

### **Price Dynamics of Crude Oil in the Short and Long Term**

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#### ABSTRACT

The drop in the price of crude oil in 2014 left no one indifferent, and motivated several researchers to analyse the nature of the relationship between the physical and the financial market of this commodity. This article discusses the issue of *Spot* and *Futures* price dynamics of crude oil in the short and long term. The originality of the present work is that we will proceed to a double analysis of the stochastic processes of the two variables *Spot* and *Futures* - with and without break - for the period from February 2015 to December 2017, using the models VAR and VECM. This last allows to understand short-term price dynamics while the VAR makes it possible to understand it in the long term. The results obtained estimate that there is a bidirectional causality between the two variables, and that their long-term dynamics undergo the same changes together. The *Spot* and *Futures* prices hold the fall of their prices on the same date of January 2016 (common break date for the 2 series), which distinguishes between two sub-periods and consequently two distinct regimes: "Tension" and "return to the average" regime. In the short term, the first one cannot reject the theory of normal backwardation, while the storage theory reasonably explains the return to the path of the equilibrium of the prices of the oil markets.

Keywords: Spot and Futures Markets, Cointegration, Autoregressive Vector Model, Vector Error Correction Model JEL Classifications: C22, G15, Q41

### **1. INTRODUCTION**

Despite the various analyses that have already been made of the hypothesis of the non-stationarity of oil prices (*Spot* and *Futures*), this hypothesis still needs to be verified. The stochastic nature of this commodity, given the way it has been generated over time, has consequences on its long-term properties, and also on the quality of forecasts. Hence the importance of recalling the importance of the study of non-stationarity.

In fact, if the tests confirm the stationarity of the price series, this means that the *Spot* and the *Futures* prices are likely to absorb the economic shocks, and that there is a long-term trend, which can be interpreted as an equilibrium path, where this price is constantly coming back. In this situation, the possible forecasts are different from those of a stationary integrated price series of order greater than zero. The econometric theory in this case suggests that there is a cointegrating relationship.

Theoretically this question has been put on the forefront of the concerns of several authors. According to Keynes, in a situation

of equilibrium between supply and demand, the commodity market is characterized by a *Futures* price that is below the *Spot* price. The observation of the prices of these commodities refers to two different situations: The first is called "Contango," when *Futures* prices are higher than the *Spot* price, in this case there is need for storage. The second is that "Backwardation," where the *Spot* price exceeds the *Futures* price, and in this case there is the obligation of destocking.

Two explanations have been developed for a theoretical assessment of these two situations. The first is storage theory, which comes down to a blueprint of the relationship between *Spot* and *Futures* markets. In this context, the actors determine the quantity of oil they will consume today (they set the *Spot* price level) in relation to the consumption of the coming days (*Futures* prices), which explains the Contango situation. The second explanation refers to the theory of normal backwardation proposed by Keynes and Hicks, it explains in general the situation where the *Futures* price is lower than the *Spot* price (Backwardation).

Our goal in this article is twofold:

- First, to understand the price dynamics (*Spot* and *Futures*) of crude oil, and to show the importance of the deterministic components, in particular the breaks.
- Next, analyse the operation of these markets and identify the relationship between *Spot* and *Futures* prices.

To achieve these goals, an "autoregressive vector model" (VAR) on daily data is used. First of all, we will test the non-stationarity of the crude oil series, and we will make a long term estimate without taking into account the breaks. Then, to test the non-stationarity of the series taking into account the existence of the structural changes in the deterministic component, while applying a "vector error correction model" (VECM) modelling on the residuals of the structural model to take into account the interaction between the *Futures* and *Spot* prices.

The choice of such econometric models is original insofar as the VAR model makes it possible to isolate the different shocks affecting the oil price in the long term, and the VECM makes it possible to take into account the short-term adjustments. They show how the latest cointegration techniques are useful including endogenous structural breaks leading to the distinction between two regimes.

The use of such econometric techniques suggests the proposition of some fundamental assumptions as to their applications. Global oil production has no impact on short-term price formation. This assumption can be justified by the persistence of major adjustment costs of short-term oil production, and the difficulty of evaluating real-time oil market information (market efficiency). The second hypothesis is related to the shock itself. The break is supposed to be independent in relation to the formation of prices on the oil markets, and it has no impact on the production of global oil. These two assumptions are therefore acceptable while taking into account the nature (maturity) and frequency (daily) of the data used.

The architecture of this work takes into account two aspects: Theoretical and empirical. The first Section provides an overview of traditional theories to explain the nature of the relationship between the two prices, while recalling the main definitions of the concepts used. In the second section, a presentation and descriptive analysis of the data used will be carried out, as well as a brief presentation of the theoretical formulations of the chosen econometric tests. In a first step, after having presented the data, a graphical analysis of the relation between the variables object of the study (*Spot* and *Futures*) will be made for the period 2006–2017 in general, and in a second step for the period 2015–2017 in particular. An application and presentation of the estimates as well as the results of the study will be conceived in the third section. And finally, in the fourth and final section, an economic analysis of the crude price dynamics will be made.

### 2. PETROLUM PRODUCTS MARKETS

To understand the nature of the relationship between the physical and financial market, two fundamental theories serve to give explanations to understand this relationship, that of normal offset and storage; according to the situations described by the context of the relationship itself. The first paragraph will deal with some definitions useful for framing the existing relationship between the prices of the petroleum market. The second one, will be focused on the theoretical foundations explaining the nature of this relationship between the prices.

#### 2.1. Markets, Contracts and Prices of Petroleum Product

As long as we are going to work on the nature of the relationship between oil prices, it is useful to define certain terms of use relating to the oil market. There are two types of markets on which black gold can be traded, namely: The financial market and the physical market.

According to J. Hull, the financial market is an organized futures market on which the exchanges between the offerers and the applicants are on paper only. The purpose of the exchanges through this market was essentially to protect them against possible price fluctuations. As such, several hedging instruments are exchanged, such as futures, options and swaps. We will be interested in the present work only in futures.

Futures contracts are concluded between the offerers and the applicants of the rough in the financial market with the help of an intermediary, at a price known as "Futures Price" or "Forward Price" agreed in advance, and over a period specified in advance. They correspond to future purchase or sale intentions at an instant price. Being linked to a term, this price systematically evokes the notion of time. It is thus the cash price that will have to be paid in the future, fixed before the date of the transaction, based on data available at a given moment.

Depending on the maturity of the contract, this period is normally set by the market authorities during the delivery month. At this time, sellers have the right to choose the delivery date that suits them throughout this month.

This type of contract can be untie in 2 ways

- Either delivery or physical receipt of the underlying, respecting the conditions set in advance;
- Or, if we do not want physical delivery, we can untie the contract by taking a position opposite to what we hold. That is to say if I am a buyer I end up take a position of seller of my future contract and vice versa. Which is called compensation.

As J. Hull had made it clear, it is because of the existence of a clearing house on the organized markets that the proper settlement of transactions on the financial market is guaranteed.

That chamber of compensation is generally composed of financial institutions. For any transaction to be made in good and due form by intermediaries who are not members, it must obligatorily go through these member institutions. It deals with the registration and management of all transactions, the maintenance of members' accounts, and also makes margin calls to them.

Contrary to the financial market, the physical market is an unorganized market, where physical exchanges occur either immediately or with delay, by mutual agreement and at auction, against a price that depends on the type of contract. The latter exists in two forms on this market: "*Spot*" cash contract and *Forward* contract.

The *Spot* contract is a contract exchanged between offerers and applicants directly without intermediary, against the current market price called "*Spot*." The latter represents the price at the time of taking the position, and thus serves to give an idea on a regular basis, on the market situation at time t. This gives cash transactions a "unique" and "random" character.

On the other hand, the Forward contract is concluded as a result of physical market transactions also, over-the-counter, against a price fixed at time t, and whose delivery of the underlying is deferred on a short-term maturity. This is a contract that has characteristics similar to those of a "*Futures*" contract, because the price is fixed in advance but the delivery is scheduled for a futur date, and those of a contract "*Spot*," since it is traded on an over-the-counter market.

In the present work, we will focus only on the dynamics of the formation of *Spot* and *Futures* prices.

## **2.2.** Theoretical Contributions of the Relation between the *Spot* and the *Futures* Prices

The purpose of this section is to expose the traditional theories that explain the relationship between the *Spot* and the *Futures* Prices. In other words, we explain the link between the physical *Spot* market and the financial market. First, we will analyse the relationship between prices through two complementary theories: Storage theory and normal backwardation theory. Then we present an extension of these two long-term theories. Finally, we carry out an economic analysis of this relationship taken from a long-term perspective.

## 2.2.1. Traditional theories of the relationship between Spot and Futures prices

The *Futures* price may be either lower or higher than the *Spot* price. In the first case, the situation is described in the context of the "Backwardation". In the second one, when the *Futures* price is higher than the *Spot* prices the situation is called "Contango."

In addition to these two situations used to explain the relationship between the *Spot* price and the *Futures* price, it is customary to also look at two complementary classical theories: That of storage and that of normal backwardation. It is therefore the two basic traditional theories, whose foundations were formulated between 1930 and 1958, which serve to clarify theoretically this relationship as best as possible.

The theory of normal backwardation explains this relationship from the equilibrium of the hedging positions of traders in the *Futures* market. It analyses the difference between the *Spot* price and the *Futures* price, based on the anticipations and the arbitrations of the *Futures* market applicants (Backwardation). As for the storage theory, it analyses the difference between *Futures* and *Spot* prices based on the study of the reasons for holding stocks of operators (offerers), (Cantango).

On the oil market, several studies have been made on the situations of Backwardation and Cantango, which led to the conclusion that Backwardation situations were more frequent. According to Keynes<sup>1</sup>, in normal conditions, the commodity market is characterized by a *Futures* price that is below the *Spot* price. An anticipation on the *Spot* market systematically leads to a situation of backwardation on both anticipation and observed bases. To keep a base lower or equal to zero on the *Futures* market can only be the result of the existence of a negative anticipated base. In addition, a study that was done by Energy Security Analysis in 1996 (Krapels, 1996), states that on all US oil markets there is increased demand for short hedging, and that the Gross *Futures* contract has become a producer contract which means that at the end consumers are not very present which converges towards a situation of imbalance in the contracts.

In sum, these are the reasons that allow us to explain why we use in a petroleum market the theory of normal backwardation to explain this relationship between *Spot* and *Futures* price: The situations of backwardation on the bases observed are frequent and the short coverage dominates.

#### 2.2.2. Long-term extension of the analysis

At the time when traditional theories explaining the relationship between commodity prices were developed, *Futures* transactions did not exceed 1 year. This explains that the relationship between the *Spot* and the *Futures* price was based essentially on the short-term price.

According to Keynes1 analysis, when there is a simultaneous presence of storage and normal backwardation along a price curve in the market, this is essentially due to an excess of supply or the demand for *Futures* contracts for different maturities. In a situation of imbalance, speculators intervene by taking positions in the *Futures* market to compensate for the position of other speculators. A risk premium is offered to them in return.

The storage theory takes into account the existence of both backwardation and cantango in a *Futures* situation, given by the inequalities of supply and demand of a product that may exist. Thus, this theory can be considered in inter-temporal relationship with the price. She will not be reliable if the expiry of a contract relating to one or more production cycles is exceeded. This is the motivation to broaden the analysis and spread it over a slightly longer horizon to see if there are other factors that influence the long-term price structure of prices.

In this case, it is necessary to leave the rigid Keynesian model in order to widen the horizon of work for the case of the theory of normal backwardation. On the other hand, this is not going to be easy for the storage theory, since the explanatory factors can be changed in the long term.

### 3. DATA, DESCRIPTIVE ANALYSIS AND METHODOLOGY

#### 3.1. Data and Descriptive Analysis

The study of short and long-term interactions between crude oil prices focuses on *Futures* and *Spot* prices. Different short-term (thirty consecutive maturities) and long-term maturities (36 to 48 months) are offered for *Futures* contract of crude oil. A daily data frequency was used, from January 2006 to December 2017

representing 3077 observations, and taking into account working days (5 days in the week). There are several sources of data available (Bloomberg, Platts, EIA, Quandl, etc.). The database used is extracted from the database of the EIA agency. It is an independent statistical agency within the United States Department of Energy. *Spot* and *Futures* prices are expressed in dollars per barrel.

## **3.2 Descriptive Analysis of the Relationship between** *Spot* and *Futures* Price

Fluctuations in crude oil prices during the 2000s attracted the interest of many researchers and led them to seek an underlying explanation for these changes. From the Figure 1, crude oil prices observed in early 2006 have increased significantly; then in autumn 2008 a remarkable collapse. The interest of most of these researchers has been rekindled in the analysis of the determinants of *Spot* and *Futures* prices traded on the market.

The analysis of the Figure 1 shows the presence of three regular phases: A period when oil prices remain relatively stable, a period when prices are falling and another one where they are rising.

The analysis of the first phase, in particular the period from February 2011 to August 2014 and the period from January 2015 to December 2017, indicates that oil prices remained relatively stable. During the first period (Figure 2), the average price persisted at \$ 110.51 per barrel for the *Spot* price and \$ 110.45 for the *Futures* price. The deviations from the average are relatively similar for both prices (including \$ 6,126 for the *Spot* price and \$ 5,859 for the *Futures* price).

The Box plot analysis (Figure 2) suggests the presence of two equality distributions, spread to the right not exceeding the normal law (with reference to Kurstosis and Assymetry coefficients) for both prices. Similarly, the Figure 3 also shows a near-stagnation observed during the period January 2015- December 2017. This period saw an average price of \$ 50,021 for the *Spot* price and \$ 51,130 for the *Futures* price. The cash market (\$ 8.16) versus (\$ 7.94) in the *Futures* market. The shape of these two distributions takes an asymmetrical form on the left and does not exceed the normal law.

The second phase is where prices on oil markets are rising. These include the periods from January 2007 to July 2008, and from June 2008 to October 2011. This is the phase where prices fallen from \$ 49.95 to \$ 143.95 for the *Spot* price, and from 51 to \$ 70 to \$ 146.08 for the *Futures* prices. Standard deviations are above the overall average over the entire period. Price distributions are asymmetrical and do not exceed the normal distribution (Figure 3).

The third phase is located between the first two, where prices are falling sharply. From 2006 to 2017, oil prices fell critically in two distinct periods (July to December 2008 and July 2014 to January 2016). The descriptive analysis of these two periods provides information on the existence of significant differences in *Spot* and *Futures* prices. Although the dramatic drop in oil prices has occurred twice since the beginning of the 2000s, the relative extent to the first half-year (2008) is minimal (5 successive months). On the other hand, it is much more important with regard to the second sub-period (20 months not successive). This period is

Figure 1: Evolution of crude oil *Spot* and *Futures* prices from January 2006 to January 2017

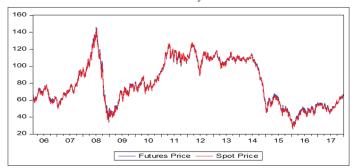


Figure 2: Stagnation of oil prices: February 2011–August 2014

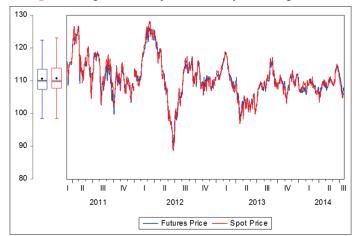
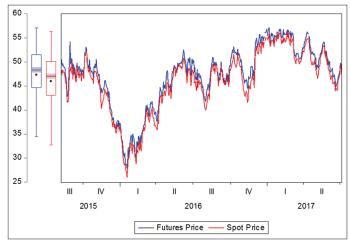


Figure 3: Stagnation of oil prices: January 2015–December 2017



characterized by a recovery (6 months) upwards, then a continuous collapse of prices.

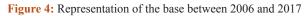
In order to better understand the evolution of the base, the Figure 4 condenses several conclusions for the collapse of prices observed on the oil markets. Figure 4 gives an overview of the difference observed between the *Futures* and the *Spot* price, it indicates that the base is mostly positive (Backwardation situation) over the entire period. The differences reach their maximum after the periods of the crises, this is the case after the fall of 2008 and that of 2014. The situation of Cantango is

observed mainly with significant differences during the year 2011. Several oil projects emerged during this period when demand was weak.

The choice of the study period was made on the basis of two complementary graphical analyses: The first concerning the evolution of the price series, and another based on the difference in crude oil prices observed.

Reading the first Figure (Figure 5), on the evolution of *Futures* and *Spot* prices during the period (from 02/02/2015 to 29/12/2017), shows that volatility of prices seems very intense. It can be assumed that these are consequences of the changes observed in the physical market through *Spot* prices. An important note from the Figure analysis is that the crude oil price series fell on 20/01/2016 to reaching respectively the value of \$ 26.01 on the *Spot* market, and \$ 27.88 on the *Futures* market. The curves of the two series had distinctly changed between the fourth quarter of 2015 and the second quarter of 2016, with 20/01/2016 as a critical date.

The second Figure (Figure 6) puts the difference between the two prices studied. This representation shows that the difference between the *Futures* and the *Spot* price is positive for most of the period, which is mean that *Futures* prices exceed *Spot* prices (Backwardation). At the end of the period (from Q3 2017), the Figure shows that *Spot* prices exceed those of paper prices (Cantango).



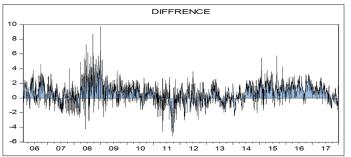
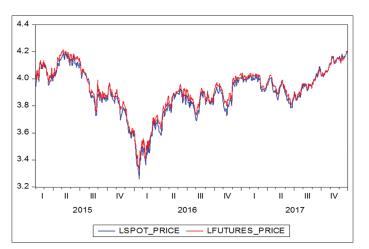


Figure 5: Graphical representation of the evolution of *Spot* and *Futures* prices



# **3.3. Methodological Approach and Econometric Techniques**

The search for answers to the initial problem requires, the use of standard econometric techniques for the identification of the generating process of the data. In this context, it is essential to test the non-stationarity of the series before making a short and long-term analysis of the dynamics of the existing relationship between the oil price series.

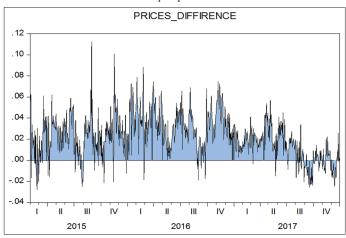
Taking into account the importance of deterministic components, particularly breaks, requires the use of non-stationarity tests more adapted to this type of situation. In fact, these are tests of non-stationarity with breaks and cointegration for a more adequate analysis of the dynamics of the relationship between the *Spot* and *Futures* series. The purpose of this methodological paragraph is to give a brief overview of these methods before their application in the paragraph which will be devoted to them.

The classical econometric literature (Gourieroux and Monfort, 1997) proposes different tests of nonstationarity (null hypothesis), such as the ADF test (Dickey-Fuller Augmented), and the PP test (Philipps and Perron). These so-called traditional tests not only make it possible to clarify the stationary nature or not of a time series through the determination of a trend (unit root test), but also to specify the correct way to stationise the series.

The ADF test is a generalization of the DF test, consisting of the introduction of the additional variables  $\Delta X_{t-p}$ , with the additif hypothesis that the autocorrelation of the residues is due to the absence of differentiated delayed variables in the equation. The test is based on the regression of three basic models: M1 model with deterministic and constant trend, M2 model without trend and with constant and M3 model without trend and without constant.

The regression is performed through the application of the OLS method, if the null hypothesis is kept in one of these three models, the process is then non-stationary.

The test is written as follows:  $H_0: 9=0$  versus  $H_1: 9<0$ , and its formulation is as follows:



## Figure 6: Graphical representation of the difference between *Futures* and *Spot* prices

Model M1: 
$$\Delta X_t = m_t + b + \vartheta X_{t-1} + \sum_{j=1}^{p} \psi_j \Delta X_{t-j} + u_t$$
 (1)

Model M2: 
$$\Delta X_t = b + \vartheta X_{t-1} + \sum_{j=1}^p \psi_j \Delta X_{t-j} + u_t$$
 (2)

Model M3: 
$$\Delta X_t = b + \vartheta X_{t-1} + \sum_{j=1}^p \psi_j \Delta X_{t-j} + u_t$$
 (3)

Unlike the ADF test (performed by the OLS), the PP test proposes a non-parametric method for the resolution of the three models following the same steps. The strategy of both tests is to test the meaning of the trend parameters in the model M3. If this test is rejected, we pass to the test of model M2 to test the meaning of the constant. And finally, we apply the M1 model to test the meaning of the unit root from the McKinnon critical value statistic.

The Granger causality test (Granger, 1969) provides a rather detailed framework for studying the direction of causality (unidirectional or bidirectional) between two variables (Bourbonnais, 2015).

The model VAR(p) for which the variables  $Y_i$  and  $X_i$  are stationary and  $\varepsilon_{1i}$ ,  $\varepsilon_{2i}$  white noises. The test can be written as follows (for any *i*):

$$Y_{t} = \alpha_{1} + \sum_{i=1}^{p} \beta_{1i} Y_{t-i} + \sum_{i=1}^{p} \gamma_{1i} X_{t-i} + \varepsilon_{1t}$$
(4)

-  $H_0: X_t$  do not cause  $Y_t, \gamma_{1i} = 0$ 

$$X_{t} = \alpha_{2} + \sum_{i=1}^{p} \beta_{2i} Y_{t-i} + \sum_{i=1}^{p} \gamma_{2i} X_{t-i} + \varepsilon_{2t}$$
(5)

#### $H_0$ : $Y_t$ do not cause $X_t \beta_{1i} = 0$

To proceed with the Granger causality test, the series must be stationary, and the number of the lags p must be determined precisely. The choice of the number of lags (mlag) of VAR is performed on the basis of the Akaik criterion (AIC), that of Schwarz (SC), the Likelihood Ration (LR) and that of Hannan-Quinn (HQ).

Introduced by (Granger, 1969), (Granger and Weiss, 1983) then (Engle and Granger, 1987), the concept of cointegration makes it possible to explain the reality and the nature of the divergences between two series theoretically related to each other.

When the series are integrated in the same order, it is possible to consider a linear combination of these series. The cointegration test of (Johansen, 1988) makes it possible to "determine the number of cointegration relationships from two stages" (Pollock, 1999): The computation of two residues, then the calculation of the matrix allowing the computation of the eigenvalues of a matrix. The Johansen model considers a Y<sub>t</sub> process in representation VAR of order p (if p = I):  $Y_t = A_I Y_{t-I} + \varepsilon_t$ 

The basic equation is written in the following form:

$$\Delta Y_t = A_0 + B_1 \Delta Y_{t-1} + B_2 \Delta Y_{t-2} + \dots + B_{p-1} \Delta Y_{t-p+1} + \pi Y_{t-1} + \varepsilon_t$$
(6)

Where matrices  $B_i$  are functions of matrices  $A_i$  and  $\pi = \sum_{i=1}^{p} A_i - I$ .

The matrix  $\pi$  can be written in the form  $\pi = \alpha\beta'$ . Where  $\alpha$  is the restoring force towards equilibrium and  $\beta$  contains the cointegration relations *r*.

This test developed by Johansen (called the test of the trace) is based on the eigen vectors corresponding to the highest eigenvalues of the matrix  $\pi$ . We will only present here the test of the trace. From the eigenvalues of the matrix  $\pi$ , the statistic is constructed:

$$\lambda_{trace} = -n \sum_{i=r+1}^{k} Ln(1 - \lambda_i)$$
<sup>(7)</sup>

With *n*, the number of observations of the *VAR*,  $\lambda_i$  the i<sup>th</sup> eigenvalue of the matrix  $\pi$ , *k* the number of variables and *r*, the rank of the matrix. This statistic follows a law of probability similar to a  $\chi^2$  (table of Johansen and Juselius, 1990).

Several cases may appear according to the following alternative hypotheses:

The first test (test of the hypothesis: No relation of cointegration versus at least one relation): The rank of the matrix is equal to 0 (r = 0), i.e.,  $H_0$ : r=0 and  $H_1$ : r>0; if we reject  $H_0$  we execute the following test;

The second test (test of the hypothesis: A cointegration relation versus at least two relations): The rank of the matrix  $\pi$  is equal to 1 (r = 1), i.e.,  $H_0$ : r=1 and  $H_1$ : r>1; if we reject  $H_0$  we execute the following test;

The third test: The rank of the matrix  $\pi$  is equal to 2 (r = 2), i.e.,  $H_0$ : r=2 and  $H_1$ : r>2; the procedure stops when  $H_0$  is accepted.

And so on until the last step (if it is necessary), whose maximum eigenvalue is based on the following statistic:

$$\lambda_{max} = -nLog(1 - \lambda_{r+1}) \quad \text{with} \qquad r = 0, 1, 2.... \tag{8}$$

The existence of a cointegrated system, according to Granger's representation theorem, implies the presence of an error correction mechanism (Bourbonnais, 2015) which reduces the deviations from the long-term equilibrium. The presence of a cointegration relationship allows the estimation of a vector error correction model (VECM). This model has ability to correct any imbalance that could impact the system from one period to another.

The error-correction model can be written as follows:

$$\Delta x_{t} = \alpha_{l} z_{t-l} + lagged(\Delta x_{t} \Delta y_{l}) + \varepsilon_{lt}$$
(9)

$$\Delta y_{t} = \alpha_{z_{t-1}} + lagged(\Delta x_{t} \Delta y_{t}) + \varepsilon_{z_{t}}$$
(10)

 $z_{t-1}$  Represents the error-correcting term derived from estimating the cointegration relationship.  $\varepsilon$  is the stationary error term  $|\alpha_1|+|\alpha_2|\neq 0. lagged(\Delta x_t \Delta y_t)$  represents the variables  $x_t$  et  $y_t$  lagged. Determining the lagging number of the VAR model is an important step in our empirical study. For this, a model VAR of order (p) is chosen if the criteria AIC (Akaike information criterion) and SC (Schwarz criterion) display a minimum values for this lags p.

The error-correction model therefore makes it possible to model jointly short-term (represented by the differentiated variables) and long-term (represented by the level variables) dynamics.

The short-term dynamic is written as follows:

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \alpha_2 x_t + \alpha_3 x_{t-1} + \varepsilon_t \tag{11}$$

The long-term dynamic is written as follows:

$$y_t = b + ax_t + \varepsilon_t \tag{12}$$

The error-correction model is obtained from short-term dynamics:

$$\Delta y_t = \gamma \Delta x_t + \delta(y_{t-1} - a_3 x_{t-1} - b) \tag{13}$$

with

$$\gamma = \alpha_2$$
 (14)

$$\delta = -(1 - \alpha_p) \tag{15}$$

$$a = \frac{\alpha_2 + \alpha_3}{1 - \alpha_1} \tag{16}$$

$$b = \frac{\alpha_0}{1 - \alpha_1} \tag{17}$$

Estimations of error-correcting models are used to analyse the parameter of the error-correction term, the dependence of the variables on the other lagged variables, and the quality of the model estimate ( $R^2$  is Fisher''s statistic).

The consideration of structural breaks has mainly been discussed in the context of univariate autoregressive time series with a unit root, in line with the work of (Perron, 1989).

A thick rupture or period's rupture is defined as any non-structural rupture involving two distinct events: The first is said to be of the additive (AO "additive outlier") or the progressive (IO "innovational outlier"), followed by a break of opposite direction, not necessarily of the same type. This type of correction can be total or partial. The break may keep the previous trend or be followed by a change in the trend. In this last case, there is both a thick and a structural break. For us, the two events are attached, they participate in one and the same phenomenon and therefore constitute only one (thick) break.

The methodology of (Papell and Prodan, 2004, 2006) proposes to test the existence of such breaks, provided to verify that the

difference between the two breaks is not too important. But, it must not be forgotten that their test is too restrictive and that the absence of rejection of the null hypothesis of nonstationarity can be due to the configuration of the rupture period. Another test must be used.

The proposed test uses the test of the detection of the rupture period with the method of (Bai and Perron, 1998) which consists in proposing three distinct models: IO1 (innovational outlier Type 1); IO2 (innovational outlier Type 2); and AO (additive outlier).

The IO1 model includes a structural change in the constant only, the change occurs gradually:

$$y_{t} = \mu + \theta DMU_{t} + \beta trend_{t} + \delta D(TB)_{t} + ayt - l + b^{l} dy_{t-l} + \dots + b^{k} dy_{t-k} + \varepsilon_{t}$$
(18)

$$y_{t} = \mu + \theta DMU_{t} + \beta trend_{t} + \gamma DT_{t} + \delta D(TB)_{t} + \alpha y_{t-1} + b^{1} dy_{t-1} + \dots + b^{k} dy_{t-k} + \varepsilon_{t}$$
(19)

The IO2 model includes a gradual change in both slope and constant.

The AO model introduces a sudden change in the slope. It is determined in two stages.

$$y_t = \mu + \theta DMU_t + \beta trend_t + \gamma DT_t^* + \overline{Y_t}$$

With

$$DT_t^* = 1(t > T_b)(t - T_b)$$
(20)

$$\overline{Y_t} = \alpha \overline{Y_{t-1}} + b^1 d\overline{Y_{t-1}} + \dots + b^k d\overline{Y_{t-k}} + \varepsilon_t$$
(21)

Where: D(TB): (TB = date of the structural break) = 1 if t = TB+1and 0 elsewhere

*Trend*: is a linear trend DMU = 1 if t > TB and 0 elsewhere DT = t if t > TB and 0 elsewhere DTS = trend - TB if t > TB and 0 elsewhere.

#### 4. RESULTS OF ESTIMATES

By having series (752 observations) of prices (*Spot* and *Futures*), the objective of this section is to apply the different tests previously developed in the methodological section. In the first, the long-term analysis of price dynamics will be carried out by applying the traditional tests of non-stationarity (without breaks). The non-stationarity tests with break will be applied to account for the persistence of shocks in the long-term dynamics of the price series. And, the analysis of short-term dynamics will be done using the application of the vector error-correction model.

### 4.1. LT Dynamics Analysis: VAR Estimation

#### 4.1.1. Traditional analyses

The non-stationarity study for the two-price series, was made by observing the respective daily series plot of the logarithm of *Spot* and *Futures* prices from the period 02/02/2015 to 29/12/2017. The

Figures analysis shows that the series looks like financial series, they are not stationary. As for the respective correlograms, they have a very particular structure; we note that autocorrelations are all significantly different from zero. This structure is therefore similar to that of a non-stationary series.

We will therefore check using the Augmented Dickey-Fuller test and the Phillips-Perron test the non-stationnarity of the series studied (Table 1).

The results demonstrate that the process is non-stationary in level for both price series. The series have a unit root, they are represented by a DS process: i.e., The coefficients are significant only for the model without tendency and without constant. The method of stationarization is the first differences. The application of the ADF and PP tests results in the stationarity of the two-time series retained after differentiation. The series are all integrated of order 1: I(1). Indeed:

- The *Futures* price series (logarithm) from 02/02/2015 to 29/12/2017 is integrated of order 1. That is to say, stationary after a simple differentiation: *Futures* Price  $\sim I(1)$ :  $\Delta$ (Future Price)  $\sim I(0)$ .
- The *Spot* price series (logarithm) from 02/02/2015 to 29/12/2017 is integrated of order 1. That is to say stationary after a simple differentiation: *Spot* Price  $\sim I(1)$ :  $\Delta(Spot$  Price)  $\sim I(0)$  (Figure 7).

The next point will take into account this type of unit root (DS), by introducing the concept of cointegration. (Hendry, 2010) suggests that several types of time series with a stochastic trend may have joint paths that verify a stable (cointegration) relationship in the long term.

But before carrying out these analyses, it is essential to examine the sign and meaning of the causal relationship between the two vectors from 02/02/2015 to 29/12/2017.

In Granger's sense, the causality tests show that, the F-statistic probabilities for each delay corresponding to the first null hypothesis (the *Spot* does not cause the *Futures* price series), all have (for both lags) a probability lower than the critical threshold. So, we can conclude, in this case, that we can accept H1: The *Spot* price causes the series of *Futures* price (Table 2).

The second Granger causality test for the null hypothesis: The *Futures* price does not cause the *Spot* price, all have a zero probability with relatively high F-stat coefficients from the theoretical value, which suggests to accept  $H_1$ : The *Futures* price causes the *Spot* price.

With reference to the causality tests, and for various lags based on Schwarz criteria, there is a two-way causal link between the two variables. The null hypothesis is rejected at the 1% significance level. In the Granger sense, it is interesting to predict the *Futures* price by taking into account the Spot price, and to predict the *Spot* price taking into account the *Futures* price.

To find the sign of causality between these two variables, we apply the following VAR model:

#### Table 1: Results of the ADF and PP non-stationarity tests

				v	
Tests	Level		Difference		
(ADF &	Spot	<b>Futures</b>	Spot	Futures	
PP)					
ADF test					
M1	4.62E-06	4.66E-06	4.15E-06	4.82E-06	
	(1.144)	(1.143)	(1.035)	(1.193)	
M2	0.028	0.034	0.0003	0.0002	
	(1.479)	(1.656)	(0.302)	(0.247)	
M3	7.21E-05	5.21E-05	-0.968***	-1.091***	
	(0.324)	(0.232)	(-26.606)	(-30.112)	
PP test					
M1	4.62E-06	4.66E-06	4.15E-06	4.82E-06	
	(1.144)	(1.143)	(1.035)	(1.193)	
M2	0.028	0.034*	0.0002	0.0002	
	(1.479)	(1.656)	(0.302)	(0.247)	
M3	7.21E-05	5.21E-05	-0.968***	-1.091 ***	
	(0.316)	(0.257)	(-26.603)	(-30.099)	

M1: Model with trend and constant, M2: Model without trend and with constant, M3: Model without trend and without constant. - Values in brackets are Student's t-values at the 5% threshold for models 1 and 2 and the critical value for the DF statistic according to the Mackinnon table at the 5% threshold. For the increased Dickey-Fuller test statistic, the determination of the McKinnon's critical value is made by deducing the number of delays that affect the series (the number of delays is determined by looking at the correlogram of the series) in first difference (study of stationarity in level) and in second difference (study of stationarity in first difference). For the Phillips-Perron test statistic, the number of lags is determined automatically according to the Newey-West approach made by Bartlett kernel. The \*\*\*\*\*indices, indicate respectively the level of significance at the 10%, 5% and 1%

#### Table 2: Tests of causality

Null hypothesiss	<i>Spot</i> do not cause <i>Futures</i> prices		<i>Futures</i> do not cause <i>Spot</i> prices		
	<b>F-Statistic</b>	Probability	<b>F-Statistic</b>	Probability	
Lag 1	9.141***	0.003	46.88***	0.000	
Lag 2	5.543***	0.004	34.53***	0.000	

Lag 1 and 2: Represente the lags chosen for the causality test between "DSPOT\_PRICE" and "DFUTURES\_PRICE" according to the criteria of Akaike (AIC), Schwarz (SC), Likelihood Ratio (LR) and Hannan Quinn (HQ). The indices \*\*\*\*\*respectively indicate the level of significance at the ddl of 10%, 5% and 1%

H<sub>0</sub>: The Futures price does not cause the Spot price

$$DPSpot_{t} = \alpha_{1} + \sum_{i=1}^{p} \beta_{1t} DSpot_{t-i} + \sum_{i=1}^{p} \gamma_{1i} DPFuture_{t-i} + \varepsilon_{1t}$$
(22)

H<sub>1</sub>: The Spot price does not cause the Futures price

$$DPFutures_{t} = \alpha_{2} + \sum_{i=1}^{p} \beta_{2i} DSpot_{t-i} + \sum_{i=1}^{p} \gamma_{1i} PFuture_{t-i} + \varepsilon_{2i}$$
(23)

The sign of the causality is determined directly by the calculation of the sum of the coefficients  $\gamma_{1i}$  and  $\beta_{2i}$  respectively associated to the lags of the *Spot* price and the *Futures* price in the following VAR system equation: +0.17-0.03+0.46+0.26=+0.86

It is therefore concluded that the causality between the *Spot* and the *Futures* price is positive (go in the same direction) for the period from 02/02/2015 to 29/12/2017.

Defining a cointegration relationship makes it possible to standardize long-term dynamic modeling. In other words, the

combinations of the non-stationary series (*Spot* and *Futures*) can be contracted in the long term in a stationary manner, which will give statistically robust results.

In our case, the tests are performed in the model VAR(p) where p represents the two variables retained the model is the following one:

$$\begin{pmatrix} Spot_t \\ Futures_t \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} Spot_{t-1} \\ Futures_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$
(24)

In reference to (Johansen, 1988) and (Johansen and Juselius, 1990), the results of trace tests and the maximum eigenvalue tests are presented in the Table 3. For the trace test, we test the hypothesis of non-cointegration (r=0), then we test the hypothesis of a cointegration relationship (r=1), and so on. All these hypotheses are tested versus the alternative hypothesis that the matrix is of full rank

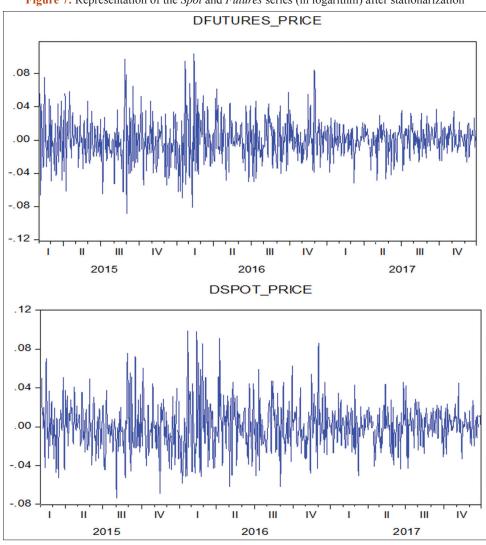
(that is to say that all the series are stationary). The tests determine the rank of the matrix and the number of cointegration relationships.

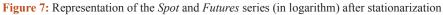
Johansen's cointegration tests reveal the existence of a cointegration relationship between the two variables, since the null hypothesis is rejected for all series. However, the number of cointegration relationships is the same according to the price considered (one relation of cointegration). In the case of multiple vectors, there is no objective rule for choosing one over the other, except that the first vector is more strongly correlated with the stationary part of the process (Johansen and Juselius, 1990).

The long-term relationship can therefore take the following form:

*LFUTURES\_PRICE* = -1,0499 *LSPOT\_PRICE* 

(0,012) SD





#### Table 3: Johansen cointegration tests between *Spot* and *Futures* prices

H	H <sub>1</sub>	Statistic of the trace	Critical value 5%	$\mathbf{H}_{0}$	H <sub>1</sub>	Maximal eigenvalue	Critical value 5%
r=0	r=1	57.257***	15.494	r=0	r>1	55.222***	14.265
r=1	r=2	2.036	3.841	r≤1	r>2	2.036	3.841

Lag 1 and 2: represente the offset chosen for the causality test between "DSPOT\_PRICE" and "DFUTURES\_PRICE" according to the criteria of Akaik (AIC), Schwarz (SC), Likelihood Ratio (LR) and Hannan Quinn (HQ). The \*\*\*\*\*indices, indicate respectively the level of significance at the 10%, 5% and 1%

## *4.1.2. Consideration of exogenous shocks in the formation of Long term price series*

The purpose of this section is to study deeply the sudden volatility of the series, in order to have arguments on these changes. The breaks can be either the fact of a discrete change of the parameters, or the fact of a gradual evolution of these parameters.

The econometric literature (Bai and Perron, 1998) argues that any estimate that does not take into account the persistence of these changes consistently leads to biased results. It is therefore essential to test the presence and persistence of these shocks in order to be able to model them.

Unit root rupture tests were performed for two types of data, extreme and outliers. Detecting and eliminating these types of data can be of benefit to time series estimates. Financial series are generally likely to contain a number of extreme values associated with rise or fall.

In this work<sup>2</sup>, we distinguish two classes of outliers:

- Outliers additives (AO): Which affect only one observation of the series and not his future values.
- The inovational outliers (IO): Which affect the series temporarily with the same dynamics of an innovation.

Generally, AO and OI are considered as atypical points related to a shock.

The results show that the unit root tests applied to the *Spot* and *Futures* series reveal AO and IO breaks at the beginning of 2016 (January). Whether it is with trend and constant, or with constant and without trend, *Spot* and *Futures* prices hold the fall of their prices on the same date of January 2016 (Table 4).

These two financial chronicles are thus realizations of nonstationary random processes. Classical analyses reveal that they do not have stochastic trends (TS) expressed in terms of unit root. On the other hand, the performed differentiation (DS) operation mainly describes the short-term variations. These movements taken in the long term that describe the properties of these chronicles, are then ignored. The cointegration test with rupture on the relationship between the *Spot* and the *Futures* price is applied. Indeed, the cointegration tests on the relation between prices with a constant (Model [2]), and with a trend and a constant (Model [1]), shows that both models were estimated by OLS. After obtaining the residues, the estimate was made by the ADF test for each observation in the interval  $[0.15\eta; 0.85\eta]$ , where  $\eta$  is the size of the sample minus the number of lags used in the initial ADF test. Therefore, the estimate is made for the period from 02/02/2015 to 29/12/2017, where we obtain the minimum t statistic from the ADF test by testing the null hypothesis (Table 5).

The results show, that the null hypothesis is rejected for both Models [1] and [2]. The *Futures* and *Spot* prices series considered during the study period are then cointegrated. Therefore, we can accept the alternative hypothesis of changing the variance of the error terms around the long-term equilibrium. Finally, it is preferable not to keep the *Spot/Futures* price balance on the full sample, but rather to consider the short-term dynamics that mark the shifts in two sub-periods: Before and after 22/01/2016 compared to the whole sample considered.

# **4.2. Short-term Dynamics Analysis: Estimation of the VECM Model**

The purpose of this section is to know if the residual term is integrated or not (autocorrelation of residues). The VECM model is estimated using the Johnson cointegration test. We can then estimate the VECM model consisting of nonstationary series; because the goal is to know after using the cointegration test whether the model residual is integrated or not. This analysis is performed taking into account the results of the long-term analysis, and also the break dates considered from the cointegration analysis of the price series.

The results show that the estimates are relatively unsatisfactory by observing the coefficients of determination and the values of the Fisher statistic. The error correction coefficients are negative and significant for the entire period of the study. Which means that in the event of a short-term imbalance, the *Spot* price seems to return rather slowly towards its equilibrium path (Table 6).

#### Table 4: Stationarity test with rupture (Perron)

Tests (Outliers)	Le	vel	First di	fference
	Spot	Futures	Spot	Futures
Test innovation outlier				
M 1	0.976	0.974	0.006***	-0.113***
	(-3.51)	(-3.608)	(-27.364)	(-30.918)
Break date	02/07/2015	06/03/2015	22/01/2016	22/01/2016
M 2	0.988	0.987	0.009***	-0.11***
	(-2.089)	(-2.208)	(-27.323)	(-30.875)
Break date	02/07/2015	30/08/2017	22/01/2016	22/01/2016
Test additive outlier				
M 1	0.976	0.974	0.006***	-0.113***
	(-3.516)	(-3.613)	(-27.416)	(-30.977)
Break date	02/07/2015	30/06/2015	22/01/2016	22/01/2016
M 2	0.988	0.987	0.010***	-0.11***
	(-2.093)	(-2.212)	(-27.358)	(-30.915)
Break date	21/07/2015	30/08/2017	22/01/2016	22/01/2016

M 1: Model with trend and with constant. M 2: Model without trend and with constant. - The \* \*\* \*\*\*indices, indicate respectively the level of significance at the 10%, 5% and 1%

The speed of return to equilibrium for the *Spot* price was more marked in the first sub-period (the error correction coefficient has a greater weight [-0.373]), but the situation for *Futures* prices was different. The values of the parameter of the error correction term are significant only for the second half-period (after the break date), but his sign (positive) reflects the idea that during this period, the *Futures* price influences only these eigenvalues. In addition, this coefficient is significant and negative during the first half-year with respect to the cointegration relationship, which means that in the short term, *Future* price volatility have a negative influence on *Spot* prices.

In general, the signs of the error terms coefficients for the *Spot* and *Futures* prices indicate that, when the *Futures* price exceeds its equilibrium level, the both series should decrease to reach the long-term equilibrium. The persistence of breaks in the cointegration relationship may affect the short-term dynamics of the two series as the *Futures* price depends on the past values of the *Spot* Price. This confirms the results already obtained with the Granger causality tests and the cointegration relationship: In the short term, price dynamics are not determined solely by their own evolutions.

### 5. BEHAVIOR OF FUTURES AND SPOT PRICES

The purpose of this work was to find explanations for the observed changes in *Futures* and *Spot* price dynamics. The analysis was conducted on crude oil, during the period from 02/02/2015 to 29/12/2017. Two complementary techniques were used to capture these changes. The first, refers to the short-term analysis of the generating process without breaks. The second aims to analyse more closely the short and long-term changes related to the price structure, applying an approach based on tests of stationarity and causality with breaks.

The results confirm that the dynamic of *Futures* price is a measure of the expected evolution of the *Spot* price. In the long term, it

 Table 5: Gregory-Hansen cointegration tests between Spot

 and Futures prices

Cointegration	Beta	<b>Z-statistic</b>	Break date	Lags
Test				
M1				
Spot	-7.030 ***	-113.936***	22/01/2016	3
Futures	-7.006***	-113.228***	22/01/2016	3
M2				
Spot	-5.955***	-83.2110***	22/01/2016	4
Futures	-5.946***	-82.765***	22/01/2016	4

M1: Model with trend and with constant. M 2 : Model without trend and with constant. The \*\*\*\*\*indices, indicate respectively the level of significance at the 10%, 5% and 1%  $\,$ 

can be used to identify expectations of continued changes in *spot* prices. The perception of these expectations naturally follows the evolution of prices on the *Futures* market i.e., positive causality.

In addition, over the period studied, the violent volatility of *Futures* prices was inexplicable, and destabilize the *Spot* market. Previous analyses show that the functioning of these markets seems efficient. However, we must not maintain the balance between the *Spot* and the *Futures* price over the entire period (given the persistence of shocks), but rather take into account the short-term dynamics marked by the changes in regimes observed over two sub-periods different.

In the short term, the results conclude that the return of crude oil price of historical average has a slow pace. And this by examining how the slope of the *Futures* curve reacts to a change in the *Spot* price. When the price on the *Futures* market exceeds its equilibrium level, a reaction to the decline in prices on the physical market, to reach the long-term equilibrium, is taken momentarily. The persistence of breaks in this relationship may also affect the short-term dynamics of the two series together, since the *Futures* price depends on the past values of the *Spot* price.

According to the models of the normal backwardation and the storage theory, a violent change in the *Spot* price curve in the face of the price volatility in the *Futures* market implies that the latter return quickly to the average. The estimates confirm this observation. In addition, it should be noted that during the study period, *Futures* prices are often higher than the observed *Spot* price, which means that *Spot* prices are subject to a postponement situation.

After the shocks, an adjustment (return to the historical average) is made by relying on the expectations of the oil players (Cantango), but without structural strategies related to the oil production, because these expectations are taken in the very short term (daily).

#### **6. CONCLUSION**

Recently, the prices have declined significantly. This is precisely the case for *Spot* and *Futures* contracts, whose volatility seem very violent. Based on this motivation, this article was interested in examining in depth the relationship between *Futures* and *Spot* prices. Indeed, the nature of the relationship between prices is complex since each purchase or sale of crude necessarily involves storage.

Finally, the analyses elaborated in this article show that the links between these markets seem to be strict in terms of estimating

#### **Table 6: VECM estimation**

VECM	VECM Whole sample		Before 2	Before 22/01/2016		After 22/01/2016	
Model	Spot	Futures	Spot	Futures	Spot	Futures	
Residues	-0.184	0.061	-0.373	-0.028	-0.066	0.163	
	(0.05)	(0.05)	(0.10)	(0.11)	(0.06)	(0.06)	
	[-3.502]	[1.103]	[-3.669]	[-0.249]	[-1.034]	[2.547]	
R <sup>2</sup>	0.101	0.023	0.144	0.059	0.096	0.026	
F-stat	16.65	3.54	8.17	3.07	10.53	2.72	

The values in (), indicate the standard deviations, and the values in [] indicate the T-statistics

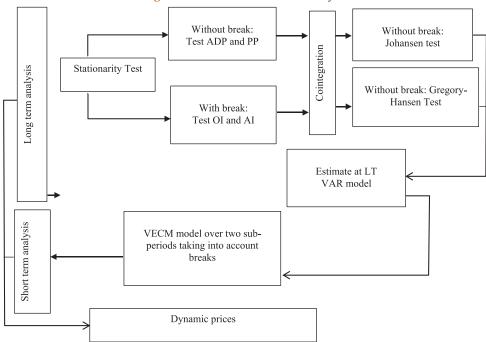


Figure 8: Overall structure of the analysis

long-term dynamics. The estimation of the short-term dynamics between the *Spot* and the *Futures* prices has shown that during the period, we have to distinguish between two regimes: The socalled "Stress" regime and the "return to the average." The first regime cannot reject the theory of the normal backwardation to explain the nature of the relationship in the short term. While the storage theory reasonably explains the return to the path of the equilibrium of oil prices.

It should be noted that this first empirical investigation, as to the determinants of price behavior on the oil markets, needs to be refined by other studies, particularly those dealing with the decomposition of price volatility, or to evaluate one of the explanatory models of the prices curves such as the Kalman filter (Figure 8).

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