

Optimal Willingness to Supply Wholesale Electricity under Asymmetric Linearized Marginal Costs

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ABSTRACT: This analysis derives the profit-maximizing willingness to supply functions for single-plant and multi-plant wholesale electricity suppliers that all incur linear marginal costs. The optimal strategy must result in linear residual demand functions in the absence of capacity constraints. This necessarily leads to a linear pricing rule structure that can be used by firm managers to construct their offer curves and to serve as a benchmark to evaluate firm profit-maximizing behavior. The procedure derives the cost functions and the residual demand curves for merged or multi-plant generators, and uses these to construct the individual generator plant offer curves for a multi-plant firm.

Keywords: Wholesale Electricity; Cost; Willingness to Supply; Linear Analysis; Multi-Plant; Asymmetric

JEL Classifications: D43; L11; L94

1. Introduction

The de-regulated wholesale electricity market is characterized by firms that submit piecewise continuous willingness to supply electricity offer curves for each hour, or half-hour, for the next day to the wholesale market operator. In the U.S., each generator plant can submit up to ten steps for each hour of the following day. Wolak (2010) notes that researchers in the wholesale electricity markets have an advantage in that the availability of bids and offers submitted by market participants can be used to recover the realized residual demand curves faced by each supplier. This alleviates the need to make assumptions about the functional form for demand and the competition structure when estimating producers' cost functions and testing the assumption of expected profit-maximizing behavior.

However, this paper presents an alternative approach. This analysis shows that it is beneficial for firms to specify a functional form for itself and its competition. Firms can utilize a benchmark rule for submitting profit-maximizing willingness to supply curves when all firms' asymmetric marginal cost curves can be approximated by continuous linear functions. This will occur when the firms' total cost functions are quadratic, and this analysis will prove that this necessarily leads to a linear residual demand curves and a linear willingness to supply curves. Since most wholesale electricity firms own multiple generation plants, this paper develops an effective method for employing its known cost parameters to construct offers curves that will maximize its expected overall profit.

Since the willingness to supply curves are increasing step functions, and since the residual demand functions are decreasing step functions, these functions can be linearized around the neighborhood of the equilibrium. Anderson and Xu (2005) note that most of the work in this area has assumed that the market supply and demand curves facing a generator can be approximated by continuous functions due to the large number of market participants that offer highly diversified prices. Holmberg et al., (2010) show that the step supply functions approaches a continuous step function as the number of steps increases.

It also commonly assumed, as in Genc and Reynolds (2011), that the market demand is perfectly inelastic up to a given price. Anderson and Hu (2008) mention that the only case of asymmetric willingness to supply curves that allows for an easy solution is when firms encounter linear marginal cost curves and face linear demand curves, but that there may also be nonlinear

solutions under these same conditions that may be analytically difficult to find. This analysis proves that under the assumptions of quadratic costs and linear marginal costs, there is only a linear solution rule for profit-maximization, and there are actually no other nonlinear solutions that exist. The simple linear-pricing rule can then be used to evaluate firms' profit maximization.

The multi-plant nature of the wholesale electricity also creates specific industry concerns for the cost structure, profit maximization, and government regulation. Green (1998) presents the Electricity Pool of England and Wales as one of the most important and controversial of the 1990s reforms in terms of pricing, costs, and regulation. Since the system software began using price bids instead of internal cost data, a System Marginal Price (SMP) was put into place to reflect the short-run marginal cost of electricity. Although Green (1998) states that the price must occasionally rise above the marginal cost for peaking capacity to cover fixed costs, this paper focuses on the fact that the imperfectly competitive structure of the electricity industry, which is based on price bids, will provide each generator plant with a profit-maximizing supply offer curve where price is always above marginal cost.

Since the many electricity markets, including the Pool of England and Wales, the PJM market in the U.S., and other electricity markets are generally characterized by a few large firms that each own several generator stations, the firms are not price takers. In order to restrain the price-cost margin and market power, regulators rely on both forward contract prices and the potential for market entry. Through a large value of forward contract obligations relative to current output, the firm's ability to raise market-clearing prices through its unilateral actions can be translated into a very small incentive to raise market-clearing prices (Kwoka and White, 2009). Additionally, if there are few entry barriers, incumbent firms have the incentive to price below the average cost of its rivals in order to avoid losing market share to other firms.

Green (2004) argues that although economists can calculate optimal prices for electricity transmission, they are rarely applied in practice. Burns et al. (2004) simulate a large number of N-plant multiple equilibria games, and then use data on prevailing price-cost margins in England and Wales to run econometric regressions in order to identify which games best explain market outcomes. They attempt to rigorously calculate electricity marginal costs using a sophisticated system dispatch model, in order to avoid the measurement bias that may arise in the marginal cost stack approach. Green (2004) uses a multi-nodal model with transmission constraints to solve for optimal uniform and nodal prices in England and Wales, and finds that nodal pricing can lead to higher welfare than uniform pricing.

2. Model Derivation

This section develops a bottom-up managerial approach to determine the optimal willingness to supply electricity that is based on the cost function of each individual electricity generation plant. During a given hour, let there be n generating firms in the regional wholesale electricity market, where each firm i produces Q_i MWh (megawatt hours) of electricity for that hour. Assume that each firm distributes its annual fixed cost across the number of hours in the year, so that for the given hour, each firm has fixed costs F_i , and faces a cost function that can be approximated by the quadratic cost function given by

$$C_i(Q_i) = F_i + c_i Q_i + d_i Q_i^2 \quad F_i, c_i, d_i > 0 \tag{1}$$

This leads to a the linear marginal cost function

$$MC_i(Q_i) = C_i'(Q_i) = c_i + 2 d_i Q_i \tag{2}$$

Let P denote the price in \$/MWh, and assume that each firm's residual demand is linear, or else can be approximated by a linear demand curve given by

$$Q_i = A_i - B_i P_i \quad A_i, B_i > 0 \tag{3}$$

The inverse residual demand curve will have the form

$$P_i = a_i + b_i Q_i \quad a_i = \frac{A_i}{B_i} \quad b_i = \frac{1}{B_i} \tag{4}$$

Using equation (4), each firm's revenue function will be:

$$R_i = P_i Q_i = a_i Q_i + b_i Q_i^2 \quad (5)$$

The firms' marginal revenue functions will be given by

$$MR_i(Q_i) = R_i'(Q_i) = a_i - 2 b_i Q_i = (a_i - b_i Q_i) - b_i Q_i \quad (6)$$

Substituting the inverse residual demand function from equation (3) into the parentheses of the expression (6), the marginal revenue curve can be rewritten as

$$MR_i(Q_i) = P_i - b_i Q_i \quad \text{or} \quad P_i = MR_i + b_i Q_i \quad (7)$$

Equation (7) provides the functional form necessary for constructing firms' willingness to supply electricity curves based on their cost function parameters. If we assume that each firm maximizes profit, then for each firm, $MR_i = MC_i$. Substitute the MC_i expression in the right hand side of equation (2) into the right hand side of equation (7) for MR_i . This yields the profit-maximizing willingness to supply for any firm facing a linear residual demand and a quadratic cost function. This offer curve is expressed as

$$P_i = MC_i + b_i Q_i = c_i + (2 d_i + b_i) Q_i \quad (8)$$

If we define the parameter $h_i = 2 d_i + b_i$, then each firm thus necessarily submits a linear willingness to supply curve of the general form

$$P_i = c_i + h_i Q_i \text{ or } Q_i = G_i + H_i P; \quad G_i < 0, c_i = \frac{-G_i}{H_i} > 0, h_i = \frac{1}{H_i} > 0 \quad (9)$$

Let Q^T denote the aggregate industry willingness to supply. The aggregate market supply curve for electricity will be the sum of all of the individual willingness to supply curves for n firms, which will only be defined for the range where $Q^T > 0$. When graphed, this market supply curve will be the vertical sum of the individual supply curves, whereas the total market inverse supply curve will be the horizontal sum of the individual inverse supply curves. The expression for the aggregate supply curve is

$$Q^T = \sum_{i=1}^n G_i + P \sum_{i=1}^n H_i \quad ; \text{ defined only for } Q^T > 0 \quad (10)$$

After rearranging equation (10) and solving for P , the inverse aggregate supply curve can thus be written as

$$P = - \frac{\sum_{i=1}^n G_i}{\sum_{i=1}^n H_i} + \frac{1}{\sum_{i=1}^n H_i} Q^T \quad (11)$$

The aggregate supply curve will be linear above the maximum value of c^{\max} , which comes from the firm that has largest value of that c_i parameter. This is consistent with the Electricity Pool of England and Wales in that the Pool based its System Marginal Price (SMP) on the bid of the most expensive station in normal operation (Green, 1998); but, the current approach only utilizes the maximum cost function intercept of all of the plants in operation. Thus, assuming that the given wholesale market is not trivially small, the aggregate willingness to supply curve will be linear across the entire relevant range of price and output.

Let Q^D denote the total quantity of electricity demanded in a given hour. The intercept coefficient a_j and the slope coefficient b_j for the residual inverse demand curve for firm j can then be expressed as a function of the other firms' supply function coefficients and the total market quantity demanded for that hour. Let c_j^{\max} denote the maximum value of c_i , excluding firm j , and let Q_j^T denote the aggregate quantity of electricity offered for supply by all firms, excluding firm j . The intercept for firm j is found by determining the minimum price that would induce the collective of all

firms, other than firm j , to supply the entire market demand, thus leaving no quantity of electricity demanded from firm j . This can be determined by using the following equation.

$$Q^D = Q_j^T = \sum_{i \neq j} G_i + P \sum_{i \neq j} H_i \tag{12}$$

Solving equation (12) for P , and then letting $a_j = P$, yields the price-intercept of the residual inverse demand curve for firm j .

$$a_j = \frac{Q^D - \sum_{i \neq j} G_i}{\sum_{i \neq j} H_i} \tag{13}$$

Once the vertical intercept (price-intercept) of the firm j residual inverse demand curve has been determined, only one additional point on the linear region of the inverse demand curve needs to be found in order to compute the slope parameter. The slope of the inverse residual demand curve can be found by

$$b_j = \frac{a_j - c_j^{\max}}{Q^D - Q_j^T(c_j^{\max})} \tag{14}$$

In the denominator, $Q_j^T(c_j^{\max})$ is the total market supply offered when $P = c_j^{\max}$, by all firms other than firm j . The residual inverse demand curve facing firm j will have a constant slope for all prices between c_j^{\max} and a_j . Thus, this provides the second point that was required in order to compute the slope. All prices below c_j^{\max} , which means quantities above $Q^D - Q_j^T(c_j^{\max})$, the inverse residual demand curve will be kinked and steeper. However, there is no practical reason that a firm in the market for wholesale electricity would ever operate in this small region, where the market price is very close to zero.

The profit-maximizing firm will set marginal revenue equal to marginal cost. For the entire smooth linear region of the inverse demand curve for prices above c_j^{\max} , this means that the profit-maximizing price and quantity for firm j can be found by setting equation (6) equal to equation (2) and then rearranging to yield:

$$Q_j = \frac{a_j - c_j}{2d_j + b_j} \quad P_j = a_j - b_j \left[\frac{a_j - c_j}{2d_j + b_j} \right] \tag{15}$$

The firm's own price elasticity of residual demand, ε_j , and its Lerner Index (or price-cost margin), L_j , are given by

$$\varepsilon_j = - \frac{1}{b_j} \frac{P_j}{Q_j} \quad L_j = \frac{P_j - c_j}{P_j} \tag{16}$$

These measures are of particular importance when addressing both profit and regulatory concerns. The price elasticity in electricity tends to be smaller than in other markets (Kwoka and White, 2009). The inverse of the elasticity, which measures the ability of the firm to raise the market-clearing price by reducing its willingness to supply electricity, is thus large compared to other markets. As a result, very large market-clearing price increases will result from a supplier's withholding a small percentage of its output (Kwoka and White, 2009). Typically, the greater is the share of total generation capacity owned by a supplier, the smaller is the elasticity of the residual demand curve it faces, and the greater is its incentive to raise prices through its unilateral actions. Newberry (2005) shows that privatization in the wholesale electricity market in Britain initially led to increasing price-cost margins and excessive firm entry.

3. Application Example

For ease of exposition, consider a wholesale electricity market that consists of only four asymmetric electric generator firms, where firm 3 must determine a willingness to supply offer curve to submit to the wholesale market operator. Suppose that the total demand for electricity during the

given hour is perfectly inelastic and given by $Q^D = 2,400\text{MWh}$. Also suppose that firms 1, 2, and 4 have submitted linear willingness supply curves of the form $Q_i = G_i + H_i P$, and inverse willingness to supply curves of the form $P_i = c_i + h_i Q_i$, as was specified in equation (9). These offer curves are specified in expression (17).

$$\begin{aligned} Q_1 &= -5.3453 + 4.85939257593P & \text{or} & & P_1 &= 1.1 + 0.205787 Q_1 & (17) \\ Q_2 &= -7.1364 + 3.568181818P & \text{or} & & P_2 &= 2 + 0.280255 Q_2 \\ Q_4 &= -59.6552 + 12.173913044P & \text{or} & & P_4 &= 4.9 + 0.082143 Q_4 \end{aligned}$$

This gives the following values for the competitors' parameters and variables:

$c_1 = 1.1$, $c_2 = 2$, $c_4 = 4.9$, $c^{\max} = 4.9$, $G_1 = -5.3453$, $G_2 = -7.1364$, $G_4 = -59.6552$, $H_1 = 4.85939257593$, $H_2 = 3.568181818$, $H_4 = 12.173913044$, $h_1 = 0.205787$, $h_2 = 0.280255$, $h_4 = 0.082143$.

In this case, $j = 3$, and the equations (12), (13), and (14) can be used to find the intercept and slope of the residual inverse demand curve. The intercept is

$$a_3 = \frac{2400 - (-5.3453 - 7.1364 - 59.6552)}{4.85939257593 + 3.568181818 + 12.173913044} = \$120 \quad (18)$$

The aggregate willingness supply function, exclusive of firm j , is

$$Q_j^T = -72.1338694 + 20.6014874376P \quad (19)$$

This produces $Q_j^T(c_j^{\max}) = Q_j^T(4.9) = 28.81342182$, so that the second point (at the lower boundary of the smooth linear section) of the firm 3 residual inverse demand curve is $Q_3(4.9) = 2,400 - 28.81342182 = 2,371.18657818$. The slope parameter is

$$b_3 = \frac{120 - 4.9}{2,400 - 28.81342182} = 0.048540183 \quad (20)$$

Thus, the firm 3 marginal revenue curve is

$$MR_3(Q_3) = a_3 - 2b_3 Q_3 = 120 - 0.097080366 Q_3 \quad (21)$$

Assume that firm 3 knows that its total cost function is given by

$$C_3(Q_3) = 3.55 + 3Q_3 + 0.046118258 Q_3^2 \quad (22)$$

After taking the derivative of the total cost function, its marginal cost function is

$$MC_3(Q_3) = 3 + 0.092236516 Q_3 \quad (23)$$

Substituting equations (20) and (23) into equations (8) and (9) yields the profit-maximizing inverse willingness to supply offer curve for firm 3:

$$P_3 = c_3 + h_3 Q_3 = 3 + 0.1407767 Q_3 \quad (24)$$

Rearranging (24) gives the willingness supply offer curve for firm 3.

$$Q_3 = -21.3103 + 7.103448276P \quad (25)$$

Thus, we obtain the parameters $c_3 = 3$, $G_3 = -21.3103$, $H_3 = 7.10344827$, and $h_3 = 0.1407767$. Combining the offer curve for firm 3 in equation (25) with the all of the other firm's offer curves in equation (17) provides the aggregate willingness to supply curve for the entire market:

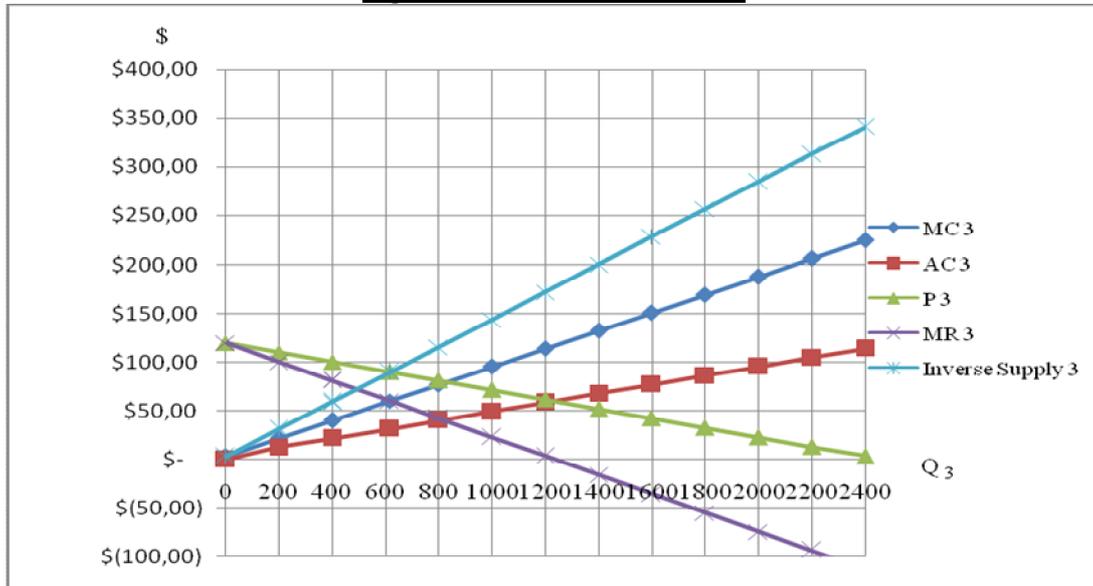
$$Q^T = -93.4442 + 27.704935713P \quad (26)$$

This can be rearranged to find the market inverse aggregate willingness to supply curve:

$$P = 3.3728 + 0.0360947Q^T \tag{27}$$

Figure 1 shows the individual willingness to supply electricity curves for each of the firms as expressed in equations (17) and (25), along with the aggregate supply function as expressed in equation (27).

Figure 1. Functions for Firm 3



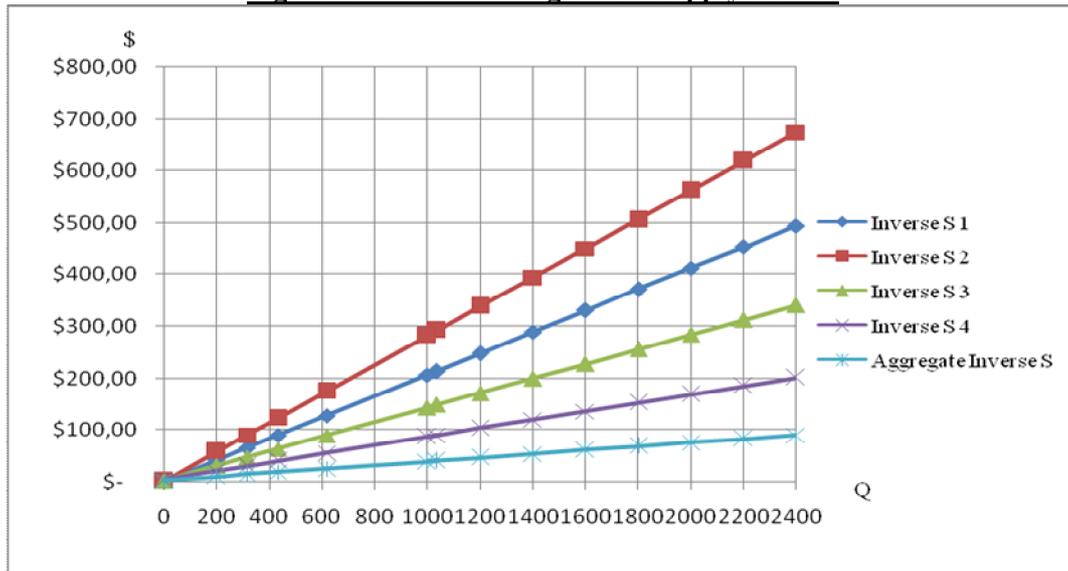
The market price of wholesale electricity for the given hour will be determined by the wholesale market operator by setting market demand equal to the market willingness to supply so that $Q^T = Q^D$. Thus, setting the right hand side of equation (26) equal to 2,400 MWh, which is the total market demand, yields a market clearing equilibrium price of $P = \$90/\text{MWh}$. Substituting this price of $\$90/\text{MWh}$ back into each of the firm’s willingness to supply curves gives the quantity supplied (in MWh) by each of the four firms as $Q = 432$, $Q = 314$, $Q = 618$, and $Q = 1,036$.

The values of all of the variables for firm 3 can be found by setting its marginal revenue function in equation (21) equal to its marginal cost function in equation (23), or by using equation (15). Setting $MR_3(Q_3) = MC_3(Q_3)$ yields the following values: $Q_3 = 618$ MWh, $P_3 = \$90/\text{MWh}$, $MR_3 = MC_3 = \$60/\text{MWh}$, $C_3 = \$19,471.22$, and $R_3 = 55,620$. The values for profit (π), average cost (AC), and average profit ($A\pi$) will be $\pi_3 = \$36,148.78$, $AC_3 = \$31.51/\text{MWh}$, and $A\pi_3 = \$58.49$. Using equation (16) yields the own-price elasticity of demand for firm 3 of $\epsilon_3 = 2.9126$, and a Lerner Index value of $L_3 = 0.333$. Figure 2 shows the market faced by firm 3.

4. Multi-Plant Firms, Mergers and Acquisitions

This section explores the situation where one of the firms is a multi-plant firm that owns m electricity generation plants. This would also be identical to a situation where there is a merger between two or more previously single plant firms. Denote the multi-plant firm as firm M , and let $\{M\}$ denote the set of the plant numbers of all plants that are included in firm M . Referring to the previous example in the above section, suppose that firm 3 and firm 2 merge, or that firm 3 acquires firm 2, thereby creating a new multi-plant firm M , so that $\{M\} = \{2, 3\}$. In this case, each individual generator plant will still submit its own individual willingness to supply curve, but the firm M will determine these offer curves by using only one total residual demand that faces the new multi-plant firm. The residual demand curve, offer curves, quantities, prices, costs, and profits can be determined using the method employed in this section.

Figure 2. Inverse Willingness to Supply Curves



Assume that plant 2 management knows that its total cost function and marginal cost functions are given by

$$C_2(Q_2) = 4.2 + 2 Q_2 + 0.119412093 Q_2^2 ; MC_2(Q_2) = 2 + 0.238824186 Q_2 \quad (28)$$

Other than the fixed cost of $F_2 = 4.2$, this cost function can be derived by first finding the inverse residual demand for plant 2 using equations (12), (13), and (14), in the same manner that was used for plant 3 above. This provides intercept and slope values, respectively, of $a_2 = 103.01$, and $b_2 = 0.041430592$. We know from the willingness supply curve in equation (17) that $c_2 = 2$. Equation (8) allows for recovery of the value of $d_2 = 0.119412093$, since $(2 d_2 + b_2) = 0.280255$, which can be solved for d_2 by using the value of b_2 obtained from the slope of the plant 2 inverse residual demand curve. After the merger, the new firm M will have full information about the cost function associated with both of the individual generating plants, alleviating the need to recover the marginal cost function from the plants' individual offer curves.

Let Q_M denote the total output of the merged multi-plant firm. In the above example, this consists of total combined output of plant 2 and plant 3. Let Q_M^T denote the aggregate quantity of electricity offered for supply by all firms, excluding firm M . The single residual demand curve for the multi-plant firm M can be found rewriting equations (12), (13), and (14) as follows, where all of the plants owned by firm M are omitted from the summations.

$$Q^D = Q_M^T = \sum_{i \notin \{M\}} G_i + P \sum_{i \notin \{M\}} H_i \quad (29)$$

Solve equation (29) for P , and then let $a_M = P$. This produces the price-intercept of the residual inverse demand curve for the multi-plant firm M .

$$a_M = \frac{Q^D - \sum_{i \notin \{M\}} G_i}{\sum_{i \notin \{M\}} H_i} \quad (30)$$

The slope of the inverse residual demand curve can be found by

$$b_M = \frac{a_M - c_M^{\max}}{Q^D - Q_M^T(c_M^{\max})} \quad (31)$$

For this scenario where $\{M\} = \{2, 3\}$, the resulting intercept for firm M is

$$a_M = \frac{2400 - (-5.3453 - 59.6552)}{4.85939257593 + 12.173913044} = \$144.72 \quad (32)$$

The aggregate willingness supply function, exclusive of the plants 2 and 3 owned by firm M , is $Q_M^T = -64.99750575 + 17.03330562P$ (33)

Note that plant 4 still has the maximum value of c_i . Thus, $Q_M^T(c_M^{\max}) = Q_M^T(4.9) = 18.46569365$, so that the second point on the smooth linear section of the firm M residual inverse demand curve is $Q_M(4.9) = 2,400 - 18.46569365 = 2,381.53430635$. The slope parameter can be computed as

$$b_M = \frac{144.72 - 4.9}{2,400 - 18.46569365} = 0.058708509 \quad (34)$$

Thus, firm M has a marginal revenue curve given by

$$MR_M(Q_M) = a_M - 2b_M Q_M = 144.72 - 0.117417018 Q_M \quad (35)$$

The next step is to find the marginal cost function and the total cost function for the multi-plant firm M . This can be achieved by adding the individual component plants' marginal cost functions as follows. First, rearrange equation (2) so that quantity of electricity produced by each plant owned by firm M is expressed as a function of its marginal cost, so that

$$Q_i = \frac{-c_i}{d_i} + \frac{1}{d_i} MC_i \quad \forall i \in \{M\} \quad (36)$$

Then, set $MC_i = MC_M \forall i \in \{M\}$, and then sum up each of the m equations in expression (36).

This leads to the equation

$$\sum_{i \in \{M\}} Q_i = - \sum_{i \in \{M\}} \frac{c_i}{d_i} + MC_M \sum_{i \in \{M\}} \frac{1}{d_i} \quad (37)$$

Solving (37) for MC_M yields

$$MC_M = c_M + 2d_M Q_M = \left[\sum_{i \in \{M\}} \left(\frac{c_i}{d_i} \right) \right] \left(\frac{\prod_{i \in \{M\}} d_i}{\sum_{i \in \{M\}} d_i} \right) + \left(\frac{\prod_{i \in \{M\}} d_i}{\sum_{i \in \{M\}} d_i} \right) \sum_{i \in \{M\}} Q_i \quad (38)$$

For the example above where $\{M\} = \{2, 3\}$, equation (38) becomes

$$MC_M = \left(\frac{c_2}{d_2} + \frac{c_3}{d_3} \right) \left(\frac{d_2 d_3}{d_2 + d_3} \right) + \left(\frac{d_2 d_3}{d_2 + d_3} \right) (Q_2 + Q_3) \quad (39)$$

Using the numerical values from the example, the marginal cost function for the combined firm becomes

$$MC_M = c_M + 2d_M Q_M = 2.721390925 + 0.066538586 Q_M \quad (40)$$

The marginal cost function for the multi-plant firm will always have a smaller value for its slope when compared to the slope of any of the individual component plants, so that $d_M < d_i$ for all $i \in \{M\}$. The marginal cost function intercept value of c_M is a weighted average of the component plant intercept parameters, and it will lie between the values of c_i from each of the individual plants operated by firm M . In the current numerical example, $c_2 < c_M < c_3$.

The total cost function for the multi-plant firm M can be recovered by deriving the integral of the marginal cost function, and then assuming that its fixed cost is the sum the individual fixed costs associated with the individual component plants. Although the fixed costs may be reduced for a multi-plant firm after a merger due to synergies, this procedure will provide an upper-bound for the maximum level of fixed costs. In the example, $F_M = F_2 + F_3 = 3.55 + 4.2 = 7.75$. Thus, the total cost function for firm M can be written as

$$C_M = F_M + c_M Q_M + d_M Q_M^2 = 7.75 + 2.721390925 Q_M + 0.033269293 Q_M^2 \quad (41)$$

The profit maximizing level of output for the multi-plant firm occurs where the marginal revenue for the merged firm equals the marginal cost of production at each of the individual plants, so that $MR_M(Q_M) = MC_{i \forall i \in \{M\}}$. This procedure requires substituting $Q_M = \sum_{i \in \{M\}} Q_i$ into the

marginal revenue function. Then, setting this same marginal revenue function equal to the marginal cost function for each individual plant yields a set of m equations of the form

$$MR_M(Q_M) = a_M - 2b_M \sum_{i \in \{M\}} Q_i = c_i + 2d_i Q_i = MC_i(Q_i) \forall i \in \{M\} \quad (42)$$

This can be written as an m -equation matrix system $AQ = B$, and then solved to find the profit-maximizing value of output for each plant. For the 2-plant example given above, the matrix system will be

$$\begin{bmatrix} 2b_M + 2d_2 & 2b_M \\ 2b_M & 2b_M + 2d_3 \end{bmatrix} \begin{bmatrix} Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} a_M - c_2 \\ a_M - c_3 \end{bmatrix} \quad (43)$$

Solving the system requires $Q = A^{-1}B$, which yields

$$\begin{bmatrix} Q_2 \\ Q_3 \end{bmatrix} = \frac{1}{2[b_M(d_2 + d_3) + d_2 d_3]} \begin{bmatrix} (b_M + d_3)(a_M - c_2) - b_M(a_M - c_3) \\ (b_M + d_2)(a_M - c_3) - b_M(a_M - c_2) \end{bmatrix} \quad (44)$$

Using the parameters from the above example results in the profit-maximizing values $Q_2 = 218.08$, $Q_3 = 553.82$, and $Q_M = 771.90$. Substituting the total quantity into the firm M inverse residual demand curve $P_M = a_M + b_M Q_M$ yields an equilibrium market price of \$99.40/MWh, which is an increase of \$9.40 above the pre-merger, or single-plant market price. Substituting these values into the firm M marginal revenue, marginal cost, total cost, total revenue, and profit functions, yields the following values: $MR_M = MC_M = \$54.08/\text{MWh}$, $C_M = \$21,931.11$, and $R_M = 76,726.15$. The profit (π), average cost (AC), and average profit ($A\pi$) will be $\pi_M = \$54,795.05$, $AC_M = \$28.41/\text{MWh}$, and $A\pi_M = \$70.99$. Using equation (16) yields the own-price elasticity of demand for firm M of $\varepsilon_M = 2.5755$, and a Lerner Index (price-cost margin) value of $L_M = 0.456$.

Using the following procedure, the price and plant quantity levels derived above can be used to construct the new willingness to supply offer curves for each of the individual plants. For each plant, the intercept of the inverse willingness to supply curve must be set equal to value of the intercept of the marginal cost function for the merged firm, so that equation (9) becomes

$$P_i = c_M + h_i Q_i \text{ or } Q_i = G_i + H_i P ; G_i < 0, c_M = \frac{-G_i}{H_i} > 0, h_i = \frac{1}{H_i} > 0 \quad (45)$$

Next, substitute the profit-maximizing quantity and price into the equation (45) for each plant owned by firm M . Solving for the parameters yields

$$P^* = c_M + h_i Q_i^*; h_i = \frac{-(c_M + P^*)}{Q_i^*} \quad G_i = \frac{-c_M}{h_i} \quad H_i = \frac{1}{h_i} \quad (46)$$

For the example above, the plant 2 and plant 3 parameters in equation (46) are $P^* = \$99.40$, $Q_2^* = 218.08$, $Q_3^* = 553.82$, $c_M = 2.72139$, $h_2 = 0.44332$, $h_3 = 0.17457$, $G_2 = -6.1387$, $G_3 = -15.5895$, $H_2 = 2.25571924$, and $H_3 = 5.72849796$.

The willingness to supply and inverse willingness to supply offer curve equations, respectively, for firm 2 and firm 3 become

$$P_2 = c_M + h_2 Q_2 = 2.72139 + 0.44332 Q_2; \quad Q_2 = -6.1387 + 2.25571924 P \quad (47)$$

$$P_3 = c_M + h_3 Q_3 = 2.72139 + 0.17457 Q_3; \quad Q_3 = -15.5895 + 5.72849796 P \quad (48)$$

Thus, we have shown that, given the aggregate willingness to supply function, the individual plant marginal cost functions can be used to derive the optimal willingness to supply offer curves for each plant in the multi-plant firm.

Since the c_M term in each of the individual plant offer curve equations is a weighted average, it is only at the current value of P^* that the individual willingness to supply offer curves will provide the profit-maximizing quantities that exactly equate $MR_M = MC_M = MC_i$ for all $i \in \{M\}$. However, the above procedure provides the closest linear approximation to the profit-maximizing quantities, and it will be close throughout the entire range of viable market prices.

This can be demonstrated with a simulated sensitivity analysis. Consider the example above, where equations (23), (28), (40), (47), and (48) can be used to compute the profit-maximizing quantities and the marginal costs for plant 2, plant 3, and for the overall merged firm. When the market price is $P = \$99.40/\text{MWh}$ as it was previously, then $MC_2 = MC_3 = MC_M = \$54.08$. If the market price is as low as $P = \$10/\text{MWh}$, then $Q_2 = 16.42$, $Q_3 = 41.69$, $Q_M = 58.11$, $MC_2 = \$5.92$, $MC_3 = \$6.85$, and $MC_M = \$6.59$. If the market price is as high as $P = \$216/\text{MWh}$, then $Q_2 = 481.10$, $Q_3 = 1,221.76$, $Q_M = 1,702.86$, $MC_2 = \$116.90$, $MC_3 = \$115.69$, and $MC_M = \$116.03$. Thus, these values for the marginal cost for each of the individual plants will be close approximations to the overall marginal costs of the combined merged firm. This allows each plant to submit these offer curves derived above to the wholesale market operator so that the multi-plant firm M will maximize its profit.

5. Conclusion

This analysis has developed a pragmatic approach for determining the optimal willingness to supply curves for single-plant and multi-plant firms when firms have quadratic total cost functions and linear marginal cost functions. The method above can be utilized by multiple-plant firms with any finite number of plants. The firm only needs to know its own cost structure and then recover the aggregate willingness to supply function for the market in order to formulate its offer curves. It can use information from competitors' previous offer curves in order estimate the cost structure of other firms. All that is needed is to approximate maximum value of the intercept of the aggregate supply curve, which is determined by the plant with the maximum intercept in its marginal cost function, and the slope of the aggregate supply curve. This information allows for the computation of the residual demand curves and the willingness to supply curves. Every firm in the market can follow this Cournot-type strategy, without the need to impose the restrictions of an arbitrarily assigned economic market structure, or a strategy game that is cooperative or non-cooperative. This is extremely useful since the plants do not need to coordinate with each other, nor with competitors, in order to formulate the optimal strategy.

The above approach does have some limitations. Since the actual firm and aggregate willingness to supply curves have discrete steps, such as in the PJM market in the northeastern U.S., this approach does not provide the exact piecewise continuous offer curve functions that would be submitted by each individual plant. However, a linear comparison line can be fitted for each stepped-function offer curve. Since each plant submits a different, multi-step curve for each hour (or half-hour in some European markets), firms have more than enough data to recover an approximate set of linear comparison curves. As long as the cost functions and number of competitors are relatively stable, each plant can formulate a stepped offer curve strategy that is consistent with the approximated linearized profit-maximization procedure explored here.

This procedure also applies when there are fixed-price forward contracts and when there are transmission constraints. If forward contracts are in place, where then net residual demand would be computed for each firm before computing the offer curves; once computed, the analysis would proceed for each plant as formulated above. If transmission constraints exist, then the residual demand curves would be kinked and steeper above the price where the constraint becomes binding, but the same procedure described above still applies with the new demand curves.

Another limitation of this approach is that it does not consider nonlinear cost functions. A further extension would involve finding the underlying offer curves when the plants' marginal cost

curves are nonlinear. This is especially relevant for cases when marginal costs eventually begin to increase at an increasing rate as the output of electricity gets closer to the maximum plant capacity. The above analysis still yields insight in this case since the nonlinear marginal cost functions can be linearized around a neighborhood of the relevant quantities, and then the method here can be applied. An even more precise approach to the nonlinear problem would be to consider specific cases, such as the cubic cost function, which results in a quadratic marginal cost function. This case is left for exploration in future papers.

Finally, it should be noted that this general approach can be used in other markets, and is not restricted to wholesale electricity. Even though the wholesale electricity market has some unique features and is uniquely characterized by an explicit wholesale market operator that determines the prices and quantities, this type of willingness to supply strategy may also characterize other oligopolistic markets where such offer curves are implicitly utilized by the industry firms. If the firm can analyze the various market equilibrium quantity and price bundles over time for itself and its competitors, then it can potentially approximate the industry aggregate willingness to supply curve, thus allowing it to derive its own optimal offer curves based on its own cost function. Utilizing this type of approach within other markets thus presents another potential avenue for future research.

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