distribution and the GPD are a good fit to the data. The density comparison of truncated intervals in Figure 2 shows that the truncated bootstrap intervals with sampling are much better to mimic the bootstrap intervals as this Figure shows that truncated bootstraps intervals are closer to the true value intervals.

Furthermore, the average computation of the risk measure indicates that Glue-VaR provides fewer bias estimates for the risk of extreme electricity consumption; giving the prediction power of 58.4% in the winter season and 84.4% in the non-winter season and finally giving the overall predictive power of 89.7%.

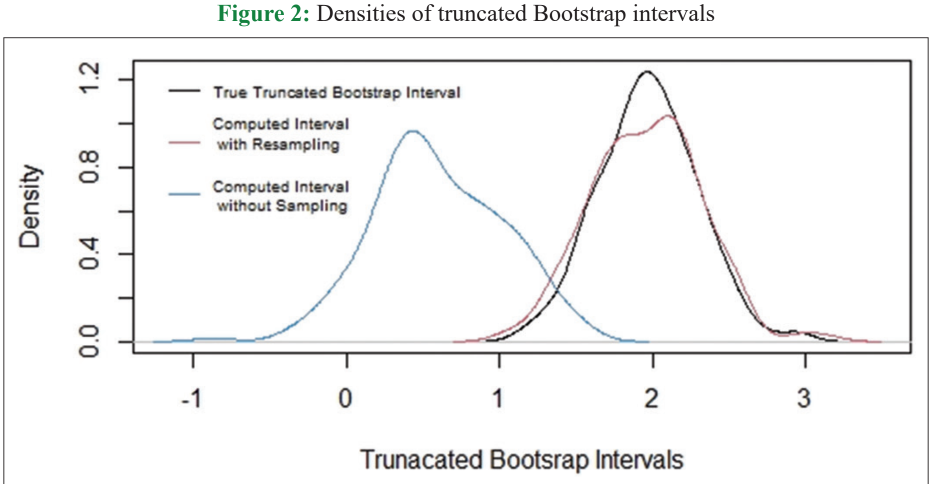
5. RETURN LEVEL PERIODS AND BOOTSTRAPPING UNCERTAINTY INTERVALS

The performance of two distributions as a function of return period *T* is explored. Persson et al. (2010), disclosed that it is not phenomenal to calculate return periods as high as 10,000 years, relating to small risk. Therefore, 10 months, 20 months and 50 months are used and the results are reported in Table 7. According to Table 7, the 20 month return period is 20799.7 kWh for winter

Table 7: Return level periods and uncertainty interval bootstraps

Period	Winter	Bootstrap Replicates	95% CI	Non-Winter	Bootstrap Replicates	95% CI
10 Months	20545.00	4000	(20544.96; 20545.04)	20834.62	2500	(20834.65; 20834.78)
20 Months	20799.70	4500	(20799.66; 20799.74)	21072.64	2500	(21072.48; 21116.60)
50 Months	21116.44	5000	(21116.40; 21116.48)	21329.57	2500	(21329.41; 21329.73)

Figure 2: Densities of truncated Bootstrap intervals



electricity consumption, which means for every 20 months an average electricity consumption is 20799.7 kWh or more in SA with a probability of 5% is expected. But with non-winter consumption, for every 20 months, only 20799.7 kWh is expected to be exceeded on monthly average with the same probability of 5%. The estimated bootstrap confidence intervals are not wide but nearer to the returns letting to conclude that the prediction performance of MCMC GPD is accurate giving a prediction power of 89.6% for winter and 85.65% for non-winter seasons respectively.

6. CONCLUSION AND RECOMMENDATIONS

Time series analysis and forecasting have been an active research area over the past decades. The accuracy of time series forecasting is fundamental to many decision processes and hence the research for improving the effectiveness of forecasting models has never stopped. This paper makes use of EVT to forecast uncertainty intervals for a return period of extreme daily electricity consumption in SA. To achieve this objective, a Bayesian MCMC is proposed and showed how it can be used in estimating the parameters of a three-parameter GPD. The importance of an MCMC approach is emphasised for parameter estimates as compared to other methods. A POT is also emphasised and the mean residual life and the Hills plots respectively are used to find an optimal threshold value. Based on the results of this study, the area of interest may have experienced too much extreme electricity consumption in the non-winter season than the winter season. This is however strange as in winter, more electricity is expected to be consumed because more people are consuming electricity for some other purposes such as heating in the house or workplaces. These empirical results showed that bootstrapping uncertainty intervals for the return period of extreme daily electricityhelp in determining critical peak months, and risk management including load shifting between transmission substations which is important for the stability of a power network.

Policy implications derived from this study are that policymakers and consumer side managers of electricity should play a fundamental role in attaining behavioural consumption of electricity, particularly during the non-winter season. In addition, there should be consumer response strategies designed for electricity consumers where the consumers will be exposed to inducements for time—of—day electricity pricing. Current developed technology for electricity billing such that is similar to mobile or landline billing technology should be improved and give a realistic day—time electricity billing.

Future studies may also adopt multivariate loss distributions and multivariate copula methods to test the interdependence and extreme relationships within the households and business sectors for the consumption of electricity and further uses machine learning methods to predict the extreme multivariate losses and the probability of interdependence and default and finally automate the models to long term solutions. Another area that requires future research is a probabilistic description and modelling of extreme peak loads using the Poison point process. This modelling approach helps in estimating the frequency of occurrence of peak losses. A sensitivity analysis concerning daily losses performed for each daily electricity consumption for winter and non-winter seasons and the development of a two-stage stochastic integer recourse model to optimise returns' distribution is an interesting future research direction with SA n data.

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