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The Probability Density Function for Wind Speed Using Modified Weibull Distribution

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ABSTRACT

Wind speed (WS) is important information to determine the potential for wind energy in an area. Wind speed has been widely expressed by the probability density function (Pdf), one of which uses the Weibull Distribution (WD). Not all WS data can be analyzed by WD because some deficiencies need to be corrected. Modified Weibull Distribution (MWD) is proposed to improve the existing WD models. In addition, this paper also compares the performance of MWD against WD using WS data measured in Medan City. To validate the two models (WD and MWD), the coefficient of determination (R-squared) and the mean square root error (RMSE) were used. In addition, data validation tests were also carried out using Chi-square and Kolmogorov-Smirnov. The result obtained is that MWD has a more acceptable fit than WD for this case.

Keywords: Wind Speed, Distribution Function, Weibull Distribution, Modified Weibull Distribution
JEL Classifications: C13, C22, C36, C93, L94, Q42

1. INTRODUCTION

The city of Medan, has a tropical rainforest climate with an unclear dry season, besides that, it has a wetter and drier month, with the driest month is February, with an average of about one-third of the wettest month in October (Bonatz et al., 2009). The average wind speed in Medan City is around 3.16 m/s with a standard deviation (SD) of 0.30433 (Suwarno et al., 2016), but analysis with other models is still needed to determine the energy potential.

Wind power density varies directly with wind speed, so small differences can lead to significant differences in energy estimates. Accurate estimation of the wind resources present at a particular location is very important (Singh et al., 2006). Modeling Pdf at WS with Weibull two parameters (W2) is more significant (Chang, 2011), (Werapun et al., 2015). The probability of occurrence of wind speed (v), shape parameters (k), and scale (c) are positive (Suwarno et al., 2021). The k factor identifies the width of the WS distribution and determines the peak wind distribution in any area (Carrasco-Díaz et al., 2015). The c factor identifies the abscissa scale of the wind distribution, and the condition of most of the wind potential in a particular location (Shu et al., 2015). Two parameters of the Weibull distribution (W2), namely the k, and c parameters are calculated using various methods in the literature (Saleh et al., 2012), (Dorvlo, 2002), (Sumair et al., 2020).

In general, WD is used to describe data while waiting for an event to occur and to express a variety of different physical phenomena that can be applied to risk analysis because it can predict component life (Suwarno et al., 2021). The probability density function (Pdf) can provide the relative likelihood that the value of a random variable will equal the sample. Pdf for wind speed (WS) provides the frequency at which certain wind speed values are observed in the studied area (Islam et al., 2011), taking into account the W2 distribution function (Gülersoy and Çetin, 2010). It was found to better match the three-parameter mixed Weibull distribution (W3) compared to the usual W2 distribution (Akdağ et al., 2010). Using a mixture of truncated normal distribution and traditional Weibull distribution to model wind speed (Akpinar and Akpinar, 2009). Using W2, W3, generalized gamma, and

4-parameter Burr distribution to describe the wind speed profile in Antakya, Turkey (Mert and Karakuş, 2015).

The probability distribution of the Weibull function (Pdf) is given in the following equation;

$$f(t)=\frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}exp\left(-\left(\frac{t}{\alpha}\right)^{\beta}\right), t>0, \alpha>0, \beta>0 \quad (1)$$

where is the observed data;
α=T=k is the scale parameter;
β=b is the shape parameter.

The probability distribution function (Pdf) model with changes in the shape and scale parameters is presented in Figures 1 and 2.

To characterize the WS probability distribution with statistics it will be simple if it is installed correctly which consists of several parameters (Kiss and Jánosi, 2008). The most common method for Pdf modeling is based on the Rayleigh and WD distribution (Kollu et al., 2012), (Arslan et al., 2014). However, Weibull's empiricists have shown low confidence in the results of the study thus encouraging researchers to carry out various alternative analyzes, such as lognormal Auwera et al (van der Auwera et al., 1980). In recent years several studies have been carried out to assess the potential of wind power using single and mixed distribution (Akpinar and Akpinar, 2007). The WD method is used to analyze the maximum and minimum monthly wind speed statistics in Zagora, Maroco (Mohammed et al., 2019). Analysis of wind characteristics for the Al-Salman site in Iraq was carried out using WD (Mahmood et al., 2020). In wind energy (WE) applications, W2 has been widely used and accepted to estimate the potential of WE in a quantifiable and flexible mathematical model of WD. WE are renewable energy that is cheap and environmentally friendly and has not been optimally utilized. Before someone utilizes this source, evaluation of wind potential (Sumair et al., 2020) and is the first input parameter that determines not only the technical feasibility but also the economic viability of a power plant project (Bilir et al., 2015). Estimating wind performance in the WE planning project with the assistance of W2, an assessment of the WE potential in Kudat and Labuan in 2006-2008 resulted in the highest monthly and annual average WS's, so this site is not suitable for large-scale WE generation (Islam et al., 2011). Three parameters of the Generalized Gamma distribution were found to more accurately describe the wind characteristics compared to W2 at different locations (Chaurasiya et al., 2019). Using a generalized extreme distribution to study wind energy variation and its potential in Debuncha, Southwestern Cameroon (Arreyndip et al., 2016). Wind speed modeling has been investigated using a modified Rayleigh model and has been tested with R^2, RMSE, and MAPE with good results (Suwarno and Rohana, 2021).

Figure 1: Density function for a = 0 and b= variation

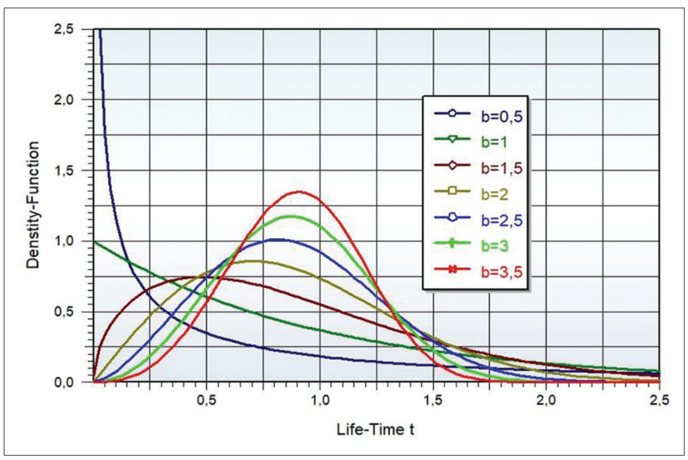
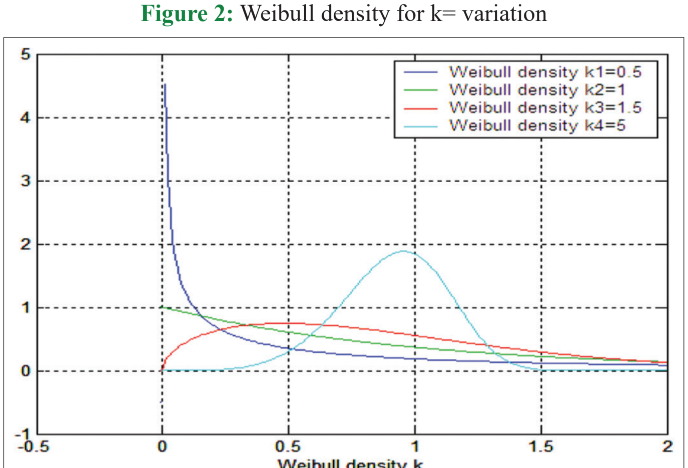


Figure 2: Weibull density for k= variation



However, there has been a heated debate regarding the appropriate selection method for its parameters, namely parameters *k* and *c*. Various methods have been adopted and used to estimate Weibull parameters by numerical approximation in the past. In this study, the proposed new model using a modification of the Weibull distribution function (MWD) with three parameters: *α*, *β* and, *λ*. The proposed new model is based on the proposal of Lai et al (2003) which is given as follows;

$$f(v)=\alpha(\beta+\lambda v)v^{\beta-1}e^{\lambda v}e^{(-av^{\beta}e^{\lambda v})}, v>0, \alpha>0, \beta>0, \lambda\geq 0 \quad (2)$$

where λ and *β* are shape parameters and α are scale parameters.

$$f(v)=1-e^{(-av^{\beta}e^{\lambda v})}, v>0, \alpha>0, \beta>0, \lambda\geq 0 \quad (3)$$

For estimating unknown parameters of the proposed new model (MWD) the least-square Method (LSM) was used. Cdf from MWD can be rewritten using equation (3) as a linear equation, namely;

$$\ln[-\ln(1-f(x)]=\ln\alpha+\beta\ln(x)+\lambda x \quad (4)$$

Equation (4) can be equated with a linear equation with two variables;

$$\ln[-\ln(1-f(x)]=y\ \ln\alpha=a;\ \beta=b_1;\ \ln(x)=x_1;\ \lambda=b_2;\ x=x_2$$

So that the new equation form becomes;

$$Y=a+b_1x_1+b_2x_2 \quad (5)$$

By using the linear regression model in equation (5), the Least Square Method (LSM) can estimate the MWD parameters without complicating the numerical calculations. Interpretation and comparison of WD and MWD models obtained parameter parameters which are estimated using the LSM model.

2. RESEARCH METHOD

2.1. Wind Speed Description

To assess the potential of WE in a particular location it can be done by analyzing and explaining the data collected from the metrology station to ensure the accuracy of the analysis. Data can be categorized in daily, monthly or annual. The renewable energy sector is encouraged to thrive because of its endless resources, lower emissions, and better economies (Owusu and Asumadu-Sarkodie, 2016). The World Wind Energy Association highlights the increased investment development of WE (https://www.iea.org/sdg.1,2019). Several distribution models have been used for this purpose. However, the most commonly used ones are the Rayleigh distribution, Gaussian distribution, and W2 (Wais, 2017), (Baseer et al., 2017).

2.2. Modelling of Wind Data

Modeling wind speed will depend on the height of the mounted instruments. Based on empirical, wind speed can be approached using the following model;

$$v_{(z)} = v_{(za)}\left(\frac{z}{z_a}\right)^{1/\pi} \tag{6}$$

where $v_{(z)}$ is the wind speed at the measured height; $v_{(za)}$ is the measured wind speed; z_a is the height of the attached instrument; z is the altitude of the wind speed to be calculated and n is the parameter specified.

2.3. Methods Used for the Determination of Weibull Parameters

The method of determining a parameter is based on the likelihood function used in statistics to calculate the parameters k and c of any probability distribution. Calculating k and c values such as equation (7) and equation (8) respectively (Khahro et al., 2014), (Chaurasiya et al., 2018). A brief explanation of the method has been given as follows;

$$k = \left[\frac{\sum_{i=1}^{N} v_i^k \ln(v_i)}{\sum_{i=1}^{N} v_1^{\,k}} - \frac{\sum_{i=1}^{N} \ln(v_i)}{N}\right]^{-1} \tag{7}$$

$$c = \left[\frac{1}{N}\sum_{i=1}^{N} v_i^k\right]^{\frac{1}{k}} \tag{8}$$

The modified maximum likelihood method (MMLM) uses wind speed data by calculating the k and c parameters as shown in equations (9) and (10) respectively (Chang, 2011), (Khahro et al., 2014), (Indhumathy et al., 2014).

$$k = \left[\frac{\sum_{i=1}^{N} v_i^k \ln(v_i) f(v_i)}{\sum_{i=1}^{N} v_1^{\,k} f(v_i)} - \frac{\sum_{i=1}^{N} \ln(v_i) f(v_i)}{f(v \geq 0)}\right]^{-1} \tag{9}$$

$$c = \left[\frac{1}{\sum f_i}\sum_{i=1}^{N} v_i^k f_i\right]^{\frac{1}{k}} \tag{10}$$

The formula for estimation and evaluation of the WD model and the proposed new MWD model uses the coefficient of determination (R squared) and the least-squares model (LSM) using equations (11) and (12) as follows;

$$R^2 = 1 - \frac{\sum\left(y_i - \overline{y_i}\right)^2}{\sum\left(y_i - \overline{y_i}\right)} \tag{11}$$

Where y_i is the amount of data from the measurement; $\overline{y_i}$ is the average of the data for each measurement; $\overline{y}$ is the average of all measurement data

$$RMSE = \frac{\sqrt{\frac{\sum_{i=1}^{n}(R_{e,i} - R_{m,i})^2}{n}}}{\overline{R_m}} \times 100\% \tag{12}$$

Where $R_{e,i}$ is the estimated data from each measurement.
$R_{m,i}$ is the data from each measurement.
n is the number of data.

2.4. Test of Goodness of Fit

Goodness-of-fit is comparing the observation frequency with the theoretical/expected frequency, whether the observed frequency deviates from the expected frequency. The Chi-Square value is small, meaning that the two data frequencies are very close and the Goodness of fit of the degree of freedom (df) model is equal to the number of categories minus the number of estimators based on the sample and subtracted by 1. A parameter estimator is a parameter whose value is estimated because the parameter value cannot be precisely determined based on available sample data. The degree of freedom is defined as:

$$df = k - m - 1;$$

where k is the number of categories of sample data. m is the number of parameter values estimated.

The results of the goodness-of-fit data using the Chi-square are shown in Table 1 and the Kolmogorov-Smirnov test is shown in Table 2.

The results of the Test of Goodness of Fit for 3-year wind speed data (2017-2019) with Chi-square and Kolmogorov-Smirnov Test can be seen from the Significance value (Sig.) Which is less than 5% (Sig. <0.05) accuracy of 95%, so that the overall wind speed data for 3 years have normally distributed data, and can be used for further research data.

3. RESULTS AND DISCUSSION

To complete this research, it is necessary to review the results for discussion using the Weibull Distribution (WD) and the Modified Weibull Distribution (MWD) which are proposed as follows;

For WD statistical descriptions for 2017, the estimated distribution parameters are shown in Table 3. For WD statistical descriptions for 2018, the estimated distribution parameters are shown in Table 4. For WD statistical descriptions for 2019, the estimated distribution parameters are shown in Table 5. The comparison of WD and MWD parameters for 3 years is shown in Table 6.

Table 6 shows that the results of the estimation and evaluation between WD and MWD in terms of the coefficient of determination (R^2) and root mean square error (RMSE) show that the MWD model provides a better value than the WD model. A special case in 2017 is that the WD model is better than the MWD model, but for 2018 and 2019 that the MWD model is better than the WD model.

The WD model only takes into account two parameters, namely scale (α) and shape (β), while MWD takes into account three parameters, namely scale (α) and two form parameters (λ and β), so this will improve the characteristic performance of the distribution function.

Based on the WD and MWD parameters obtained from wind speed data, the characteristics of the Pdf can be described in Figures 3 and 4, while the comparison characteristics of the two models are shown in Figures 5-7.

Figure 3 shows the comparison of WD between Pdf for the respective 3 years, while Figure 4 shows the MWD comparison between Pdf for the respective 3 years. Figures 5-7 is a comparison between WD and MWD Pdf proposed model. The results of the comparison to the two models show that the proposed model provides Pdf MWD better when compared with

Table 1: Test Statistics by Chi-Square

	Wind_ Speed_17	Wind_ Speed_18	Wind_ Speed_19
Chi-square	207.055	234.000	331.255
df	159	179	150
Asymp. Sig.	0.006	0.004	0.000

Table 2: Kolmogorov-Smirnov Test

	Wind_ Speed_17	Wind_ Speed_18	Wind_ Speed_19
n	365	366	365
Normal Parameters			
Mean	4.9626	4.9060	5.0625
Std. Deviation	1.06733	1.17450	1.13909
Most Extreme Differences			
Absolute	0.116	0.078	0.081
Positive	0.080	0.078	0.080
Negative	-0.116	-0.068	-0.081
Kolmogorov-Smirnov Z	2.223	1.499	1.557
Asymp. Sig. (2-tailed)	0.000	0.022	0.016

Figure 3: Pdf WD for 3 years

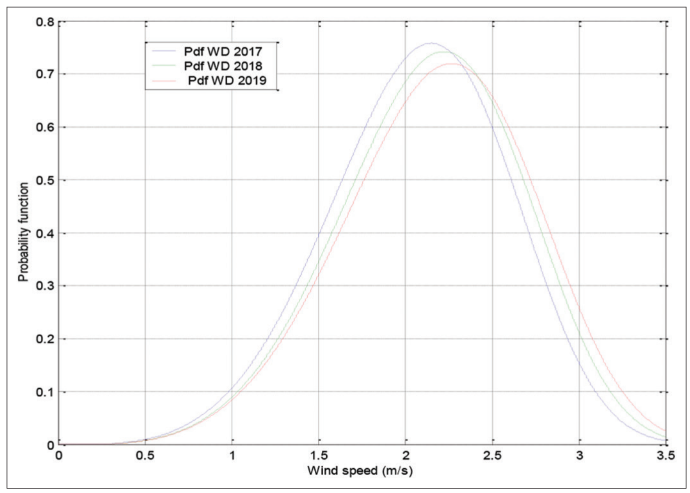


Table 3: WD Parameters in 2017

	January	February	March	April	May	June	July	August	September	october	November	December
WD												
Scale	4.849	5.343	5.549	5.049	5.717	5.834	5.848	5.536	5.261	5.186	5.449	4.585
Shape	4.897	4.667	4.930	5.729	5.820	4.783	4.373	5.056	4.306	4.631	4.572	4.556

Table 4: WD Parameters in 2018

	January	February	March	April	May	June	July	August	September	october	November	December
WD												
Scale	4.849	5.318	5.573	5.029	5.757	5.857	5.867	5.456	5.354	5.097	5.490	4.611
Shape	4.897	4.744	4.959	5.722	5.857	4.747	4.341	5.187	4.161	4.834	4.540	4.459

Table 5: WD Parameters in 2019

	January	February	March	April	May	June	July	August	September	october	November	December
WD												
Scale	5.017	5.389	5.053	5.228	5.303	5.866	5.095	5.740	5.702	5.833	5.596	6.113
Shape	4.887	4.053	3.833	5.303	4.813	5.122	6.952	5.253	6.714	5.725	5.688	6.048

Figure 4: Pdf MWD for 3 years

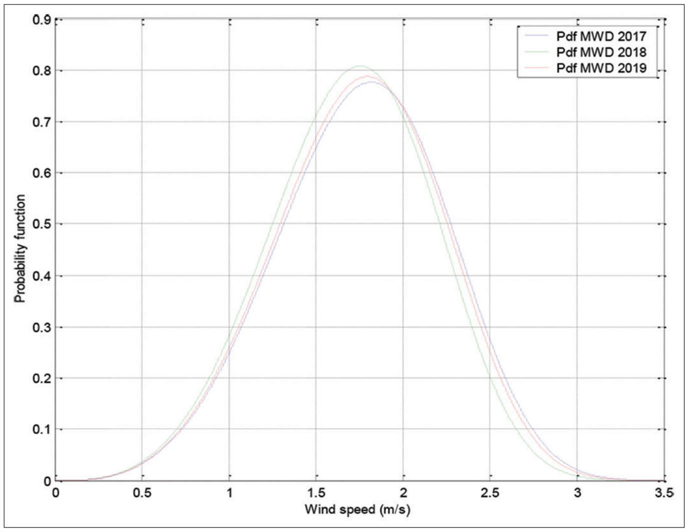


Figure 5: Pdf of WD and MWD for 2017

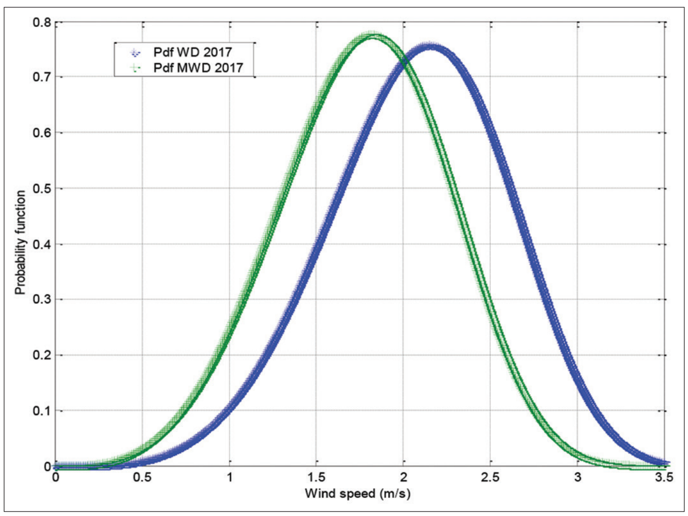
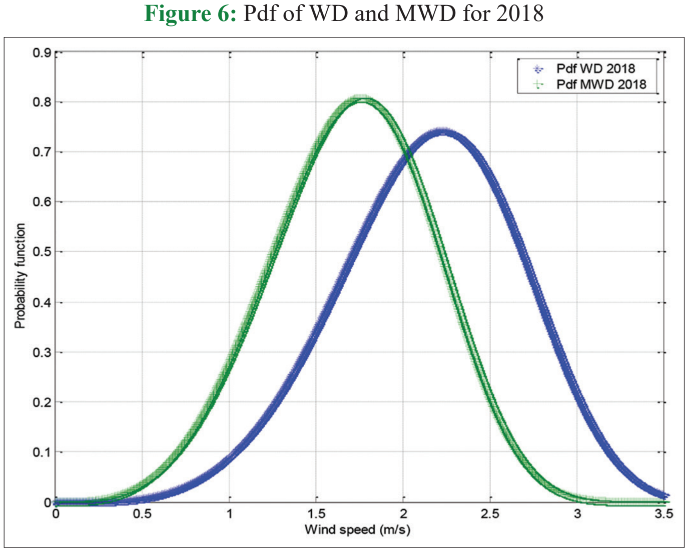


Figure 6: Pdf of WD and MWD for 2018



the model WD commonly used in analyzing the distribution of wind speed.

Figure 7: Pdf of WD and MWD for 2019

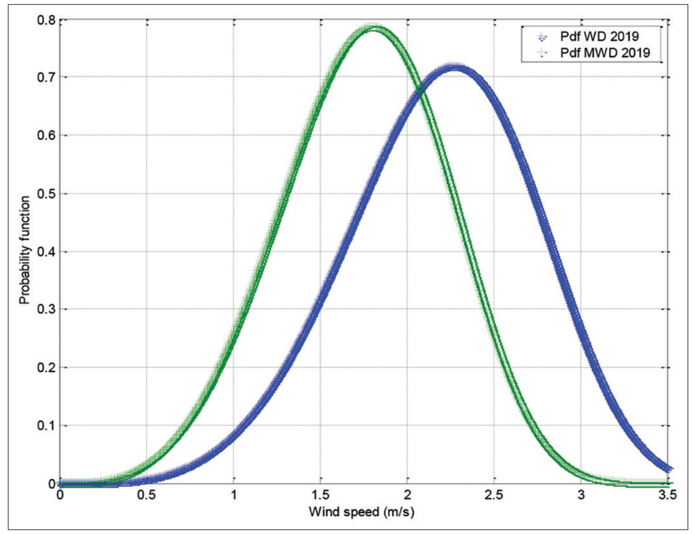


Figure 8: Pdf of data with WD and MWD for 2017

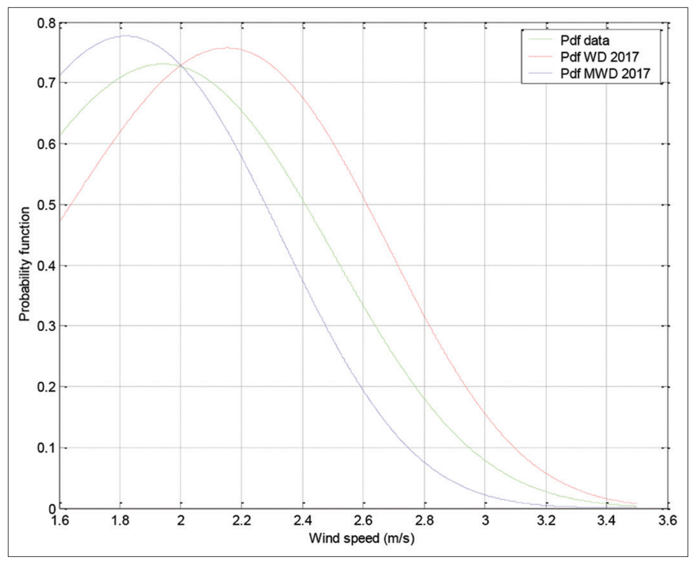


Table 6: Comparison of parameters tree years

Model	Years	α=k	β=b	λ	R^2	RMSE
	2017	5.56	5.37	…	0.9975	0.0503
WD	2018	4.92	5.35	…	0.9839	0.1268
	2019	5.27	5.49	…	0.9945	0.0740
	2017	2.65	3.47	0.57	0.9971	0.0541
MWD	2018	3.76	2.59	0.77	0.9968	0.0566
	2019	1.75	2.61	0.78	0.9969	0.0559

The Pdf comparison between the wind speed data obtained from the Government station and the two observed models is shown in Figure 8-10 for each observation year. The probability distribution function (Pdf) of the data station is colored "magenta" while the Pdf WD is colored "red" and the proposed Pdf MWD is colored "blue". Comparison of the three Pdf characteristics for 3 years (2017-2019), in which the proposed model (MWD) approaches the Pdf characteristics of the data obtained from observations with errors between 1,868 and 2,412, while the WD Pdf model has a fairly large error with errors between 10,168 up to 12,286.

Figure 8: Pdf of data with WD and MWD for 2017

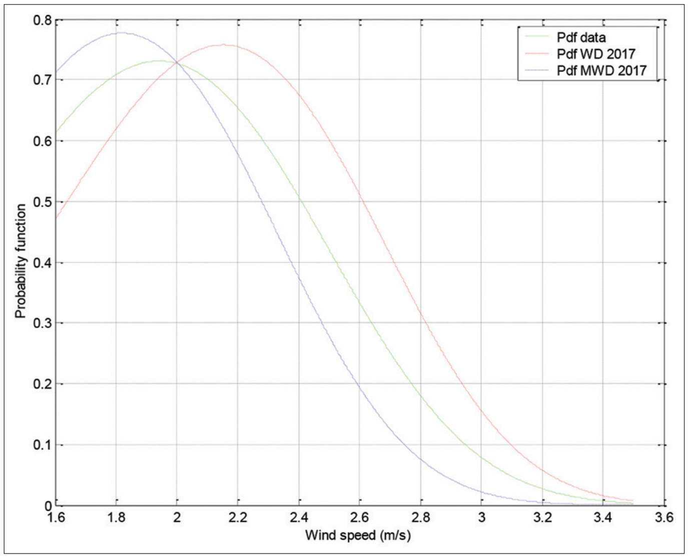


Figure 9: Pdf of data with WD and MWD for 2018

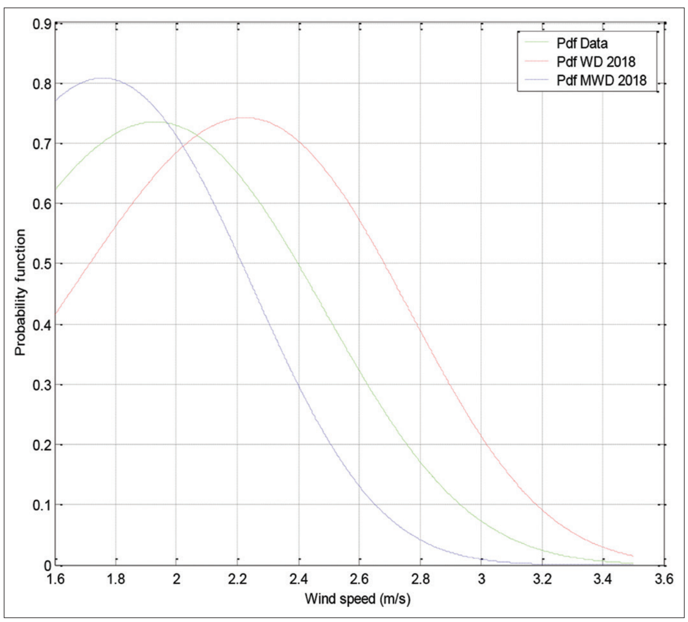
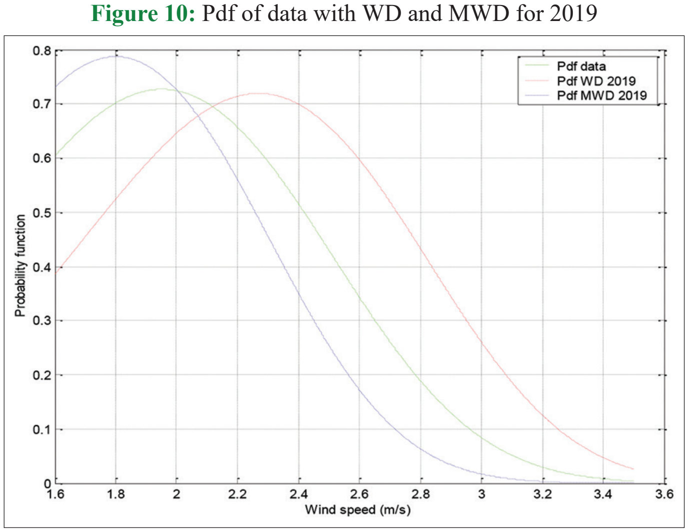


Figure 10: Pdf of data with WD and MWD for 2019



4. CONCLUSION

The results and discussion that has been done can be concluded as follows;

1. Comparison of the WD and MWD models using the evaluation coefficient of determination (R^2) and RMSE in general results that the proposed MWD model is better than WD.
2. The two models of WD and MWD have similar characteristics for Pdf.
3. For low wind speeds, you should use the MWD model because it has better Pdf and Cdf values when compared to the WD model.
4. For wind speeds greater than 2 m/s it is recommended to use the WD model because the Pdf value is better than MWD.

REFERENCES

Akdağ, S.A., Bagiorgas, H.S., Mihalakakou, G. (2010), Use of two-component Weibull mixtures in the analysis of wind speed in the Eastern Mediterranean. Applied Energy, 87(8), 2566-2573.

Akpinar, S., Akpinar, E.K. (2007), Wind energy analysis based on maximum entropy principle (MEP)-type distribution function. Energy Conversion and Management, 48(4), 1140-1149.

Akpinar, S., Akpinar, E.K. (2009), Estimation of wind energy potential using finite mixture distribution models. Energy Conversion and Management, 50(4), 877-884.

Arreyndip, N.A., Joseph, E., David, A. (2016), Wind energy potential assessment of Cameroon's coastal regions for the installation of an onshore wind farm. Heliyon, 2(11), e00187.

Arslan, T., Bulut, Y.M., Yavuz, A.A. (2014), Comparative study of numerical methods for determining Weibull parameters for wind energy potential. Renewable and Sustainable Energy Reviews, 40, 820-825.

Baseer, M.A., Meyer, J.P., Rehman, S., Alam, M.M. (2017), Wind power characteristics of seven data collection sites in Jubail, Saudi Arabia using Weibull parameters. Renewable Energy, 102, 35-49.

Bilir, L., Imir, M., Devrim, Y., Albostan, A. (2015), An investigation on wind energy potential and small scale wind turbine performance at İncek region-Ankara, Turkey. Energy Conversion and Management, 103, 910-923.

Bonatz, D., Miksic, J., Neidel, J.D. (2009), From Distant Tales: Archaeology and Ethnohistory in the Highlands of Sumatra. United Kingdom: Cambridge Scholars Publishing.

Carrasco-Díaz, M., Rivas, D., Orozco-Contreras, M., Sánchez-Montante, O. (2015), An assessment of wind power potential along the coast of Tamaulipas, Northeastern Mexico. Renewable Energy, 78, 295-305.

Chang, T.P. (2011), Estimation of wind energy potential using different probability density functions. Applied Energy, 88(5), 1848-1856.

Chaurasiya, P.K., Ahmed, S., Warudkar, V. (2018), Comparative analysis of weibull parameters for wind data measured from met-mast and remote sensing techniques. Renewable Energy, 115, 1153-1165.

Chaurasiya, P.K., Kumar, V.K., Warudkar, V., Ahmed, S. (2019), Evaluation of wind energy potential and estimation of wind turbine characteristics for two different sites. International Journal of Ambient Energy, 42, 1-11.

Dorvlo, A.S.S. (2002), Estimating wind speed distribution. Energy Conversion and Management, 43(17), 2311-2318.

Gülersoy, T., Çetin, N.S. (2010), Menemen bölgesinde rüzgar türbinleri için Rayleigh ve Weibull dağılımlarının kullanılması. Politeknik Dergisi, 13(3), 209-213.

Indhumathy, D., Seshaiah, C.V, Sukkiramathi, K. (2014), Estimation of weibull parameters for wind speed calculation at kanyakumari in India. International Journal of Innovative Research in Science Engineering and Technology, 3(1), 1-6.

Islam, M.R., Saidur, R., Rahim, N.A. (2011), Assessment of wind energy potentiality at Kudat and Labuan, Malaysia using weibull distribution function. Energy, 36(2), 985-992.

Khahro, S.F., Tabbassum, K., Soomro, A.M., Dong, L., Liao, X. (2014), Evaluation of wind power production prospective and weibull parameter estimation methods for Babaurband, Sindh Pakistan. Energy Conversion and Management, 78, 956-967.

Kiss, P., Jánosi, I.M. (2008), Comprehensive empirical analysis of ERA-40 surface wind speed distribution over Europe. Energy Conversion and Management, 49(8), 2142-2151.

Kollu, R., Rayapudi, S.R., Narasimham, S.V.L., Pakkurthi, K.M. (2012), Mixture probability distribution functions to model wind speed distributions. International Journal of Energy and Environmental Engineering, 3(1), 27-39.

Mahmood, F.H., Resen, A.K., Khamees, A.B. (2020), Wind characteristic analysis based on weibull distribution of Al-Salman site, Iraq. Energy Reports, 6, 79-87.

Mert, I., Karakuş, C. (2015), A statistical analysis of wind speed data using Burr, generalized gamma, and weibull distributions in Antakya, Turkey. Turkish Journal of Electrical Engineering and Computer Sciences, 23(6), 1571-1586.

Mohammed, D., Mou Abdelaziz, A.S., Mohammed, E., Elmostapha, E. (2019), Wind speed data and wind energy potential using weibull distribution in Zagora, Morocco. International Journal of Renewable Energy Development, 8(3), 267-273.

Owusu, P.A., Asumadu-Sarkodie, S. (2016), A review of renewable energy sources, sustainability issues and climate change mitigation. Cogent Engineering, 3(1), 1167990.

Saleh, H., Aly, A.A.E., Abdel-Hady, S. (2012), Assessment of different methods used to estimate weibull distribution parameters for wind speed in Zafarana wind farm, Suez Gulf, Egypt. Energy, 44(1), 710-719.

Shu, Z.R., Li, Q.S., Chan, P.W. (2015), Statistical analysis of wind characteristics and wind energy potential in Hong Kong. Energy Conversion and Management, 101, 644-657.

Singh, S., Bhatti, T.S., Kothari, D.P. (2006), A review of wind-resource-assessment technology. Journal of Energy Engineering, 132(1), 8-14.

Sumair, M., Aized, T., Gardezi, S.A.R., Ur Rehman, S.U., Rehman, S.M.S. (2020), A novel method developed to estimate weibull parameters. Energy Reports, 6, 1715-1733.

Suwarno, Hwai, L.J., Zambak, M.F., Nisja, I. (2016), Assessment of wind energy potential using weibull distribution function as wind power plant in Medan, North Sumatra. International Journal of Simulation-Systems Science and Technology, 17(41), 1-5.

Suwarno, S., Rohana, R. (2021), Wind speed modeling based on measurement data to predict future wind speed with modified Rayleigh model. International Journal of Power Electronics and Drive System, 12(3), 1823-1831.

Suwarno, S., Yusuf, I., Irwanto, M., Hiendro, A. (2021), Analysis of wind speed characteristics using different distribution models in Medan city, Indonesia. International Journal of Power Electronics and Drive Systems, 12(2), 1102-1113.

van der Auwera, L., de Meyer, F., Malet, L.M. (1980), The use of the weibull three-parameter model for estimating mean wind power densities. Journal of Applied Meteorology, 19(7), 819-825.

Wais, P. (2017), Two and three-parameter Weibull distribution in available wind power analysis. Renewable Energy, 103, 15-29.

Werapun, W., Tirawanichakul, Y., Waewsak, J. (2015), Comparative study of five methods to estimate weibull parameters for wind speed on Phangan Island, Thailand. Energy Procedia, 79, 976-981.